Calculation of Hole Pressure for Newtonian Fluids

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Introduction

The purpose of this project was to calculate the hole pressure of a Newtonian fluid when fluid is flowing through two flat parallel plates. The geometry of the hole pressure problem in two dimensions is shown below in Figure 1. All of the parameters shown below are dimensionless.

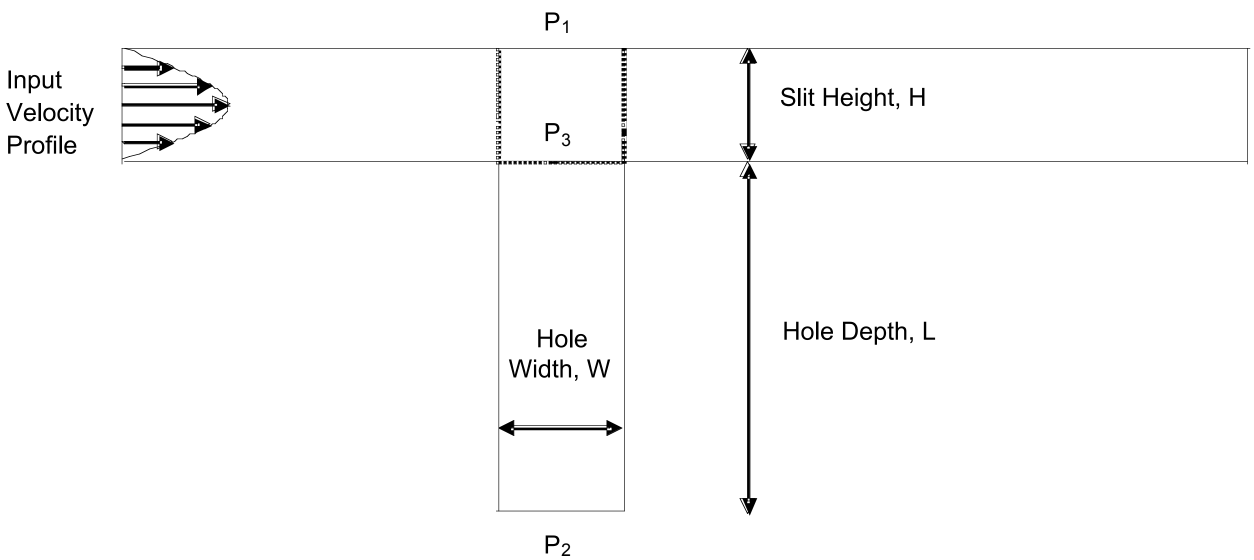

Figure 1. Geometry used to model the 2-D hole pressure problem

Experimental measurements have been made in the past by placing a pressure transducer at the exit of the hole. The hole pressure was determined by comparing the pressure forces given by the transducers. Results from these experiments were inconsistent with one another and a different method of solving the hole pressure problem was implemented. Although this problem has been previously solved in two dimensions, the goal was to prepare a solution in COMSOL Multiphysics for three dimensions and compare the results from the two models.

Materials and Method

In order to model the two dimensional problem to simulate the three dimensional problem, the hole on the bottom of the flat plate was represented as a circle. This circle was divided into many slices as shown in Figure 2. The hole pressure was calculated in 2D for a slit of the width of a slice, and the results of all the slits were added to estimate the hole pressure in 3D. This result was compared to the hole pressure derived with a 3D simulation.

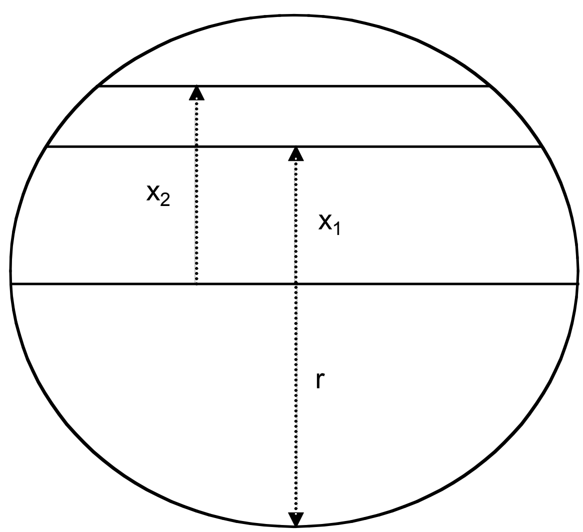

Figure 2. Circular hole divided into several slices

A value of 0.5 was used for the radius and the x_2 and x_1 values varied depending on the number of slices. The area of each slice was then determined using Eqn. 1:

$$Area = x_2\sqrt{r^2 - x_2^2} + r^2 \sin^{-1}\left(\frac{x_2}{r}\right) - x_1\sqrt{r^2 - x_1^2} + r^2 \sin^{-1}\left(\frac{x_1}{r}\right) \tag{1}$$

As a check, the sum of the segmented areas was compared to the area of the whole circle with a radius of 0.5 and the values were equal.

A two dimensional model was created for each slice of the circle. The geometry for the two and three dimensional models were created using COMSOL Multiphysics. As shown in Figure 1 above, the dimensions used for the two dimensional model were a slit height of 1, a hole depth of 3, a hole width of 3, and a plate length of 8. The incompressible Navier-Stokes model was used to solve this problem. The equation governing the incompressible Navier-Stokes model is given by Eqn. 2:

$$\rho u \cdot \nabla u = \nabla \cdot \left(-pI + \eta\left(\nabla u + (\nabla u)^T\right)\right) + F \tag{2}$$

The parameters applied to Eqn. 2 are as follows: ρ =1, μ =1, F_x=0, F_y=0, where the Reynolds number is represented by ρ. Boundary conditions were also implemented for the two dimensional case. At the entrance of the flat plates, a velocity profile represented by Eqn. 3 was used.

$$u_o = 1.5\left[1-\left(\frac{y-4.5}{0.5}\right)^2\right] \tag{3}$$

At the exit of the flat plates, a condition of zero pressure (p_o=0) was implemented. A boundary condition of no slip was applied to the remaining surfaces.

To obtain the pressures represented by P_1, P_2, and P_3 as shown in Figure 1, the use of numerical integration was applied at the three boundaries. The values obtained from COMSOL were then used to calculate the pressures for each individual segment of the circle from Eqn. 4:

$$P_i = \frac{\int P dA}{\int dA} \tag{4}$$

The total pressure at each location was then calculated using Eqn. 5:

$$P_H = \frac{\sum P_i A_i}{\sum A_i} \tag{5}$$

where P_i is the calculated pressure of each segment and A_i is the area of each segment.

For the three dimensional case, the same parameters and boundary conditions were applied in COMSOL. The total hole pressure was obtained by integrating the three boundaries.

Results

All cases for the two dimensional and three dimensional models converged in COMSOL.

Figure 3 displays a solution for the two dimensional case with a width of 1 and a Reynolds number of 1. For this case, the solution was solved with 2624 elements and 12291 degrees of freedom. The different colors within the model represent velocities.

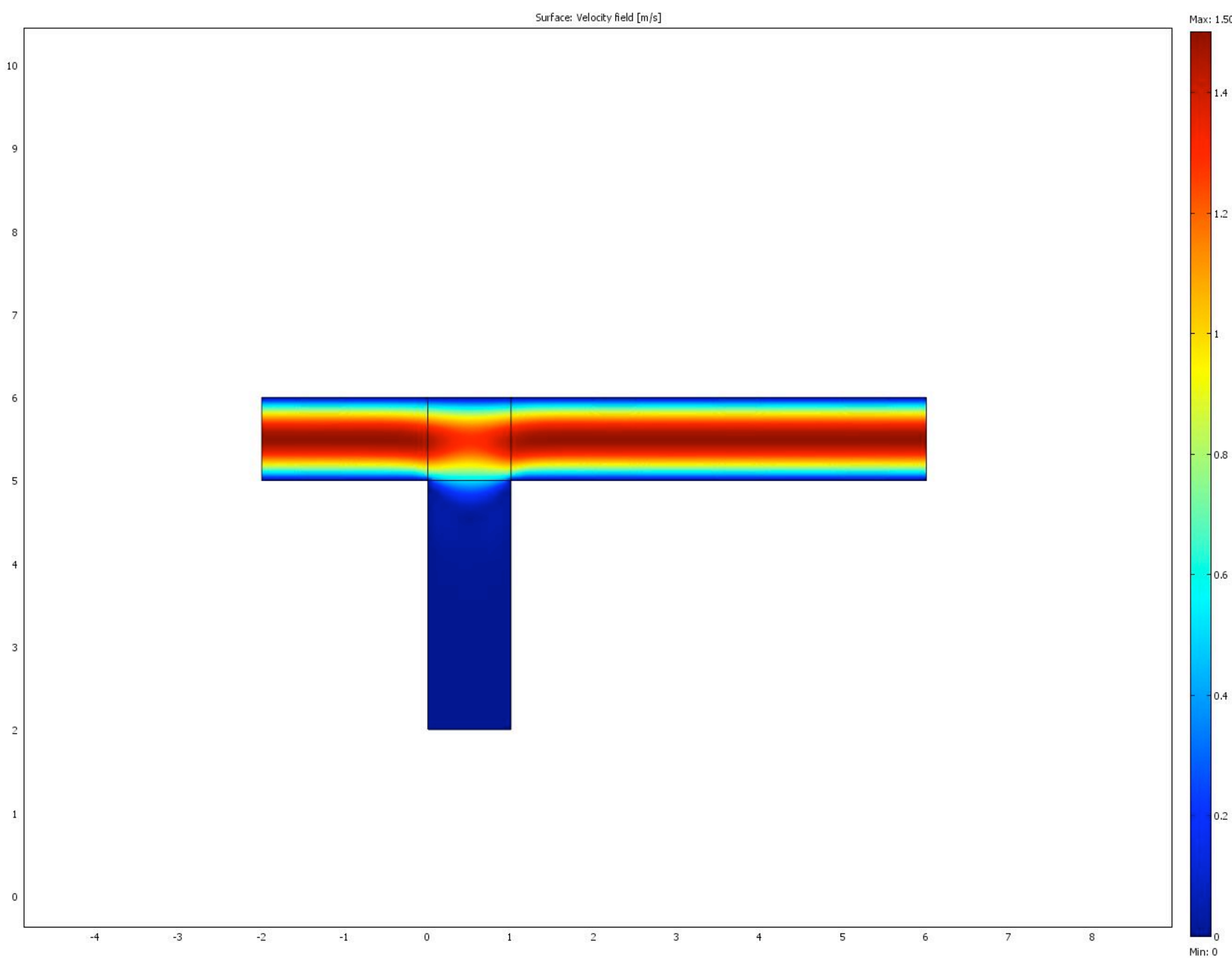

Figure 3. Sample solution for the two dimensional model

Figure 4 presents the numerical values obtained for the two dimensional case for a Reynolds number of 1.

Slice	Radius	Width	x_2	x_1	P_1	P_2	P_3	Area	P1*Area	P2*Area	P3*Area	$Area_{Calc}$
1	0.5	2/19	0.50	0.45	60.62	60.39	60.65	0.02	0.96	0.96	0.96	0.39
2	0.5	4/19	0.45	0.39	61.25	61.25	61.20	0.03	1.73	1.73	1.73	
3	0.5	6/19	0.39	0.34	61.77	61.86	61.85	0.04	2.19	2.20	2.20	
4	0.5	8/19	0.34	0.29	62.29	62.43	62.42	0.04	2.54	2.54	2.54	
5	0.5	10/19	0.29	0.24	62.79	63.47	63.88	0.04	2.81	2.84	2.86	
6	0.5	12/19	0.24	0.18	63.69	63.93	63.45	0.05	3.04	3.05	3.03	
7	0.5	14/19	0.18	0.13	63.69	63.93	63.88	0.05	3.18	3.19	3.19	
8	0.5	16/19	0.13	0.08	64.09	64.36	64.30	0.05	3.30	3.31	3.31	
9	0.5	18/19	0.08	0.03	64.47	64.75	64.67	0.05	3.37	3.39	3.38	
10	0.5	1	0.03	0.00	64.64	64.93	64.84	0.03	1.70	1.71	1.71	
							Total Area	0.39	24.81	24.91	24.89	
							P_H		63.19	63.43	63.39	

Figure 4. Numerical values for the two dimensional model

Figure 5 displays the solution for the three dimensional case with a radius of 0.5 and a Reynolds number of 1. For this case, the solution was solved with 3669 elements and 19383 degrees of freedom.

Reynolds Number	Radius	P_1	P_2	P_3	Number of Elements	DOF
1	0.5	69.50	69.74	69.54	3669	19383
10	0.5	69.82	69.90	69.87	3669	19383

Figure 5. Numerical values for the three dimensional model

After solving for both the two and three dimensional models, a comparison of the two models were made by taking a cross-sectional plot of the three dimensional case. Figure 6 shows this comparison.

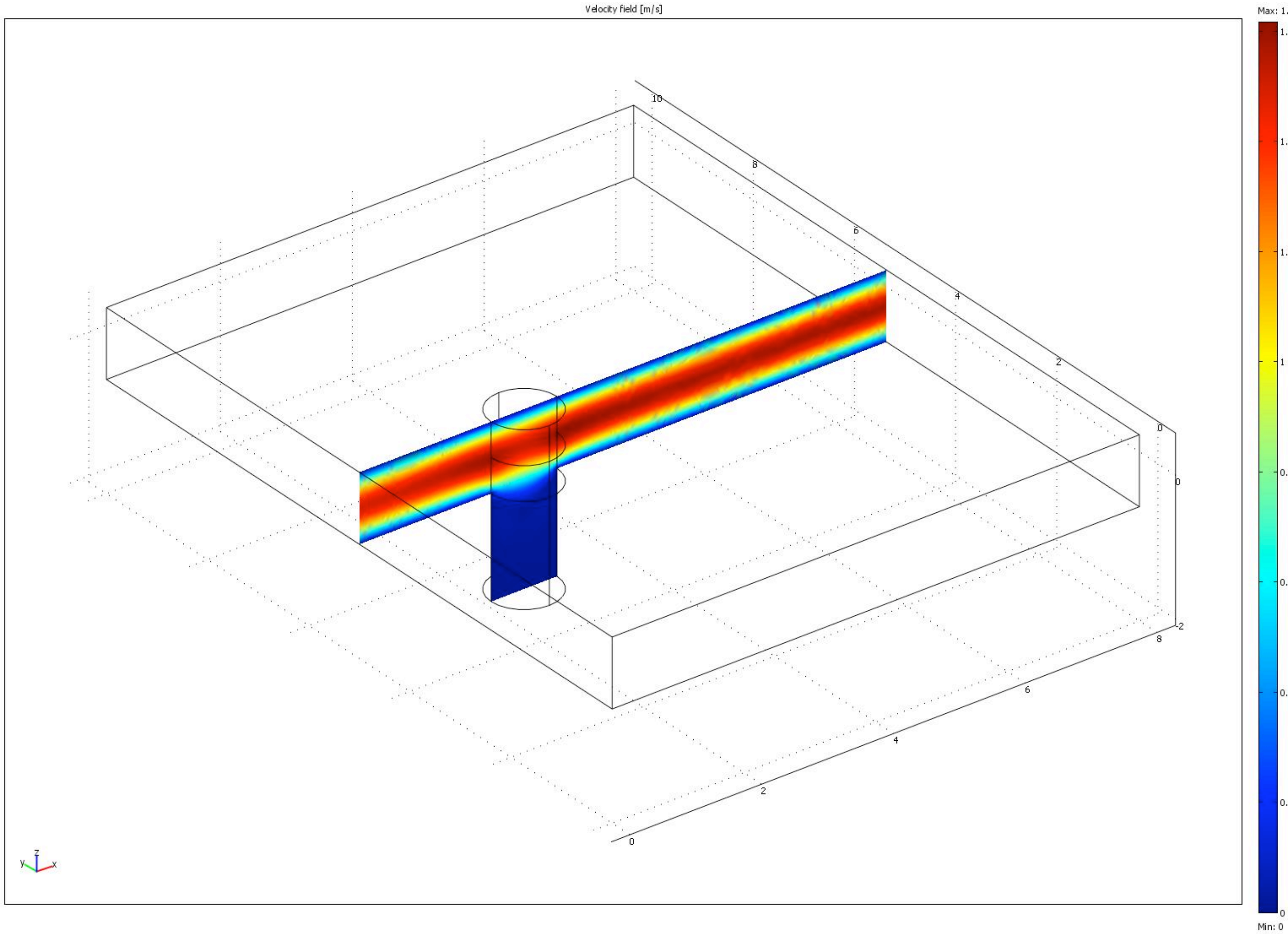

Figure 6. Cross sectional plot of the three dimensional model

Conclusion

The hole pressures were obtained for P_1, P_2, and P_3 for both two and three dimensions. Comparing the numerical values obtained, the total hole pressures for the two dimensional case are slightly lower than the values obtained for the three dimensional case. This difference in pressure is due to the velocity at the walls. When taken into account, the two dimensional pressures increased to a value closer to the values obtained for the three dimensional case. For both cases, several solutions using various Reynolds numbers were solved for. A noticeable trend in the results was an increase in hole pressure with increased Reynolds number.

To ensure that the values obtained are valid, a cross-sectional plot of the three dimensional model was made. The velocity profile from Figure 6 matches very closely to the velocity profile of Figure 2.

Appendicies

Appendix A: Incompressible Navier-Stokes Equation

$$\rho u \cdot \nabla u = \nabla \cdot \left(-pI + \eta \left(\nabla u + (\nabla u)^T \right) \right) + F$$

where _ is the Reynolds number, _ is the viscosity and F is the volume force

Appendix B: Sample Calculations

Calculating area for each segment:

$$Area = x_2 \sqrt{r^2 - x_2^2} + r^2 \sin^{-1}\left(\frac{x_2}{r}\right) - x_1 \sqrt{r^2 - x_1^2} + r^2 \sin^{-1}\left(\frac{x_1}{r}\right)$$

$$Area = .50\sqrt{.50^2 - .50^2} + .50^2 \sin^{-1}\left(\frac{.50}{.50}\right) - .45\sqrt{.50^2 - .45^2} + .50^2 \sin^{-1}($$

$$Area = .02$$

Calculating the pressure for each segment:

$$P_i = \frac{\int P dA}{\int dA}$$

$$P_i = \frac{6.38}{0.105} = 60.76$$

Calculating the total pressure for each boundary:

$$P_H = \frac{\sum P_i A_i}{\sum A_i}$$

$$P_H = \frac{24.81}{0.39} = 63.61$$