Pressure Drop in Expansion Flow

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Introduction

The purpose of this research is to develop a correlation that allows one to estimate the pressure drop due to sudden expansion in the micro-fluidic devices since information is not available for expansion flow in the laminar region. The two objectives to this research are to develop a correlation for predicting the pressure drop for a 3:1 laminar expansion flow of the micro-fluidic devices and to investigate the effects of mesh refinement on solutions obtained from FEMLAB simulations. The expansion flow was modeled in COMSOL Multiphysics Software (previously known as FEMLAB).

Materials and Method

In this model, a straight circular pipe (channel) empties into another straight circular pipe. The dimensionless diameter of the small channel is 1 and the dimensionless diameter of the larger channel is 3. The diameter ratio of the large channel to the small channel is 3:1. Figure 1 shows the schematic of the expansion flow. Because of the symmetry line, only half of the geometry is needed in FEMLAB for modeling. The length of the smaller channel was kept constant at 4 and the length of the bigger channel was 8. For each Reynolds number, the non-dimensional Incompressible Navier-Stokes equation was solved.

Below is the detailed layout of what was done in FEMLAB.

(For sample calculations for obtaining the excess pressure drop, refer to Appendix A.)

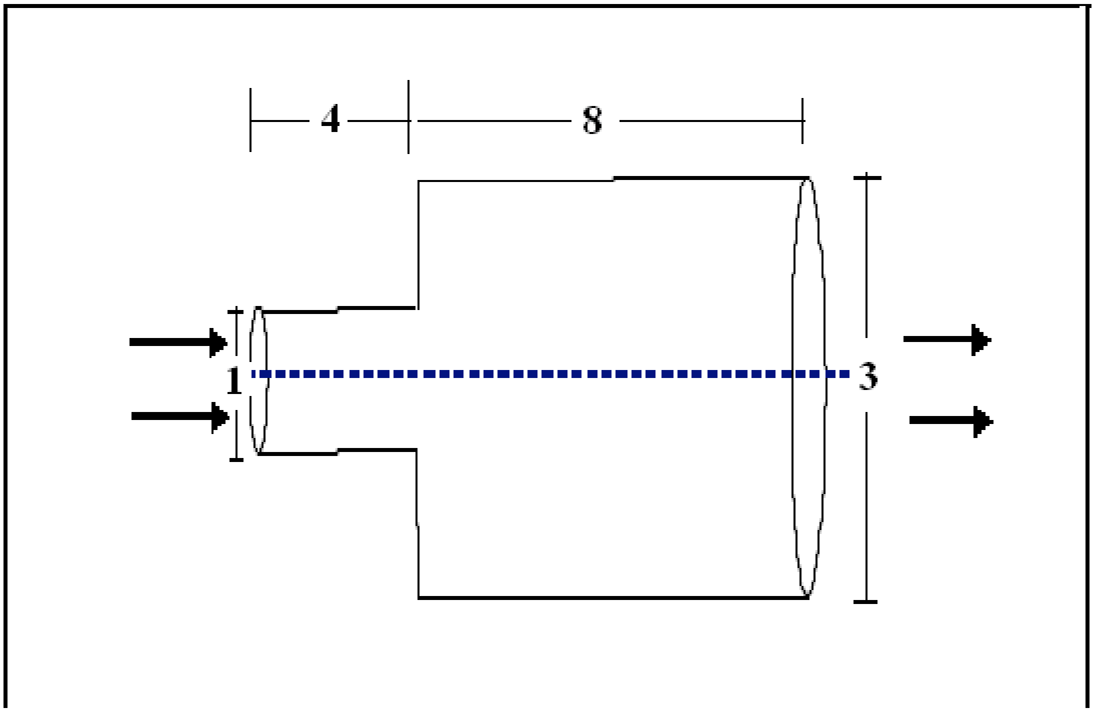

Figure1: Schematic of the expansion flow

Note: Figure 1 was not the actual set up of the model in FEMLAB. Because of the symmetry, it was only required to solve half of the geometry in FEMLAB (performed axisymmetric simulations in FEMLAB.)

Set-up in FEMLAB

In FEMLAB, Chemical Engineering/Momentum Balance/ Incompressible Navier-Stokes, Steady-state analysis was selected. As shown in figure 6, a composite object was drawn, and the mesh was initialized and refined as shown in figure 4.

Solving the Non-dimensional Incompressible Navier Stokes

The governing equation used in the research is the Incompressible Navier Stokes equation [1, p.80].

$$\rho \frac{\partial \mathrm{u}}{\partial \mathrm{t}} + \rho u . \nabla u = -\nabla P + \mu \nabla^2 u$$

The Incompressible Navier Stokes equation was non-dimensionalized in order to fully converge the solutions from simulations modeled in FEMLAB. The simulation was modeled for two different cases: one for low Re (Re ≤ 10) and one for high Re (Re > 10).

In solving for the non-dimensional Incompressible Navier-Stokes equation, the equations, parameters and boundary conditions used were:

For Re ≤ 10

Non-dimensional N.S.equation resembles the normal N.S. Equation. Below is the non-dimensional Navier-Stokes equation used for low Reynolds number [2, p.202].

$$\mathrm{Re}\frac{\partial \mathrm{u}'}{\partial \mathrm{t}'} + \mathrm{Re}\,\mathrm{u}'.\nabla'\mathrm{u}' = -\nabla' P' + \nabla'^2 \mathrm{u}'$$

The following parameters were specified for low Reynolds number in FEMLAB by going to Physics> Subdomain Settings.

Parameters:

_ = 1

_ = Re

Figure 2 shows specifying the density and viscosity parameters in FEMLAB.

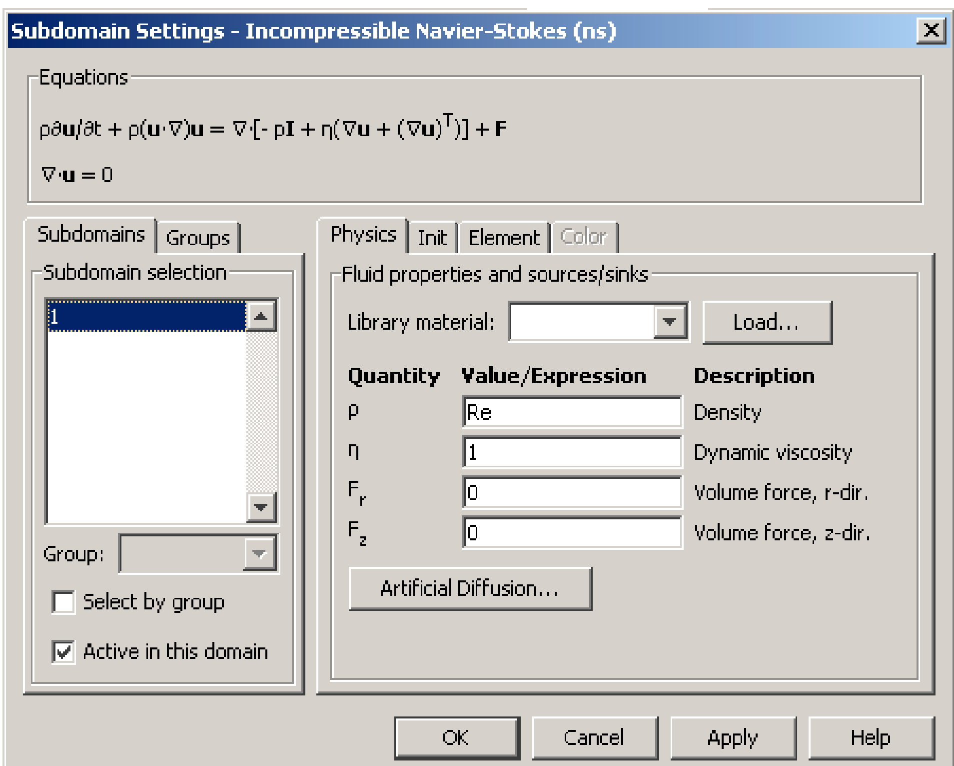

Figure2: Specifying Subdomain parameters in FEMLAB (Physics> Subdomain Settings)

For Re > 10:

Below is the non-dimensional Navier Stokes equation used for high Reynolds number [2, p.194].

$$\frac{\partial \mathrm{u}'}{\partial \mathrm{t}'} + \mathrm{u}'.\nabla'\mathrm{u}' = -\nabla' P'' + \frac{1}{\mathrm{Re}}\nabla'^2\mathrm{u}'$$

The following parameters were specified for high Reynolds number in FEMLAB by going to Physics> Subdomain Settings.

Parameters:

_ = 1/Re = $1/10^x$
_ = 1

Figure 3 shows specifying the density and viscosity parameters in FEMLAB.

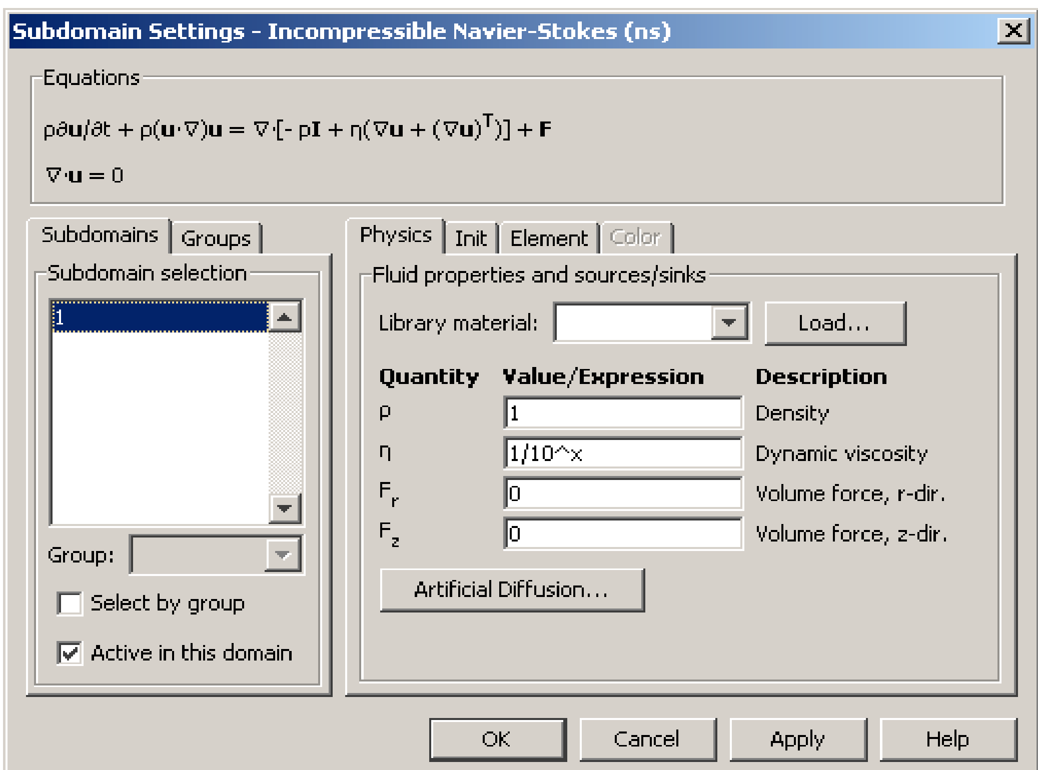

Figure3: Specifying Subdomain parameters in FEMLAB (Physics> Subdomain Settings)

Each selected Reynolds number was run three times using three different mesh sizes ranging from smaller mesh size to larger mesh size.

For each Reynolds number,

First Mesh Size

Initial mesh size = Normal
Degrees of freedom = 8303
Number of elements = **1760**

Second Mesh Size

Initial mesh size = Normal
Degrees of freedom = 32443
Number of elements = **7040**

Third Mesh Size

Initial mesh size = Normal
Degrees of freedom = 128243
Number of elements = **28160**

For each Reynolds number, it was run three times using three different mesh refinements to investigate if the value obtained from FEMLAB would fluctuate. Therefore, the mesh was increased from small mesh and then refined to larger mesh. The detailed mesh refinements of three chosen mesh elements for each Reynolds number are shown in figure 4 below.

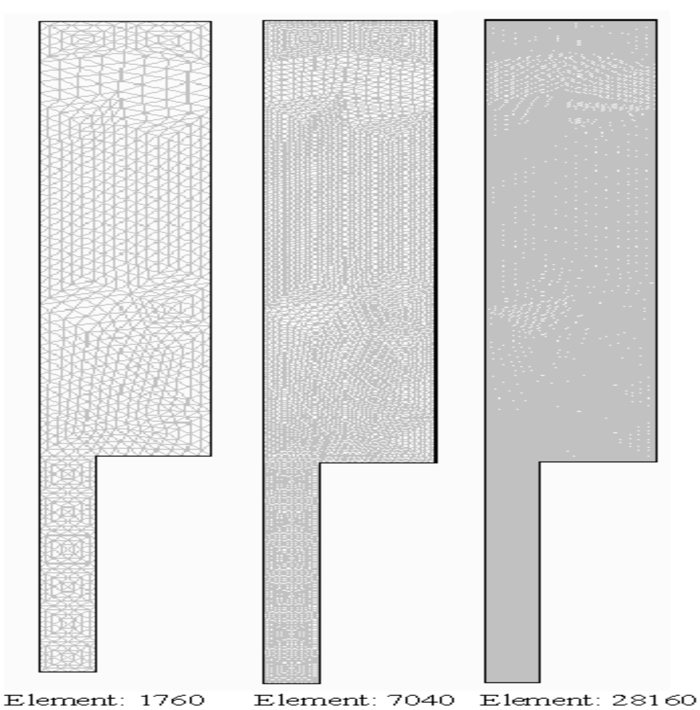

Figure4: Specifying mesh elements in FEMLAB.

The statistical information about mesh (number of elements) and number of degrees of freedom could be determined in FEMLAB by going to Mesh> Mesh Statistics. Figure 5 shows the mesh statistics window in FEMLAB.

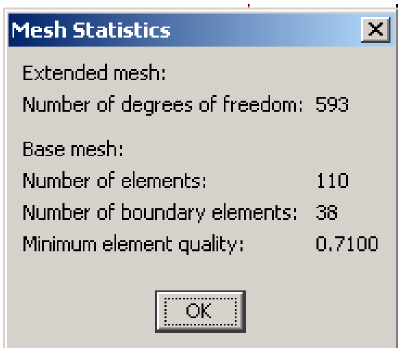

Figure5: Mesh Statistics in FEMLAB. (Mesh> Mesh Statistics)

Figure6 below shows the boundaries specified in FEMLAB. The boundary conditions of the geometry were specified in FEMLAB by going to Physics> Boundary Settings.

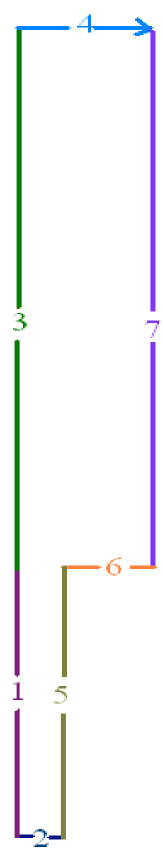

Figure6: The boundaries of the geometry in FEMLAB

Boundary Conditions:

1) Slip Symmetry at the centerline.
2) Normal flow/pressure.
3) Slip Symmetry at the centerline.
4) Inflow/outflow velocity set equal to#[–2/9 * $(1-(r/1.5)^2)$] (m/s)
5) No slip at the boundary wall.
6) No slip at the boundary wall.
7) No slip at the boundary wall.

The following figures illustrate how the boundary conditions were specified in FEMLAB at boundary 1(symmetry at the centerline), boundary 2(inlet flow), boundary 4(outlet flow), and boundary 5(at the outside wall).

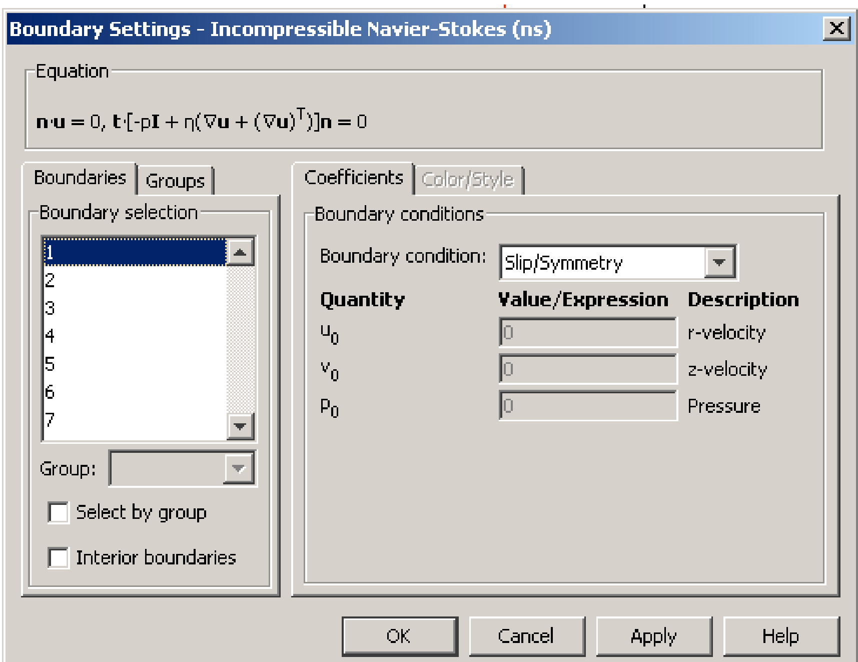

Figure7: Specifying Boundary Condition in FEMLAB. (Physics>Boundary Settings)

Note: The inlet velocity was found by using the fully developed velocity profile [2, p.183]: $v = v_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$

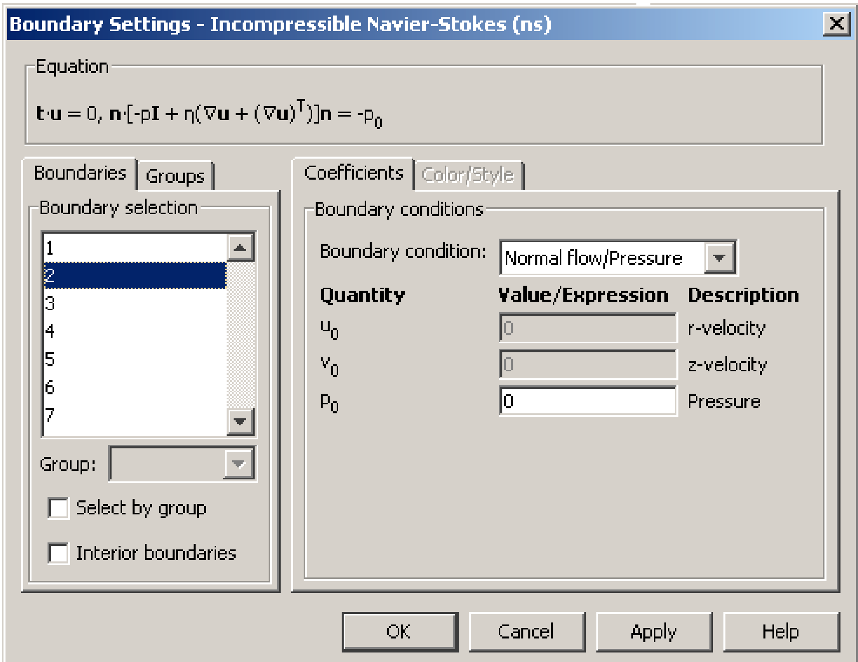

Figure8: Specifying Boundary Condition in FEMLAB. (Physics>Boundary Settings)

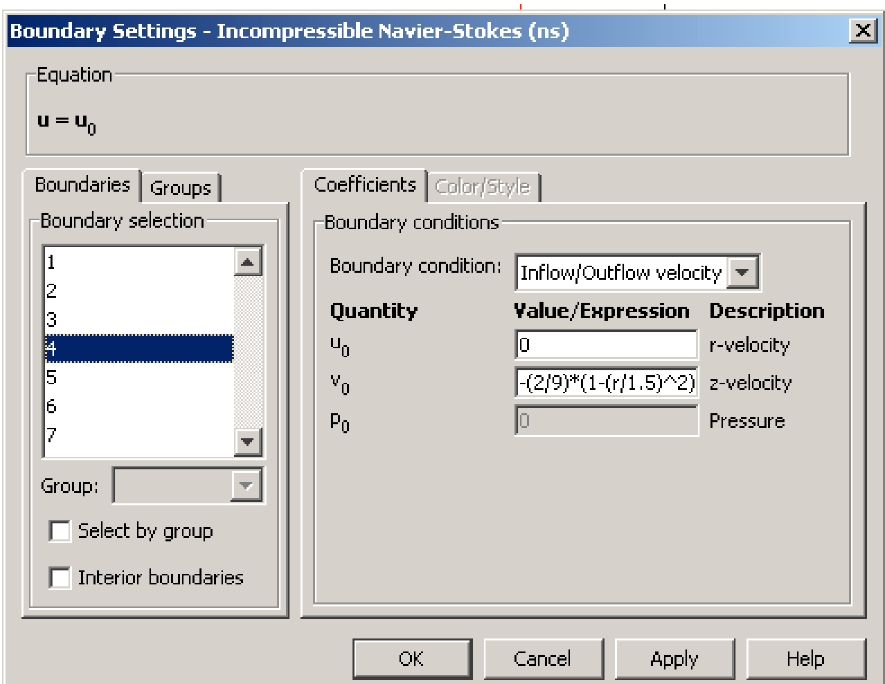

Figure9: Specifying Boundary Condition in FEMLAB. (Physics>Boundary Settings)

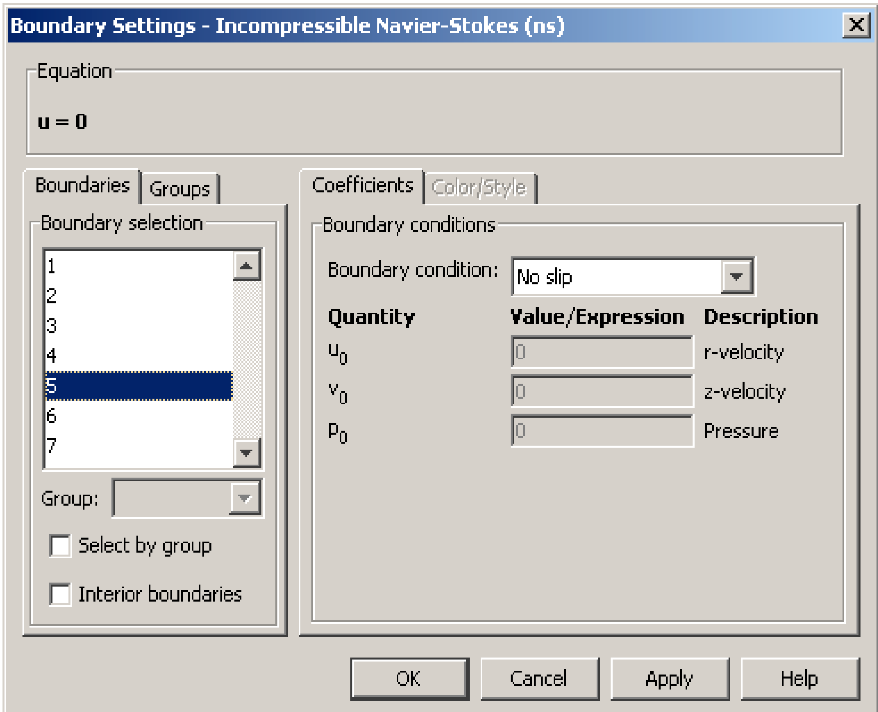

Figure10: Specifying Boundary Condition in FEMLAB. (Physics>Boundary Settings)

In solving for the case of larger Reynolds number, Reynolds number was defined as 10^x. And in order to converge the solutions in FEMLAB, x can be specified from low value and incremented to a higher value.

Parameters for the case of large Re:

$\_ = 1/Re = 1/10^x$
$\_ = 1$

x can be specified in FEMLAB by going to Solve> Solver Parameters.

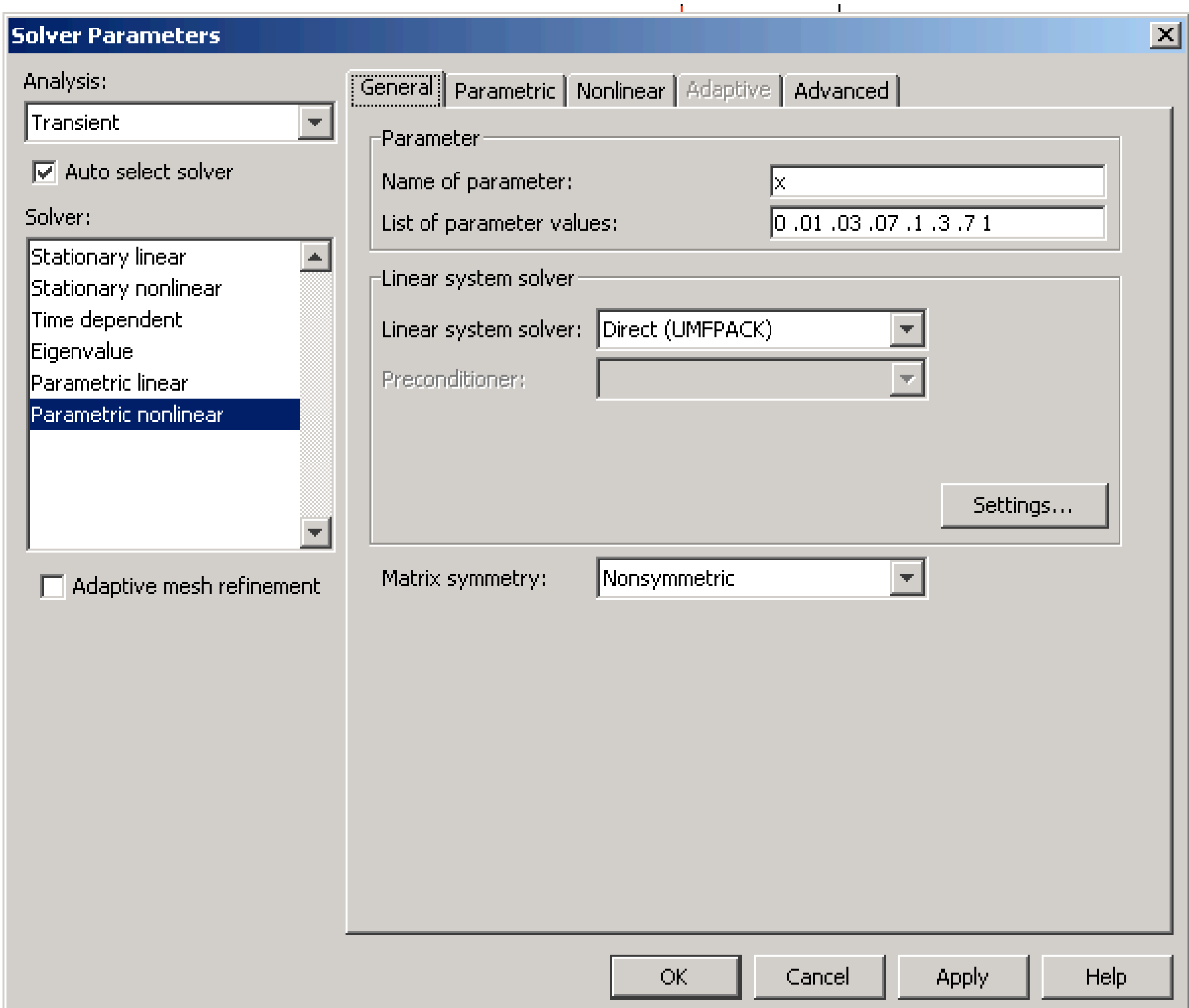

Figure11: Specifying x (x was in Re= 10^x for large Reynolds number) in FEMLAB. (Solve>Solver Parameters)

Background

For laminar flow, the excess pressure is defined as the non-dimensional excess pressure drop times the standard pressure drop [3, p.7].

$$\Delta P_{excess} = K_L \frac{\eta < v >}{D}$$

Using the above equation, the non-dimensional excess pressure drop (K_L) is defined as [3, p.7]:

$$K_L = \frac{\Delta P_{excess}}{\frac{\eta < v >}{D}}$$

For laminar flow, the dimensionless excess pressure drop is also given by the following equation [3, p.8].

$$\mathrm{K}_{\mathrm{L}} = \Delta P_{excess} = \Delta P_{Total} - \Delta P_{\text{Large channel}} - \Delta P_{\text{Small channel}}$$

When using the above equation, the pressure drop in the small and large channel can be analytically calculated, and it is the total pressure drop that is obtained from the computer simulations in FEMLAB.

After the FEMLAB simulation has been converged, the total pressure drop can be obtained from FEMLAB simulations. In FEMLAB, first the cross sectional area of inlet and outlet boundaries was found. Then, using the boundary integration option in FEMLAB, it was specified as p/area at the inlet and outlet boundaries and obtained the pressure drop at those boundaries (Post Processing> Boundary Integration). When doing the boundary integration, the check box for “Compute surface integral (for axisymmetric modes)” needed to be checked.

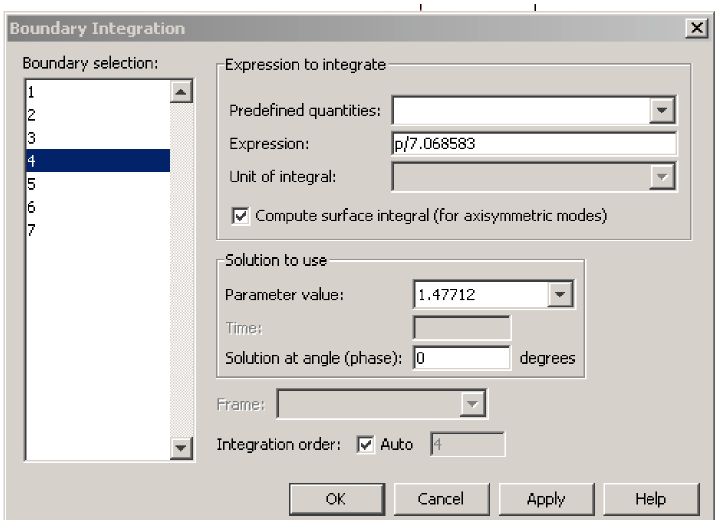

Figure11: Boundary integration in FEMLAB for obtaining the total pressure drop. (Post Processing> Boundary Integration)

To analytically calculate the pressure drop in the small channel and the large channel, the following correlations of friction factor were used to obtain the pressure drop formula which correlates with the flow geometry and characteristics.

In the laminar region (for Re < 2200), the correlation between the friction factor and the Reynolds number is given by [1, p.187]:

$$f = \frac{16}{\mathrm{Re}}$$

And the Reynolds number of the flow is given by [1, p.191]:

$$\mathrm{Re} = \frac{\rho < v > D}{\mu}$$

where $\rho = density$

$< v >=$ average velocity

$D = diameter$

$\mu = vis\cos ity$

For laminar flow, the friction factor for this flow characteristics and flow geometry is defined by [3, p.1]:

$$f \equiv \frac{1}{4}\frac{D}{L}\frac{\Delta P}{\frac{1}{2}\rho < v >^2}$$

Setting the two friction factor correlations and substituting for the Reynolds number definition provide the pressure drop formula which correlates the pressure drop with the flow characteristics [3, p.2]. For the analytical calculations of pressure drop in the small and large channels, the following pressure drop equation, in which the pressure drop is a function of the density and viscosity of the fluid, the average velocity of the fluid, and the diameter and length of the pipe, is used.

$$\Delta P = 32\frac{L}{D}\frac{\eta < v >}{D}$$

The equation below was used to identify the components contributing to the total pressure drop. The first term on the right side of the equation is the term for the change in kinetic energy change due to expansion and the second term is the viscous dissipation term. By calculating the kinetic energy change term, the viscous dissipation term can be readily available by using the following equation [3, p. 11].

$$P_1' - P_2' = \mathrm{Re}\langle v_1' \rangle^2 \left(\frac{1}{\beta^2} - 1 \right) + \hat{E}_v'$$

where $\beta = \frac{D_2^2}{D_1^2} = \frac{3^2}{1} = 9$

and $\langle v_1' \rangle = 1$

By simplifying the above equation, the following equation was produced. This equation was used to prepare for a plot of Re vs. the pressure drop, as shown in figure 12 in the results and discussion section of the report.

$$P_1' - P_2' = \mathrm{Re}\left(\frac{1}{9} - 1 \right) + \hat{E}_v'$$

Results and Discussion

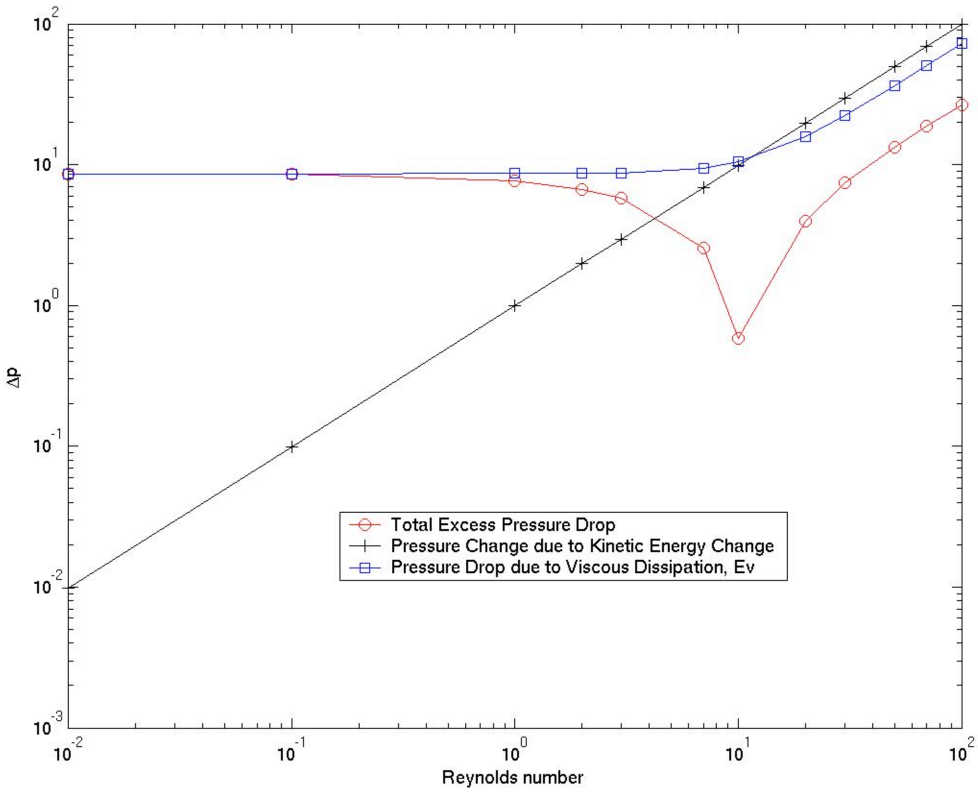

Figure12: A plot of Re vs. pressure drop

(Note: The total excess pressure drop becomes negative at a Reynolds number of about 10, and the absolute value of the pressure drop is plotted for Reynolds numbers higher than that. This happens because the pressure drop for fully developed flow in the large pipe is much larger than actual pressure drop, due to the recirculation, which reduces the friction. Thus, for Reynolds numbers higher than 10, the viscous dissipation is partly balanced by the decrease in pressure due to the kinetic energy change of an expansion.)

The variation of the total excess pressure drop with the Reynolds number can be obtained using the cumulative effect of the pressure drop due to viscous dissipation and pressure drop due to kinetic energy change for the specified Reynolds number range of $0.01 < Re < 100$. Figure 12 shows a linear relationship between the pressure drop due to kinetic energy change and the Reynolds number. The steady slope of the pressure drop due to viscous dissipation indicates the minimal effect of the Reynolds number on the pressure drop due to viscous dissipation. There is also a steady relationship between the total excess pressure drop and the Reynolds number until 10. The excess pressure drop curve's sloping negatively and positively near Reynolds number 10 indicates that there exists an inflection point near Reynolds number 10. For Reynolds number less than 1, the effect on the excess pressure drop is primarily due to the pressure drop due to viscous dissipation. For Reynolds number greater than 1, the effect of the pressure drop due to the kinetic energy change becomes more prominent on the excess pressure drop. For

Reynolds numbers greater than 10, the total excess pressure increases linearly with the pressure drop due to the kinetic energy change.

The following tables show the effect of mesh refinement on the value of K_L for the Reynolds numbers ranging from 0 to 100.

Table1: K_L for three different mesh refinements for 0< Re< 0.1.

Re	**0**	**0.01**	**0.1**
Element	**K_L**	**K_L**	**K_L**
1760	8.50	8.49	8.40
7040	8.59	8.58	8.49
28160	8.63	8.62	8.53

Table2: K_L for three different mesh refinements for 1< Re< 7.

Re	**1**	**3**	**7**
Element	**K_L**	**K_L**	**K_L**
1760	7.52	5.66	2.41
7040	7.61	5.76	2.51
28160	7.65	5.80	2.56

Table3: K_L for three different mesh refinements for 10< Re< 30.

Re	**10**	**20**	**30**
Element	**K_L**	**K_L**	**K_L**
1760	0.43	-4.18	-7.60
7040	0.54	-4.05	-7.46
28160	0.59	-4.00	-7.39

Table4: K_L for three different mesh refinements for 50< Re< 100.

Re	**50**	**70**	**100**
Element	**K_L**	**K_L**	**K_L**
5696	-13.35	-18.77	-26.51
22784	-13.26	-18.67	-26.40

Comparing the values of K_L for different mesh refinements shows that even with the more refined mesh, the value of K_L fluctuates within 1 percent of each other. Therefore, the mesh elements are negligible and the degree of mesh refinement does not have an effect on the value of K_L. In addition, K_L is positive for Reynolds number up to 10 and becomes negative for larger Reynolds number.

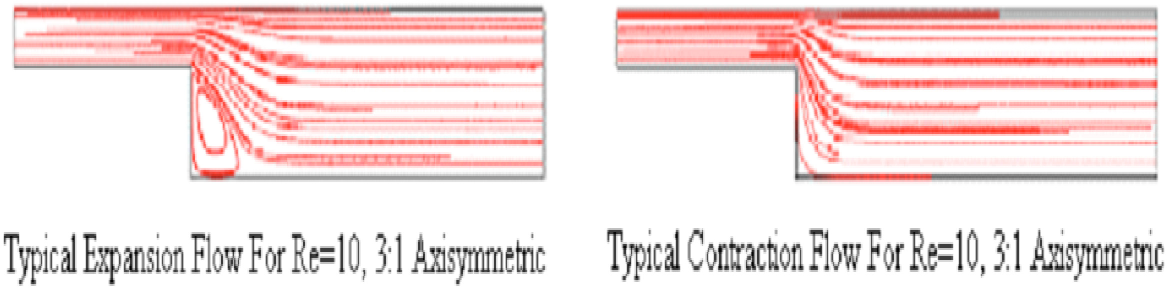

Figure13: Comparison of typical Expansion and Contraction Flows

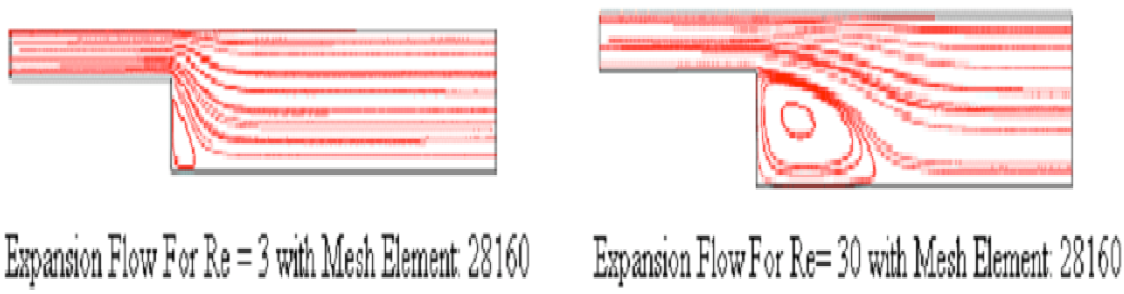

Figure14: Comparison of typical Expansion and Contraction Flows

Figure13 above compares the expansion and contraction flow patterns at a Reynolds number of 10. The expansion flow pattern at larger Re shows a bigger recirculation zone. The contraction flow experiences little or no recirculation because the fluid gets pushed up against the wall rather than flowing along a long path which could result into recirculation. Figure 14 compares the expansion flows at a small Reynolds numbers of 3 and a larger Reynolds number of 30. The comparison shows a pronounced recirculation zone at the larger Reynolds number. This could be explained with the presence of a relaxed no slip condition.

Conclusions

The method for finding the non-dimensional excess pressure drop, K_L, has been established, and K_L could assist as a correlation for predicting the pressure drop in the laminar expansion flow. Moreover, the results show that the mesh refinement does not have an effect on the values of K_L and therefore, the mesh refinement is negligible when obtaining the solutions from FEMLAB simulations.

Appendix A: Sample Calculations for K_L

Re=10
<u>Mesh Statistics</u>
d.o.f = 128,243
No. of elements=28160
From Post Processing > Boundary integration,
At small channel, P_2=-7.81 10^{-16} (got with P/0.785398).
At large channel, P_4=-131.7474216 (got with P/7.068583).
_$\mathbf{P_{total}}$ = P_2- P_4 = 131.7472

For large channel,
L=8
_= 1
<v> =1/9
D=3

$$\Delta P_{l\arg e} = 32\frac{L}{D}\frac{\eta < v >}{D} = 32\frac{8*1*\frac{1}{9}}{3*3} = 3.16049$$

For small channel,
L=4
_= 1
<v> =1
D=1

$$\Delta P_{small} = 32\frac{L}{D}\frac{\eta < v >}{D} = 32\frac{4*1*1}{1*1} = 128$$

K_L = 131.7472 – 3.160 – 128 = 0.5867.
The kinetic energy change is 10(1/81-1)= 9.876.
$\hat{E}_v{}'$= 0.5867+9.876 = 10.46.

Work Cited:

1. Bird, R. *Transport Phenomenon.* John Wiley & Sons. 1960.
2. Finlayson, Bruce. *Introduction to Chemical Engineering Computing*. 2006.
3. Finlayson, Bruce. Research Report. Micro-component flow characterization.2006.

Appendix B: Original Data

From Case 1 to Case 8, Length of large channel= 8, density= Re, _=1, and for case 8, two simulations were run(one with density = Re,,_=1 and another with density =1, _=1/Re=1/10^x) and KL in both cases were the same.

Case1: Re 0 **Re = 0, density = 0, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-139.657208	-2.92E-15	1.40E+02	3.16049383	128	8.496714
7040	32443	-139.746686	-6.64E-15	1.40E+02	3.16049383	128	8.586192
28160	128243	-139.786027	-1.44E-15	1.40E+02	3.16049383	128	8.625533

Case2: Re 0.01 **Re = 0.01, density = Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-139.647325	-5.22E-15	1.3965E+02	3.16049383	128	8.486831
7040	32443	-139.73681	-8.85E-15	1.3974E+02	3.16049383	128	8.576316
28160	128243	-139.776151	-3.83E-15	1.3978E+02	3.16049383	128	8.615657

Case3: Re 0.1 **Re = 0.1, density = Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-139.558493	-6.97E-15	1.40E+02	3.16049383	128	8.397999
7040	32443	-139.64804	-9.95E-15	1.40E+02	3.16049383	128	8.487546
28160	128243	-139.687392	-1.09E-15	1.40E+02	3.16049383	128	8.526898

Case4: Re 1 **Re = 1, density =Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-138.682359	-3.98E-15	1.39E+02	3.16049383	128	7.521865
7040	32443	-138.772612	-1.02E-14	1.39E+02	3.16049383	128	7.612118
28160	128243	-138.812083	-4.01E-15	1.39E+02	3.16049383	128	7.651589

Case5: Re 2 **Re =2, density =Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-137.737135	-7.76E-15	1.38E+02	3.16049383	128	6.58E+00
7040	32443	-137.828322	-1.49E-14	1.38E+02	3.16049383	128	6.67E+00
28160	128243	-137.868034	-7.68E-15	1.38E+02	3.16049383	128	6.71E+00

Case6: Re 3 **Re = 3, density = Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-136.825188	-8.48E-15	1.37E+02	3.16049383	128	5.66E+00
7040	32443	-136.917511	-8.68E-15	1.37E+02	3.16049383	128	5.76E+00
28160	128243	-136.957594	1.40E-15	1.37E+02	3.16049383	128	5.80E+00

Case7: Re 7 **Re = 7, density = Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-133.573979	-3.71E-15	133.5740	3.1605	128.0000	2.4135
7040	32443	-133.673472	-1.63E-14	133.6735	3.1605	128.0000	2.5130
28160	128243	-133.716493	-6.72E-15	133.7165	3.1605	128.0000	2.5560

Case8: Re 10 **Re = 10, density = Re, _ = 1** L = 8

Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1760	8303	-131.593454	-3.45E-15	131.5935	3.1605	128.0000	0.4330
7040	32443	-131.700705	-4.39E-15	131.7007	3.1605	128.0000	0.5402
28160	128243	-131.747216	-7.81E-16	131.7472	3.1605	128.0000	0.5867

Starting from Case9 (Re =20),density =1, _=1/Re=1/10^x.(For case 9 and 10, Length of the large channel =8)

Case9:

Re	**20**	**Re = 20 density = 1, _ = 1/Re = 1/ 10^x**	L =	8			
Element	**d.o.f.**	**P_4**	**P_2**	**P_{total}**	**$P_{large\ channel}$**	**$P_{small\ channel}$**	**K_L**
1760	8303	-6.348807	-2.79E-16	126.9761	3.1605	128.0000	-4.1844
7040	32443	-6.355314	-4.71E-16	127.1063	3.1605	128.0000	-4.0542
28160	128243	-6.358222	1.80E-16	127.1644	3.1605	128.0000	-3.9961

Case10:

Re	**30**	**Re =30 density = 1, _ = 1/Re = 1/ 10^x**	**L =**	8			
Element	**d.o.f.**	**P_4**	**P_2**	**P_{total}**	**$P_{large\ channel}$**	**$P_{small\ channel}$**	**K_L**
1760	8303	-4.118585	-1.98E-16	123.5576	3.1605	128.0000	-7.6029
7040	32443	-4.123314	-5.16E-16	123.6994	3.1605	128.0000	-7.4611
28160	128243	-4.125527	-5.59E-17	123.7658	3.1605	128.0000	-7.3947

From case11 to case 13, Length of the large channel =16, density =1, _=1/Re=1/10^x.

Case11:

Re	**50**	**Re =50 density = 1, _ = 1/Re = 1/ 10^x**	**L=**	16			
Element	**d.o.f.**	**P_4**	**P_2**	**P_{total}**	**$P_{large\ channel}$**	**$P_{small\ channel}$**	**K_L**
5696	26495	-2.419402	-1.20E-16	120.9701	6.3210	128.0000	-13.3509
22784	104251	-2.421209	1.31E-16	121.0605	6.3210	128.0000	-13.2605

Case12:

Re	**70**	Re =70 density = 1, _ = 1/Re = 1/ 10^x	L=	16			
Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
5696	26495	-1.650667	-1.12E-16	115.5467	6.3210	128.0000	-18.7743
22784	104251	-1.652125	1.28E-16	115.6488	6.3210	128.0000	-18.6722

Re	**100**	Re =100 density = 1, _ = 1/Re = 1/ 10^x	L =	16			
Element	d.o.f.	P_4	P_2	P_{total}	$P_{large\ channel}$	$P_{small\ channel}$	K_L
1424	6841	-1.076181	-2.56E-17	107.6181	6.3210	128.0000	-26.7029
5696	26495	-1.078064	-7.76E-17	107.8064	6.3210	128.0000	-26.5146
22784	104251	-1.079235	2.46E-17	107.9235	6.3210	128.0000	-26.3975

Pressure drop in small channel = 128
Pressure drop in large channel =3.16049

$L_{large\ channel}$=	8		For Re = 10
Element	d.o.f.	P_4	K_L
1760	8303	-131.593454	0.432962
5696	32443	-131.700705	0.540506
28160	128243	-131.747216	0.540506