Money Formulas

Compound interest formulas

The **compound interest formula** applies if you make a deposit into an account that bears interest and you leave it there untouched for some period of time. You don't make any additional deposits or withdrawals

If P is the principal, r is the annual percentage rate, n is the number of compoundings per year, t is the number of years, and A is the amount in the account after t years, then

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}.$$

Accounts typically compound yearly ($n = 1$), quarterly (every three months, $n = 4$), monthly ($n = 12$), daily ($n = 365$), or continuously (you have to use a different formula – see below).

In a way, you can think of **continuous compounding** as compounding infinitely often during the year. (Of course, each time they pay the interest, they pay only an infinitesimal part.) Because we can't plug $n = \infty$ into the compound interest formula, we need a different formula for continuous compounding:

Again, P is the principal, r is the annual percentage rate, and t is the number of years. For continuous compounding, the amount in the account after t years is

$$A = Pe^{rt}.$$

e is a number, approximately equal to 2.71828. It appears in mathematics about as often as π, which is why it gets its own symbol. Your calculator knows e; don't punch in the numbers by hand. Use the e^x key to calculate these.

The compound interest formulas apply when we make one deposit into an interest-bearing account and leave it alone for a period of time. But this is a fairly unrealistic setting – we're not likely to have that big lump of money to start with. In most real-life savings accounts, the saver continues to make deposits and withdrawals over a period of time. This is a more complicated situation, and the compound interest formula doesn't apply directly.

If the deposits and withdrawals are irregular (as in most real-life situations), modeling the situation can be quite complicated. For example, in most credit card accounts, they figure your balance every day and compute your finance charge on your average daily balance. This method is often used in savings accounts also.

One common scenario is simple enough to model with a formula. An *annuity* is a savings plan where the saver makes equal deposits at regular intervals. For example, you might save $50 per month for a vacation next spring, or put away $250 per month until you retire.

The Savings Plan Formulas

Use for accumulating money, balances that increase.

If *PMT* is the regular payment (deposit) amount, *r* is the annual percentage rate expressed as a decimal, *m* is the number of payment periods per year, and *t* is the number of years, the **Savings Plan Formula** can tell you *A*, the accumulated balance or amount in the account:

$$A = PMT \times \frac{\left[\left(1+\frac{r}{m}\right)^{(mt)} - 1\right]}{\left(\frac{r}{m}\right)}$$

Sometimes you need to solve for the payment. In this case, the payment formula may be more useful:

$$PMT = \frac{A \times \frac{r}{m}}{\left[\left(1+\frac{r}{m}\right)^{(mt)} - 1\right]}$$

Example: Suppose we want to save for a vacation 2 years away. If we put $50 per month into an account earning 4% per year, how much money will we have at the end of 2 years?

Solution: This is an annuity, because we're making regular, equal deposits, and we're accumulating money. We want the amount in the account, so we want to use the Savings Plan formula:

$$A = PMT \times \frac{\left[\left(1+\frac{r}{m}\right)^{(mt)} - 1\right]}{\left(\frac{r}{m}\right)} = 50 \times \frac{\left[\left(1+\frac{.04}{12}\right)^{12\times 2} - 1\right]}{\left(\frac{.04}{12}\right)} = 1247.14.$$

Does this answer make sense? Even if we weren't earning any interest at all, we still would have made 24 deposits of $50, so we'd have at least $1200 in the account. This is bigger, but not a lot bigger, reflecting the fact that 4% isn't a very high interest rate and 2 years isn't a very long time.

Example: Suppose we want to have a million dollars at the end of 25 years. How much do we need to deposit each month into an account paying 8.5% per year to have a million dollars at the end of 25 years?

Solution: This is an annuity, because we're making regular, equal deposits, and we're accumulating money. We want the payment, so the payment formula will be more convenient:

$$PMT = \frac{A \times \frac{r}{m}}{\left[\left(1+\frac{r}{m}\right)^{(mt)} - 1\right]} = \frac{1000000 \times \frac{.085}{12}}{\left[\left(1+\frac{.085}{12}\right)^{12\times 25} - 1\right]} = 968.94.$$

We'd have to put \$968.94 into this account each and every month for 25 years. Does this answer make sense? Our deposits put about \$12000 per year × 25 years = \$300,000 into the account. The rest of the million comes from the interest!

Loan Formulas

Use for dwindling money, balances that decrease.

The other common scenario that's easy to model is where we start with a sum of money in the account and make regular withdrawals (or payments) that reduce the balance. The situation that comes to mind first is a loan – you borrow \$18000 to buy a car, and pay it off with regular monthly payments over 5 years. But this could also be a different kind of situation – putting a lump sum in an account so your grandson could withdraw exactly \$200 per month during his four years in college.

The initial amount *P* (the principal) can be found with the formula:

$$P = PMT\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\left(\frac{r}{m}\right)}\right]$$

The loan payment can be found with the Loan Payment Formula:

$$PMT = \frac{P \times \frac{r}{m}}{\left[1-\left(1+\frac{r}{m}\right)^{-mt}\right]}$$

Example: You borrow $18000 at 12.9% per year. What will your monthly payment be if you want to pay off the loan in 5 years?

Solution: The balance here is dwindling with our regular payments, so we'll use the loan payment formula:

$$PMT = \frac{P \times \frac{r}{m}}{\left[1 - \left(1 + \frac{r}{m}\right)^{-mt}\right]} = \frac{18000 \times \frac{.129}{12}}{\left[1 - \left(1 + \frac{.129}{12}\right)^{-12 \times 5}\right]} = 408.63.$$

In order to pay off the loan, we'll need to pay $408.63 per month. Does this make sense? If we pay about $4800 per year for 5 years, that's about $24000 – that would pay back the $18000 plus the interest.

Example: You want to deposit a lump sum into an account that pays 6% annual interest so that your grandson can withdraw $200 per month for the four years he's in college (and then the money will be gone). How much do you need to deposit?

Solution: Even though this isn't a loan, the balance is dwindling because of regular payments, so we use the same formulas. In this case, we want to know the Principal:

$$P = PMT\left[\frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\left(\frac{r}{m}\right)}\right] = 200\left[\frac{1 - \left(1 + \frac{.06}{12}\right)^{-12 \times 4}}{\left(\frac{.06}{12}\right)}\right] = 8516.06.$$

You'll need to deposit $8516.06 into the account. Does this answer make sense? If you deposited the full 48 × $200 = $9600 into the account, it would earn interest and you'd still have money left at the end of the four years. The interest means you can deposit somewhat less than $9600.

How do you know which formula to use?

If the balance is changing because of regular payments, deposits, or withdrawals, you know you can use either the savings plan or the loan formulas. If the balance is increasing, use the savings plan formulas. If the balance is decreasing, use the loan formulas.