Basic Algebra of Sets

	Algebra of sets	*Algebra of numbers*
	Union $\cup$	sum "+"
	Intersection $\cap$	product "·"
1	$A\cup B = B\cup A$	$a+b=b+a$
2	$A\cap B = B\cap A$	$a\cdot b = b\cdot a$
3	$A\cup(B\cup C) = A\cup B\cup C$	$a+(b+c)=a+b+c$
4	$A\cap(B\cap C) = A\cap B\cap C$	$a\cdot(b\cdot c) = a\cdot b\cdot c$
5	$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$	$a\cdot(b+c) = a\cdot b + a\cdot c$
6	$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$	see below

Since $A\cap A = A$, $(A\cap B)\subset A$, $(A\cap C)\subset A$, and

$$(a+b)(a+c) = a\cdot a + a\cdot b + a\cdot c + b\cdot c$$

Line 6 in the table above follows from

$$(A\cup B)\cap(A\cup C) = \underbrace{(A\cap A)\cup(A\cap B)\cup(A\cap C)}_{=A}\cup(B\cap C) = A\cup(B\cap C)$$

This illustrates that set algebra has its own rules.

De Morgan's laws:

$$\overline{A\cup B} = \overline{A}\cap\overline{B} \tag{2.1}$$

$$\overline{A\cap B} = \overline{A}\cup\overline{B} \tag{2.2}$$

Similarly,

$$\overline{A\cup B\cup C} = \overline{A\cup D} = \overline{A}\cap\overline{D} = \overline{A}\cap\overline{B\cup C} = \overline{A}\cap(\overline{B}\cap\overline{C}) = \overline{A}\cap\overline{B}\cap\overline{C}$$

$$\overline{A_1\cup\cdots\cup A_n} = \overline{A_1}\cap\cdots\cap\overline{A_n}$$

$$\overline{A_1\cap\cdots\cap A_n} = \overline{A_1}\cup\cdots\cup\overline{A_n}$$

$$\overline{(A_1\cup A_2)\cap(A_3\cup A_4)} = (\overline{A_1}\cap\overline{A_2})\cup(\overline{A_3}\cap\overline{A_4})$$

Rules: (1) interchange $\cup$ and $\cap$; (2) interchange $(*)$ and $\overline{(*)}$. However, care should be taken when dealing with multiple nests, as demonstrated below.

Example

$$\overline{\underbrace{(\overline{A}\cap B)}_{D}\cup\overline{C}} = \overline{\overline{D}\cup\overline{C}} = D\cap C = (\overline{A}\cap B)\cap C = \overline{A\cup\overline{B}}\cap C \tag{2.3}$$