Winning Strategy for a TV Game

A TV game that was popular in some European countries is as follow. A car is behind one of three doors. A player chooses one door. The player wins the car if it turns out to be behind the selected door. After the player chooses, the TV host will open another door which does not have the car because the host knows where the car is. After the host opens that door, the player is allowed to switch to choose the third door or stick to the original choice.

Question: Is it better to switch to the third door?

Assume for notational simplicity that the player has chosen door B and the host has opened door A. Then, since by now we know that door A does not have the car, switching to choose door C will win if and only if door B does not have the car, denoted by $\overline{B}$; that is,

$$P\{\text{Winning by switching}\} = P\{C|\overline{A}, \overline{B}\}P\{\overline{B}\} = P\{\overline{B}\} = 1 - P\{B\} = 2/3$$

learly, switching and not switching are mutually exclusive because they cannot oth win. Thus, we have

$$P\{\text{Winning by not switching}\} \leq 1 - P\{\text{Winning by switching}\} = 1/3$$

n fact, the probability of winning by choosing door B in the first place is $1/3$. y not switching, the probability of winning stays unchanged because the new nformation that door A does not have the car is not utilized since it would be e same if instead door C was opened by the host. Thus,

$$P\{\text{Winning by not switching}\} = P\{B\} = 1/3$$

onsequently, the chance of winning is doubled by switching! This answer would hard to come by without a good understanding of probability concepts.

The above analysis can be extended to the general n-door problem as follows. ssume for simplicity that the original choice of the player was door B, door A as opened by the host, and door C is chosen if the player changes his or her oice. Then, switching will win if and only if door B does not have the car d door C turns out to have the car among the $n-2$ remaining doors; that is,

$$P\{\text{Winning by switching}\} = P\{C|\overline{A}, \overline{B}\}P\{\overline{B}\} = \frac{1 - P\{B\}}{n-2} = \frac{1}{n-2}\frac{n-1}{n}$$

hich is greater than $P\{\text{Winning by not switching}\} = P\{C\} = \frac{1}{n}$.