These problems will be collected, and graded for credit.

5. A system is composed of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

  (a) How many outcomes are in the sample space of this experiment?

  (b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

8. Suppose that A and B are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that

  (a) either A or B occurs;

  (b) A occurs but B does not;

  (c) both A and B occur?

11. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.

  (a) What percentage of males smoke neither cigars nor cigarettes?

  (b) What percentage smoke cigars but not cigarettes?

12. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. These classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

  (a) If a student is chosen randomly, what is the probability that he or she is not in any of these classes?

  (b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?

  (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

23. A pair of fair dice are rolled. What is the probability that the second die lands on a higher value than does the first?

27. An urn contains 3 red and 7 black balls. Players A and B withdraw balls from the urn consecutively until a red ball is selected. Find the probability that A selects the red ball. (A draws the first ball, then B, and so on. There is no replacement of the balls drawn.)

28. An urn contains 5 red, 6 blue, and 8 green balls. If a set of 3 balls is randomly selected, what is the probability that each of the balls will be (a) of the same color; (b) of different colors? Repeat under the assumption that whenever a ball is selected, its color is noted and it is then replaced in the urn before the next selection. This is known as *sampling with replacement.*

47. If there are 12 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?