

# **Perturbation Method applied to Pipe Flow of Ferrofluid in an Oscillating Magnetic Field**

**By:  
Yusyani Setyono**

A ferrofluid can respond to a magnetic field. It is frequently made from small, nano-sized particles of magnetite, suspended in an oil with a surfactant added to keep the particles from agglomerating. When such a fluid flows through a tube with an axial magnetic field applied, oscillating in time, the pressure drop increases from that expected for a Newtonian fluid. This has been measured and studied.<sup>1</sup> In the analysis, though, a time-averaged torque is used in order to calculate a time-averaged velocity. The goals of this project are to derive perturbation equations for the time-averaged solution and a small, time-varying solution, to solve it in Comsol, and to compare the results to determine if the time-averaged equations are accurate. There are two perturbation methods done in this project.

### **Perturbation Method I**

In perturbation method I, all the variables for  $u$ ,  $\omega$ ,  $M$ , and Torque need to be written as a steady value plus a time varying value. The format is as the following.

$$u = u_0 + u^1$$

$$M = M_0 + M^1$$

$$\omega = \omega_0 + \omega^1$$

$$Torque = Torque_0 + Torque^1$$

The following are the original equations before changing the format of the variables.

The first two equations are steady state:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r(u + \xi) \frac{\partial u_0}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega_0 r) - \left( \frac{\partial p}{\partial z} \right)_{\cos \tan t} = 0 \quad (1)$$

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<sup>1</sup> Schumacher, K. R., Sellien, I., Knoke, G. S., Cader, T., Finlayson, B.A., “Experiment and simulation of laminar and turbulent ferrofluid pipe flow in an oscillating magnetic field,” Phys. Rev. E **67** 026308 (2003).

$$2\xi \left( -\frac{\partial u_0}{\partial r} - 2\omega_0 \right) + torque_0 = 0 \quad (2)$$

The last four equations are transient:

$$\rho \frac{\partial u'}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u'}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega' r) \quad (3)$$

$$2\xi \left( -\frac{\partial u'}{\partial r} - 2\omega' \right) + torque' = 0 \quad (4)$$

$$\frac{\partial M_R'}{\partial t} - \omega_0 M_z' + \frac{M_R'}{\tau} = \frac{\chi_0}{\tau} \left[ H_0 \sin \Omega t - M_R' \right] \quad (5)$$

$$\frac{\partial M_z'}{\partial t} + \omega_0 M_z' + \frac{M_z'}{\tau} = \frac{\chi_0}{\tau} H_0 \sin \Omega t \quad (6)$$

### In COMSOL:

We have to solve as follows:

Solve the steady state equations, equation (1) and (2). To do this, under Solver Parameters choose stationary. Under Solver Manager, choose Solve For tab, and highlight just the equations (1) and (2). Then say solve.

$$1. \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u_0}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega_0 r) - \left( \frac{\partial p}{\partial z} \right)_{\text{constant}} = 0$$

$$D = \text{miu} + \text{zeta}$$

$$R = \text{dp} + 2 * \text{zeta} * (\text{om}/r + \text{diff}(\text{om}, r))$$

$$\text{BC: at } r = 0, \text{ symmetry; at } r = 0.5, u_0 = 0$$

$$2. 2\xi \left( -\frac{\partial u_0}{\partial r} - 2\omega_0 \right) + torque_0 = 0$$

$$D = 0$$

$$R = 2 * \text{zeta} * (-ur - 2 * \text{om}) + \text{torque}$$

BC: at  $r = 0$ ,  $\omega_0 = 0$  (symmetry); at  $r = 0.5$  no boundary condition

Then solve the transient ones only. Under Solver Parameters choose transient. Under Solver Manager, choose Current Solution for variables not solved for, and under the Solver For tab highlight just the last four equations. Then say solve.

$$3. \rho \frac{\partial u'}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u'}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega' r)$$

\_ts = rho

$$D = \text{miu} + \text{zeta}$$

$$R = (2 * \text{zeta} * (\text{omp}/r) + \text{diff}(\text{omp} * r))$$

BC: At  $r = 0$ , Insulation/ symmetry; at  $r = 0.5$ ,  $u' = 0$

$$4. 2\xi \left( -\frac{\partial u'}{\partial r} - 2\omega' \right) + \text{torque}' = 0$$

$$2 * \text{zeta} * ((-\text{diff}(u', r)) - (2 * \text{omp})) + \text{torque}'$$

BC: at  $r = 0$ ,  $\omega' = 0$  (symmetry); at  $r = 0.5$  no boundary condition

$$5. \frac{\partial M'_R}{\partial t} - \omega_0 M'_z + \frac{M'_R}{\tau} = \frac{\chi_0}{\tau} \left[ -M'_R \right]$$

$$(\text{om} + \text{omp}) * \text{mzp} - (\text{mrp}/\tau) + (\chi_0/\tau) * (h_0 * \sin(\text{rot} * t) - \text{mrp})$$

BC:  $M'_R(0) = 0$  (symmetry); at  $r = 0.5$ , no applied flux

$$6. \frac{\partial M'_z}{\partial t} + \omega_0 M'_z + \frac{M'_z}{\tau} = \frac{\chi_0}{\tau}$$

$$-(\text{om} + \text{omp}) * \text{mrp} - (\text{mzp}/\tau) + (\chi_0/\tau) * h_0 * \sin(\text{rot} * t)$$

BC: at  $r = 0$ , Axial symmetry; at  $r = 0.5$ , no applied flux

**Constants (under options) (for case Q = 342.7 ml/min)**

Symbol	Name	Expression	Value	Description
$\zeta$	zeta	1.93E-03	1.93E-03	vortex viscosity - Table II*
$\mu$	miu	3.85E-03	3.85E-03	viscosity - Table II*
$\tau$	tau	6.10E-05	6.10E-05	time constant, varies with case - Table IV
$\chi_0$	chi0	0.0043	0.0043	susceptibility, varies with case -Table III
$\Omega$	rot	$60 \cdot 2 \cdot \pi$	376.8	rotation rate, varies with case
$H_0$	h0	1264	1264	applied magnetic field, varies with case
$\mu$	miu0	$4 \cdot \pi \cdot E-7$	0.000001256	magnetic permeability of free space
dp	dp	3	3	
$\rho$	rho	1187.4	1187.4	density

**Results**

The solutions are shown in Figures 1-8. The perturbation velocity is very small compared with the time-averaged velocity. The gradual reduction in the value of the perturbation velocity with time is something that needs examination. The perturbation spin is likewise very small compared with the time-averaged spin. The components of the magnetic field have a period that is twice that of the velocity. The time-varying torque is also large, at least at the centerline ( $r = 0$ ).

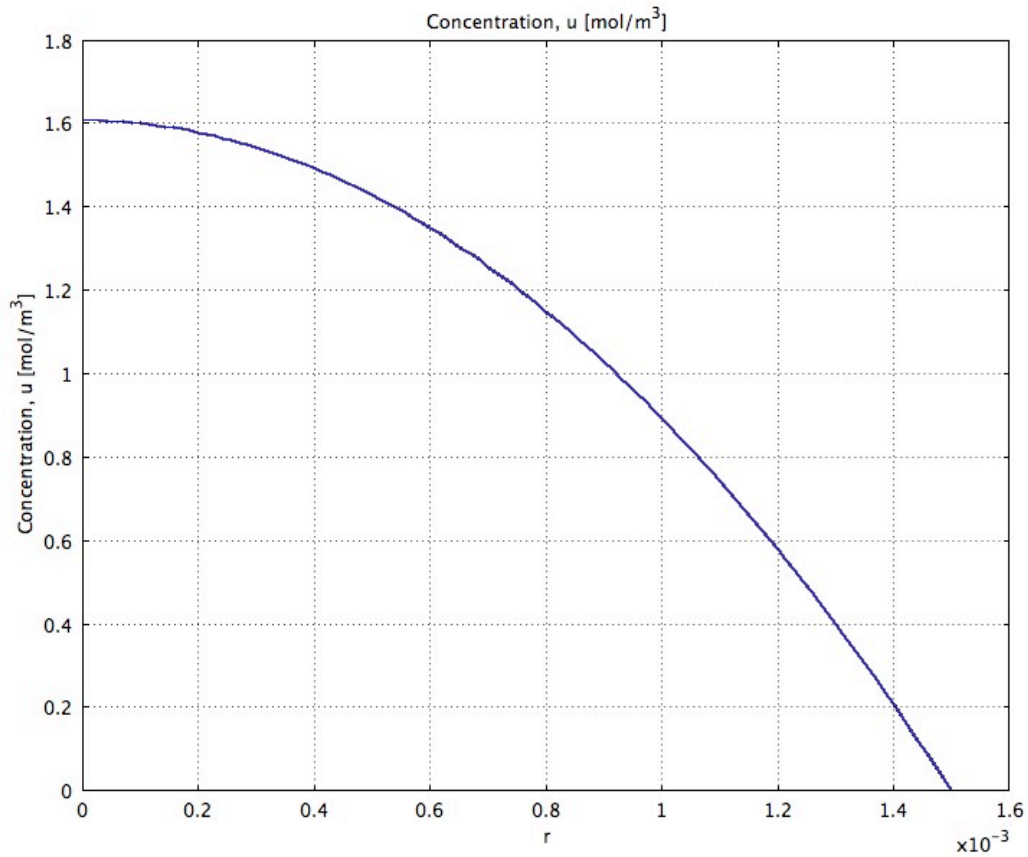


Figure 1: u (axial velocity, m/s)

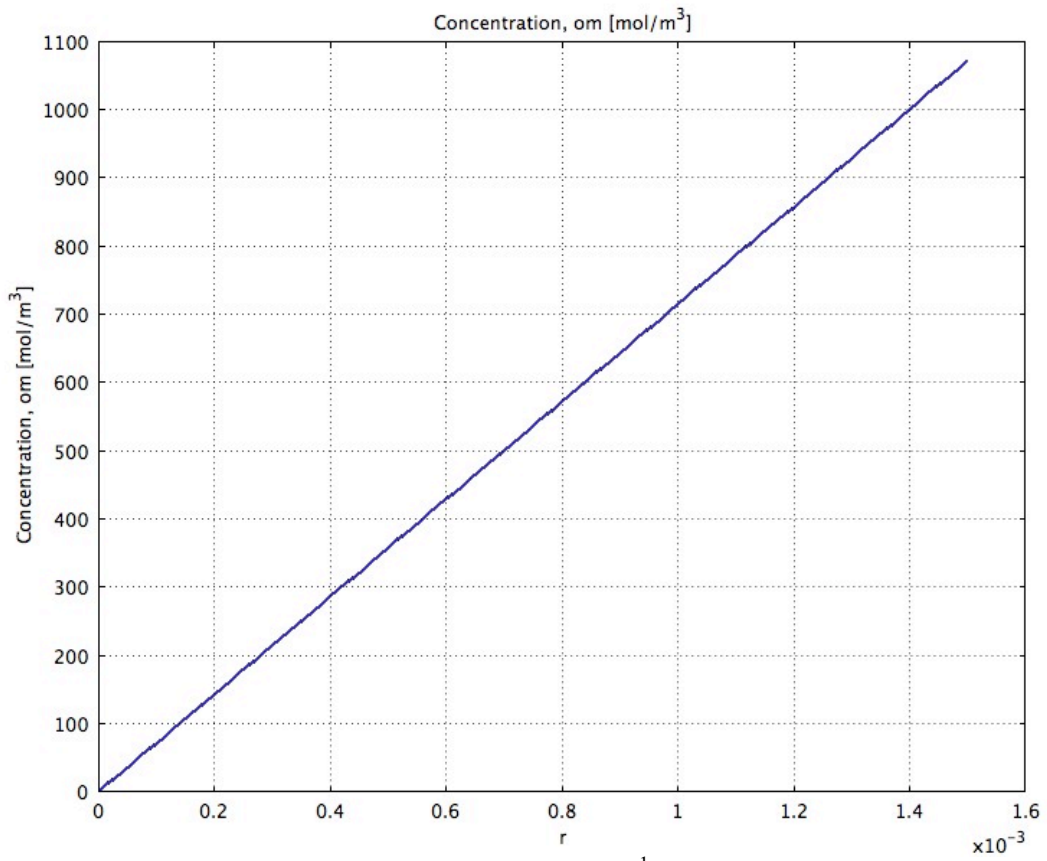


Figure 2:  $\omega$  (spin, s<sup>-1</sup>)

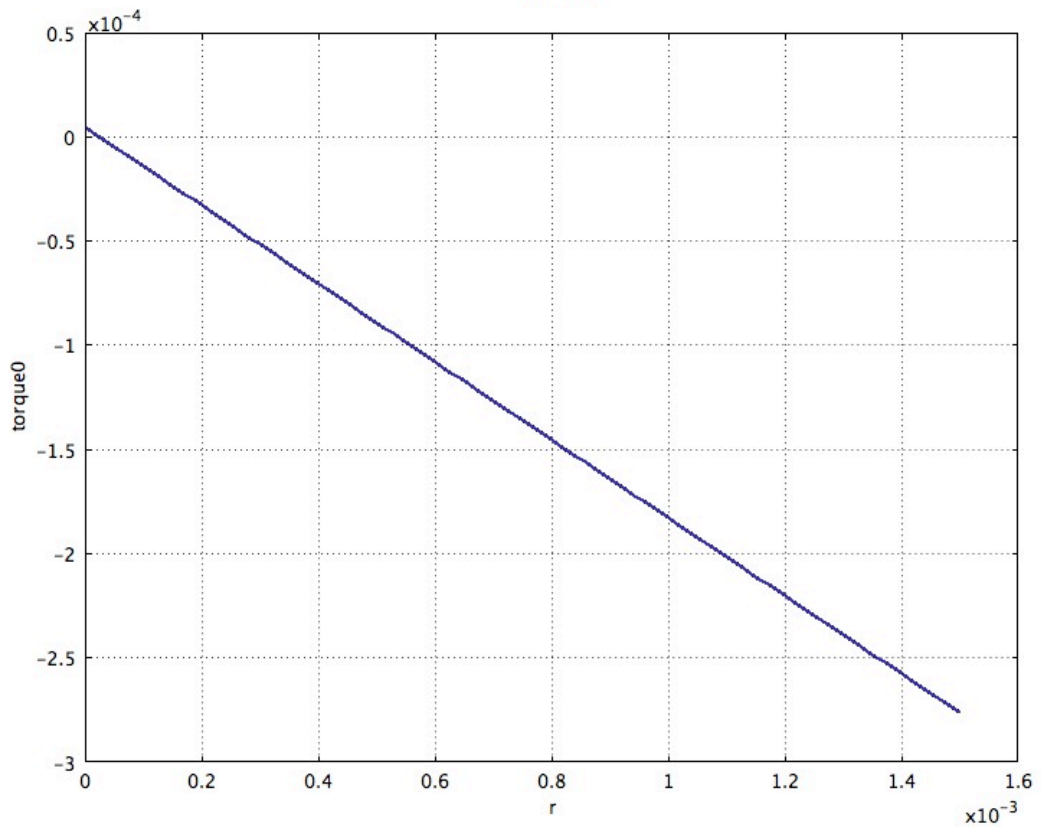


Figure 3: torque0

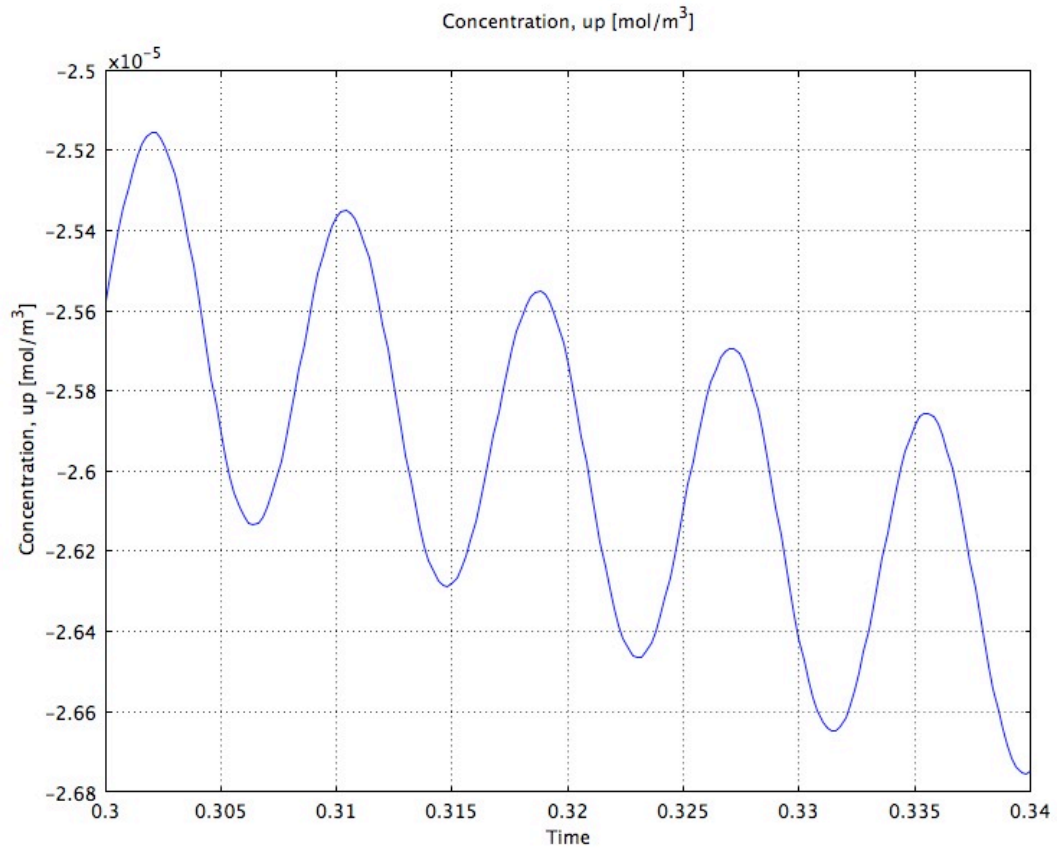


Figure 4. Perturbation velocity (m/s)

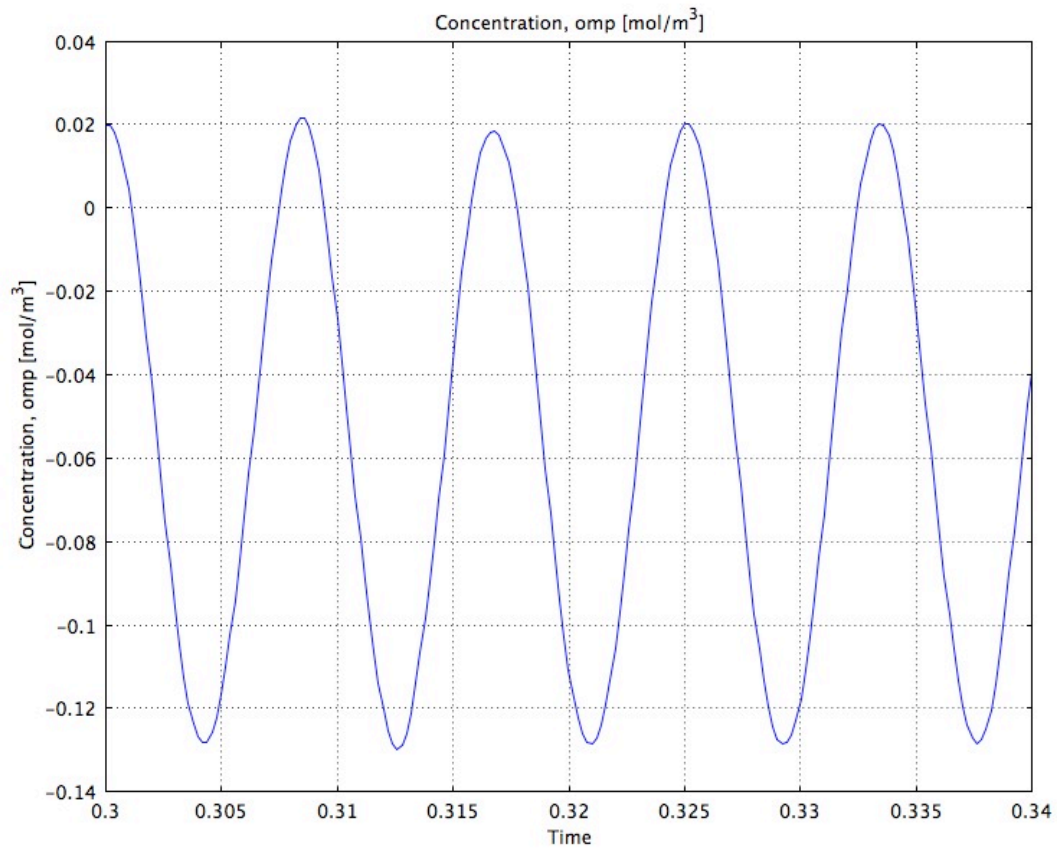


Figure 5: Perturbation spin (s<sup>-1</sup>)

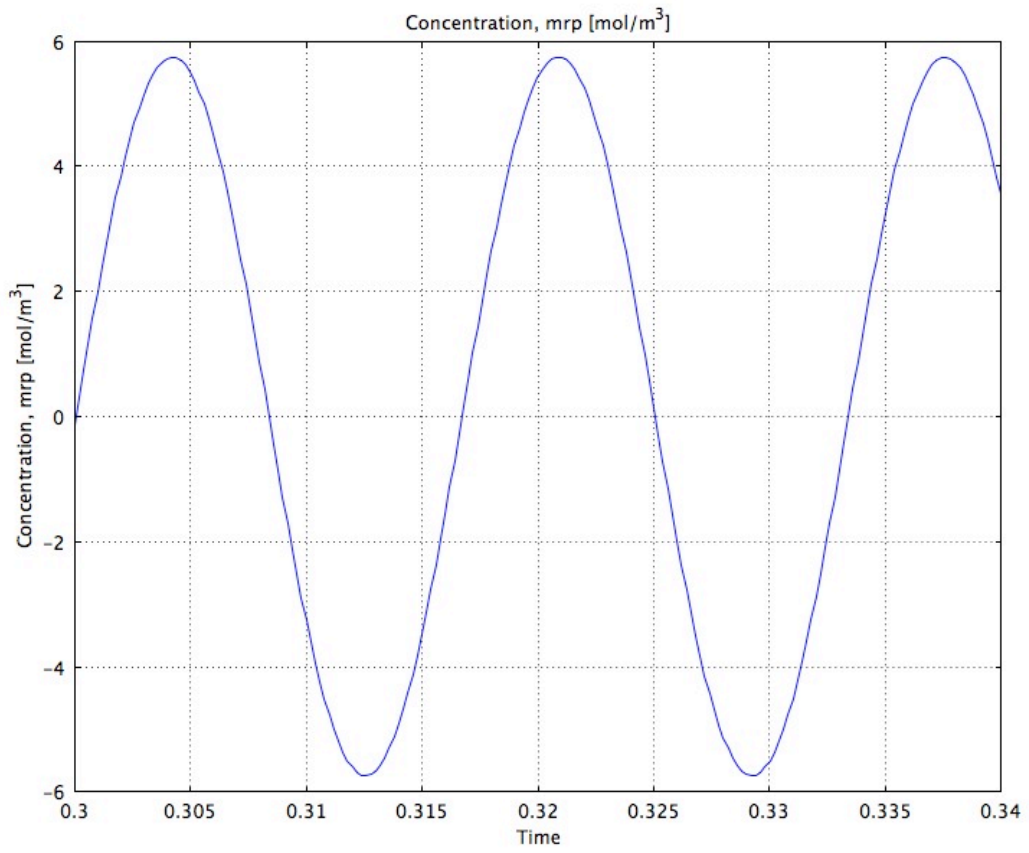


Figure 6.  $M_R$

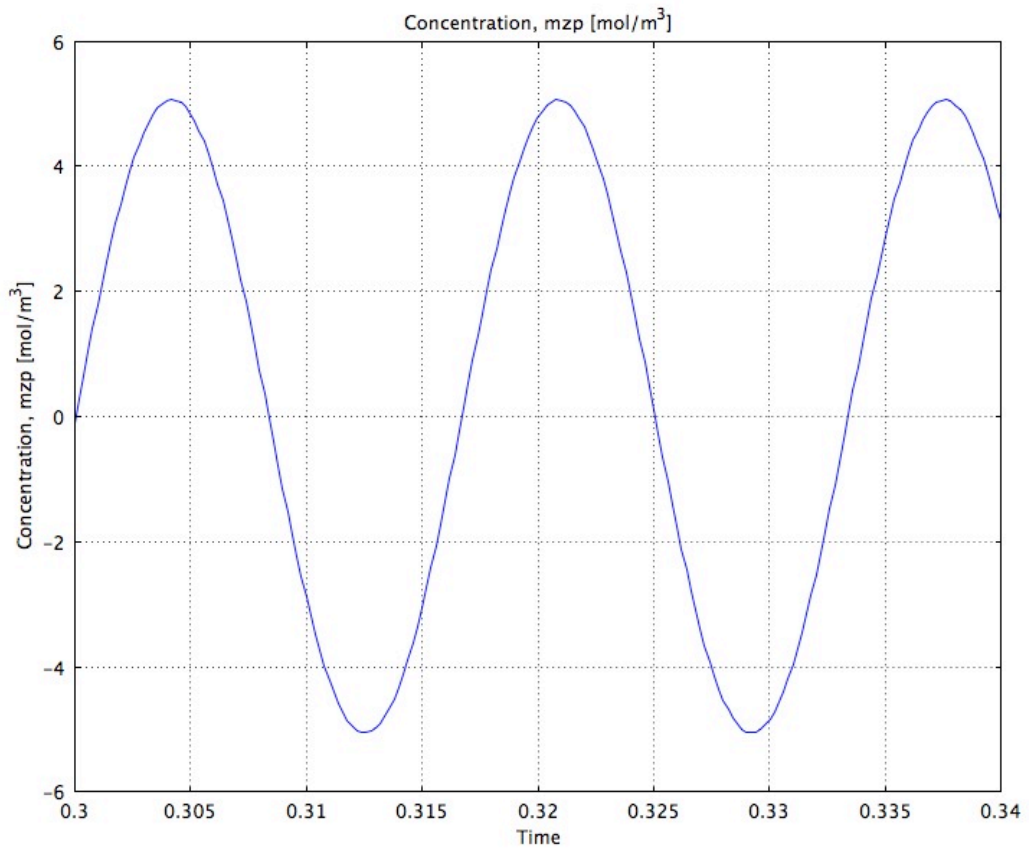


Figure 7.  $M_Z$



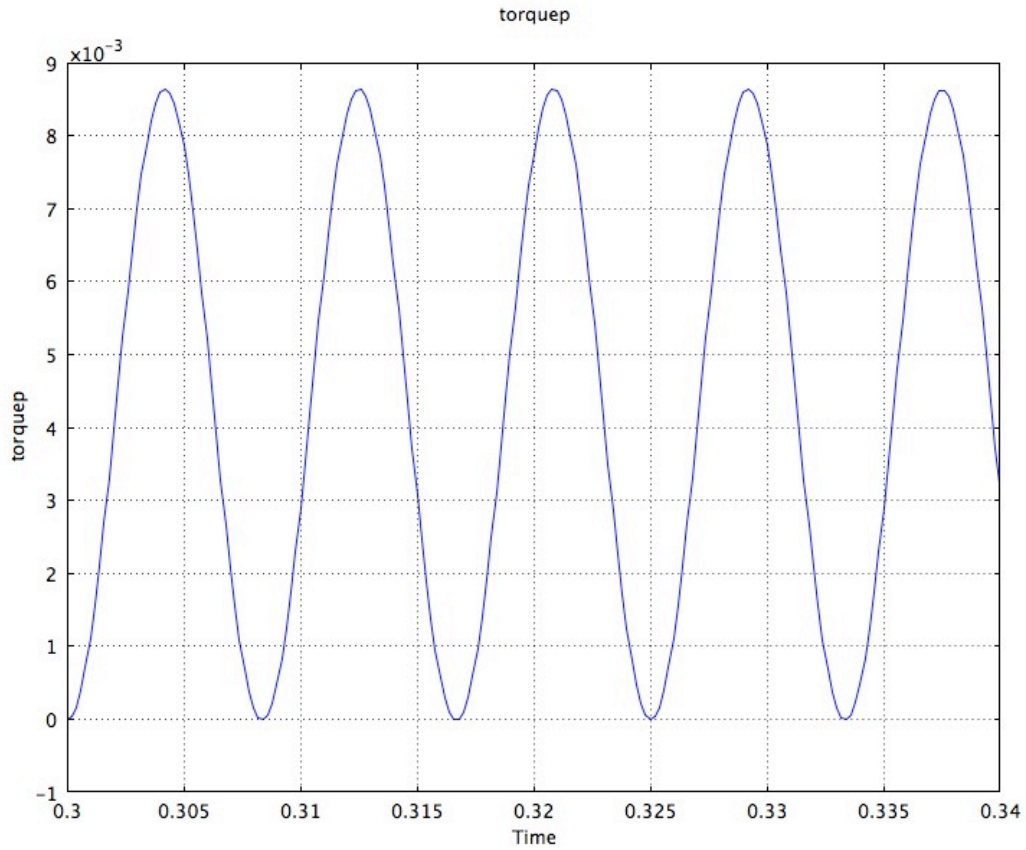


Figure 8. Perturbation in torque

## Conclusion

In conclusion, the perturbed quantities for velocity and spin can be neglected compared with the time-averaged velocity and spin..

## Appendix - Perturbation Method II

Expanding all the variables in a perturbation series in terms of  $\Omega\tau$  is the method for second perturbation.

These equations done under the following assumptions:

$$M_R = \Omega\tau M'$$

$$M_z = M_0$$

(From Krelchov, A.P., Shilomis, M, I, Kamiyama, S. , Phys Fluids 17 033105 (2005))

Eq. (7)

$$\rho \frac{\partial u}{\partial t} = - \left( \frac{\partial p}{\partial z} \right)_{\text{cons tan } t} + \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega r)$$

$$\rho \frac{\partial u_0}{\partial t} + \Omega \tau \rho \frac{\partial u^1}{\partial t} = - \left( \frac{\partial p}{\partial z} \right)_{\text{cons tan } t} + \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u_0}{\partial r} + \Omega \tau r(\mu + \xi) \frac{\partial u^1}{\partial r} \right] + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega_0 r) + 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\Omega$$

$$(\Omega \tau)^0 : \quad \rho \frac{\partial u_0}{\partial t}, \left( \frac{\partial p}{\partial z} \right)_{\text{cons tan } t}, \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u_0}{\partial r} \right], 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega_0 r)$$

$$(\Omega \tau)^1 : \quad \rho \frac{\partial u^1}{\partial t}, \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\mu + \xi) \frac{\partial u^1}{\partial r} \right], 2\xi \frac{1}{r} \frac{\partial}{\partial r} (\omega^1 r)$$

Eq.(8)

$$2\xi \left( -\frac{\partial u_0}{\partial r} - 2\omega \right) + \text{torque} = 0$$

$$2\xi \left[ \left( -\frac{\partial u_0}{\partial r} - \Omega \tau \frac{\partial u^1}{\partial r} \right) - 2(\omega_0 - \Omega \tau \omega^1) \right] + \text{torque} = 0$$

$$(\Omega \tau)^0 : \quad 2\xi \frac{\partial u_0}{\partial r}, 2\omega_0, 2\xi \left( -\frac{\partial u_0}{\partial r} - 2\omega_0 \right) + \text{torque}_0$$

$$(\Omega \tau)^1 : \quad 2\xi \frac{\partial u^1}{\partial r}, 2\omega^1, 2\xi \left( -\frac{\partial u^1}{\partial r} - 2\omega^1 \right) + \text{torque}^1$$

where,

$$\text{Torque} = \mu_0 [M_z H_0 \sin \Omega t - M_r (H_0 \sin \Omega t + M_z)]$$

$$\text{Torque}_0 = \mu_0 [M_{z0} H_0 \sin \Omega t - M_{r0} H_0 \sin \Omega t - M_{r0} M_{z0}]$$

$$\text{Torque}^1 = \mu_0 [M_z^1 H_0 \sin \Omega t - M_r^1 (H_0 \sin \Omega t + M_z) - M_{r0} M_z^1]$$

Eq. (9)

$$\frac{\partial M_R}{\partial t} - \omega_0 M_z + \frac{M_R}{\tau} = \frac{\chi_0}{\tau} [H_0 \sin \Omega t - M_R]$$

$$\left( \frac{\partial M_{R0}}{\partial t} + \Omega \tau \frac{\partial M_R^1}{\partial t} \right) - \left[ (\omega_0 + \Omega \tau \omega^1) (M_{z0} + \Omega \tau M_z^1) \right] = \frac{\chi_0}{\tau} [H_0 \sin \Omega t - M_R^1]$$

$$(\Omega \tau)^1 : \quad \left( \frac{\partial M_R^1}{\partial t} \right), \omega_0 M_z^1$$

Eq.(10)

$$\frac{\partial M_z}{\partial t} + \omega_0 M_z + \frac{M_z}{\tau} = \frac{\chi_0}{\tau} H_0 \sin \Omega t$$

$$\left( \frac{\partial M_{z0}}{\partial t} + \Omega \tau \frac{\partial M_z^1}{\partial t} \right) - \left[ (\omega_0 + \Omega \tau \omega^1) (M_{z0} + \Omega \tau M_z^1) \right] + M_{z0} + \Omega \tau \frac{M_z^1}{\tau} = \frac{\chi_0}{\tau} H_0 \sin \Omega t$$

$$(\Omega\tau)^{\dagger} : \left( \frac{\partial M_z^1}{\partial t} \right), \omega_0 M_z^1$$

Due to the time constrains, all the equations for second perturbation were not run using Comsol.