Prove the following relations.

1. $EF \subset E \subset E \cup F$.
2. If $E \subset F$, then $F^c \subset E^c$.
3. $F = FE \cup FE^c$, and $E \cup F = E \cup E^c F$.
4. $\left(\bigcup_1^\infty E_i\right) F = \bigcup_1^\infty E_i F$, and $\left(\bigcap_1^\infty E_i\right) \cup F = \bigcap_1^\infty (E_i \cup F)$.
5. For any sequence of events $E_1, E_2, \ldots$, define a new sequence $F_1, F_2, \ldots$ of disjoint events (that is, events such that $F_i F_j = \emptyset$ whenever $i \neq j$) such that for all $n \geq 1$,

$$\bigcup_1^n F_i = \bigcup_1^n E_i$$

7. Find the simplest expression for the following events:
   (a) $(E \cup F)(E \cup F^c)$;
   (b) $(E \cup F)(E^c \cup F)(E \cup F^c)$;
   (c) $(E \cup F)(F \cup G)$.

9. Suppose that an experiment is performed n times. For any event E of the sample space, let $n(E)$ denote the number of times that event E occurs, and define $f(E) = n(E)/n$. Show that $f(\cdot)$ satisfies Axioms 1, 2, and 3.

10. Prove that $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c FG) - P(EF^c G) - P(EFG^c) - 2P(EFG)$.

11. If $P(E) = .9$ and $P(F) = .8$, show that $P(EF) \geq .7$. In general, prove Bonferroni's inequality, namely,

$$P(EF) \geq P(E) + P(F) - 1$$

12. Show that the probability that exactly one of the events E or F occurs equals $P(E) + P(F) - 2P(EF)$.

13. Prove that $P(EF^c) = P(E) - P(EF)$.