Re-circulation and recovery length for expansion flow

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Introduction

There are many interesting things, which occur when a sudden expansion is reached in a rectangular duct. Since no current literature values exist for the behavior of fluids in laminar flow through a sudden expansion the problem addressed in this report was the behavior of fluids through a sudden expansion and what can be expected to occur at said expansion. Fem-lab was used to model the behavior of fluids in the laminar region through an expansion in both 2-D and 3-D. When modeling this behavior many things must be considered before the problem and the situation can be examined. The first thing to be considered is the geometry to be used for both the 2-D and the 3-D. The second would be the mesh selection for all the applicable Geometries. The third and final step would be to examine the behavior of the fluids over a range of Reynolds numbers to characterize their behavior in the sudden expansion.

Set-up

In any engineering situation the first obstacle to be overcome is the set-up of the problem. In this instance this includes the geometries of the different situations, the boundary conditions, and the mesh selection for the analysis. These must be done independently for both the 2-D and the 3-D.

2-D

For this situation to get the best idea of what is happening in the sudden expansion three different situations were examined in 2-D axisymmetric geometry, including a 2:1, 3:1 and a 4:1 expansion ratio. In this report I will concentrate on the 2:1 contraction ratio and how the geometry, boundary conditions and the mesh selection were made for this ratio.

In the 2-D examination the symmetry of the pipe can be used to our advantage by only having to model half the pipe as can be seen in figure 1.:
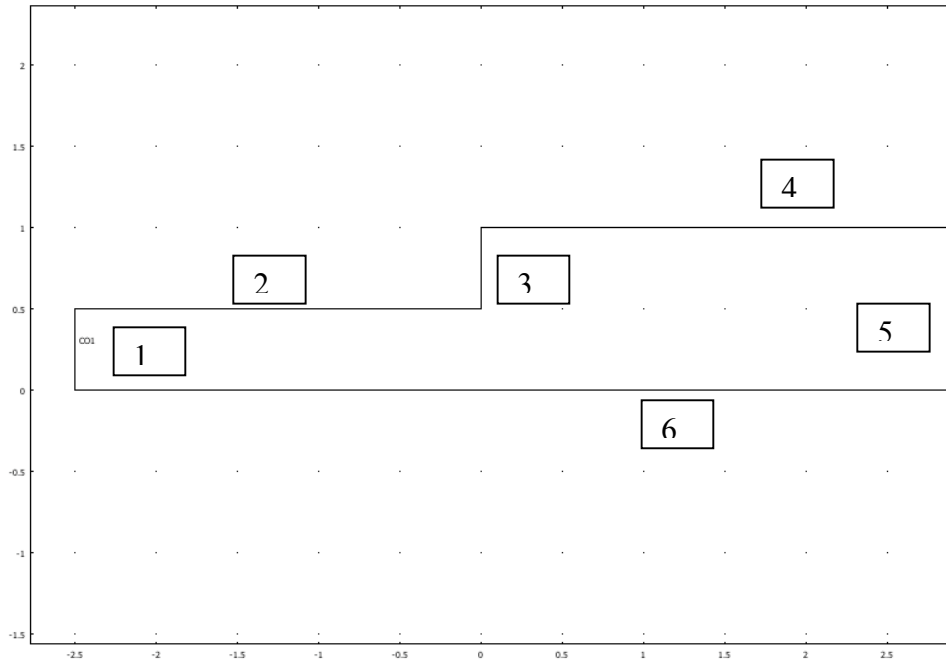

Figure 1

The entrance size for the duct was chosen such that the entrance of the duct would be that of the unit size in Fem-Lab(Appendix A). The next step was to select the boundary conditions. For the inlet and outlet streams the following conditions were chosen:

1. $u = 1.5*(1-(y/.5)\text{^}2)$

5. Normal Outflow/Pressure

The inlet condition was chosen from this equation simply because we want a parabolic velocity profile coming in, or fully-developed flow. Due to the fact that we chose to use the symmetry of the situation to our advantage in Figure 1, boundaries 2, 3 and 4 were chosen to be no slip while boundary 6 was chosen to be slip/symmetry.

The next step of solution process would be to make the appropriate mesh selection so that the problem could be solved and accurate values can be gotten. For the initial selection of the sub-domain settings the following choices were made.

1. Density(Re Number)= 10

2. Mu=1

For the 3:1 expansion ratio several refinements were made to the mesh before an appropriate mesh number was determined that would allow the problem to be solved accurately. For the purposes of mesh selection the pressure drop across the expansion was examined and once the change between meshes was insignificant enough the final mesh was chosen. For the 2:1 expansion ratio five mesh patterns were examined. The fourth mesh was chosen because it exhibited only a .005% difference from the fifth mesh and this is small enough to get the values needed for our purposes (Figure2).
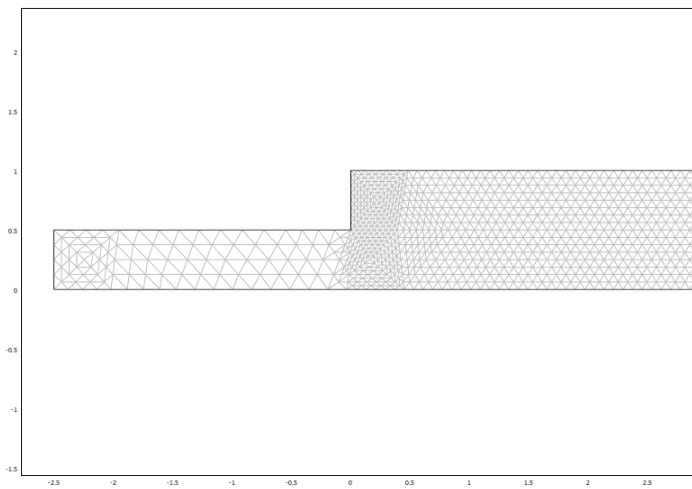

Figure 2 (2430 Elements)

The Third and final part of the problem would be to solve the problem for a wide range of Reynolds numbers. Due to the visual nature of the solution and the fact that the re-

circulation length and recovery length cannot be gotten in any numerical method each

solution for each Reynolds number had to examined to determine both of these values.

To solve over a wide range of Reynolds numbers the only change made was to that of the

sub-domain settings. The change made was:

1. Density (Re number)= 10^x

In this study x went from –2 to 2 in increments of .2. In figures 3 and 4 one can see, what

is meant by the re-circulation length and the recovery length. The re-circulation length or

the length of the vortex formed after the expansion, was determined using streamlines

(Figure 3) and for the recovery length an arrow plot was used to determine when the flow
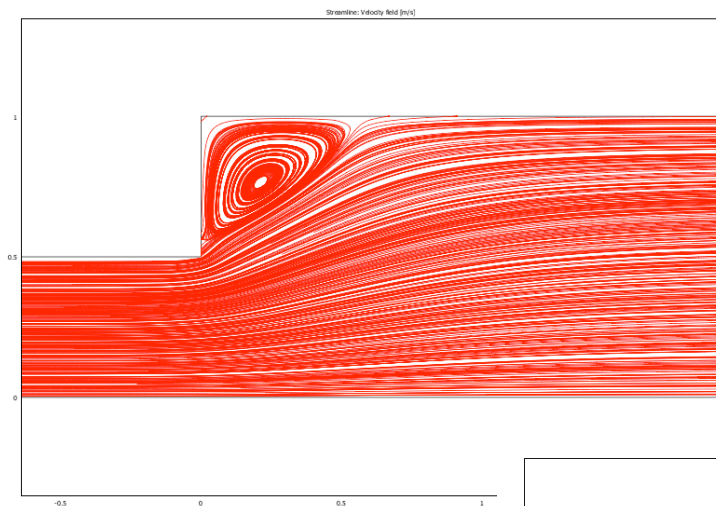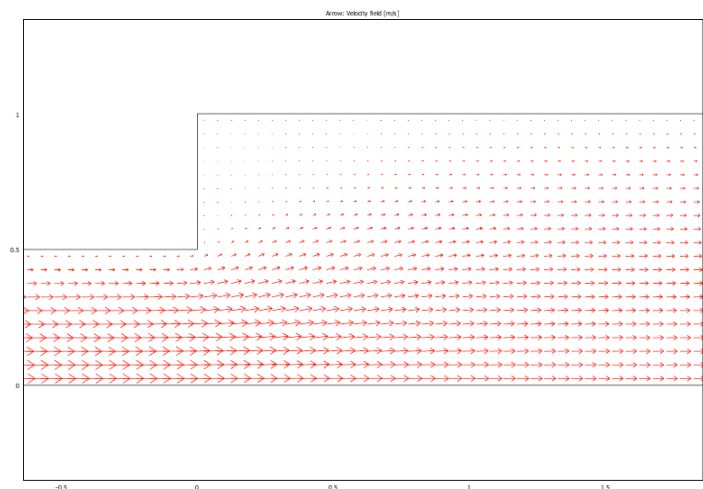
had returned to being fully-developed (Figure4).

Figure 3

Figure 4

3-D

For a 3-D contraction the best method was to once again use the symmetry of the situation. Instead of using the entire contraction one-quarter of the rectangular expansion can be used as can be seen in Figure 5.
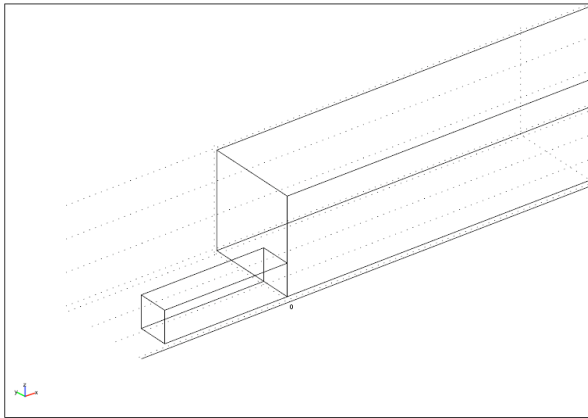

Figure 5. 3:1 square contraction

In choosing the boundary conditions the extra dimension must be taken into account. The inlet and outlet conditions were chosen as follows:

1. Normal/Outflow Presssure

2. $u = .25*(1-(y/1.5)^2)*(1-(z/1.5)^2)$

The fully-developed velocity profile was chosen to be the outlet condition because this would guaranteed that the flow would have recovered by the time the end of the duct was reached. For all the outside boundaries no slip conditions were chosen while all the inside boundaries were chosen to be slip/symmetry. The last part of the set-up for the 3-D version was to choose the mesh necessary to solve the problem accurately. Four meshes were examined and after three refinements the third was chosen to be sufficient because it only exhibited a .06% difference from the fourth mesh (Figure 6).

Figure 6
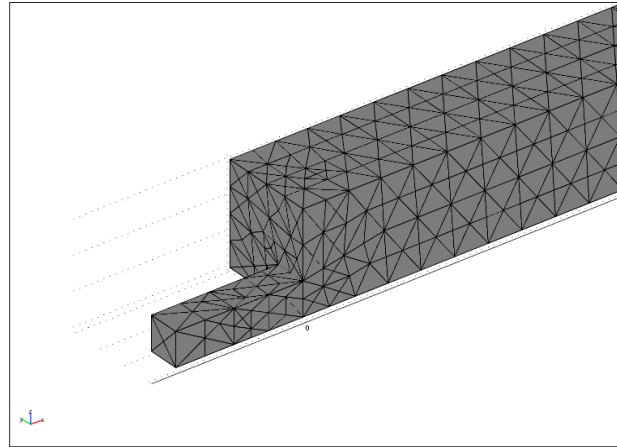
The same settings were chosen for the sub-domain settings when selecting the mesh as were used in the selection of the 2-D mesh.

Due to the complicated nature of the 3-D situation the solution of the this expansion was only able to be examined from a Reynolds number of 10^-2 to 10^1. Anymore than this and the memory requirements became to high and the solution was unable to converge.

As in the 2-D case the vortex length was determined using streamlines (Figure 7). One difference though is that due to the complicated nature of the expansion and the difficulty in determining the recovery length, both the streamline option as well as the arrow plot was used to determine the recovery length. .
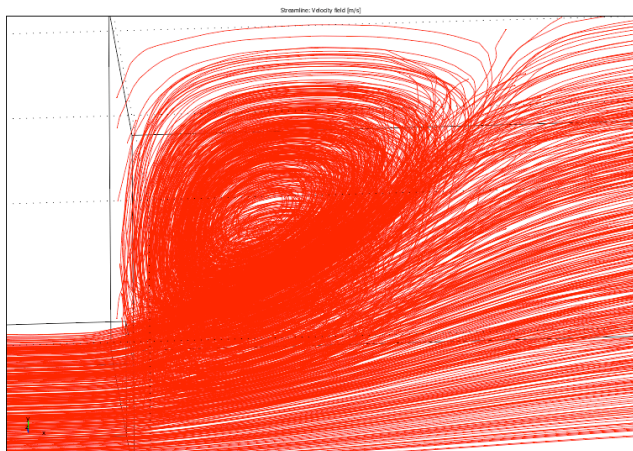

Figure 7

Results

The results for both the 2-d and the 3-d simulation are rather similar but there are a few differences. For the 2-D simulation the re-circulation length starts out as a constant and for each of the three cases: 2:1, 3:1, and 4:1 that constant is almost the same. This is to rather intuitive because for low enough Reynolds numbers the re-circulation length can be expected to be the same for the different ratio sizes. As you move up in Reynolds number the re-circulation length starts to increase exponentially at about 10^.6 (Figure8).
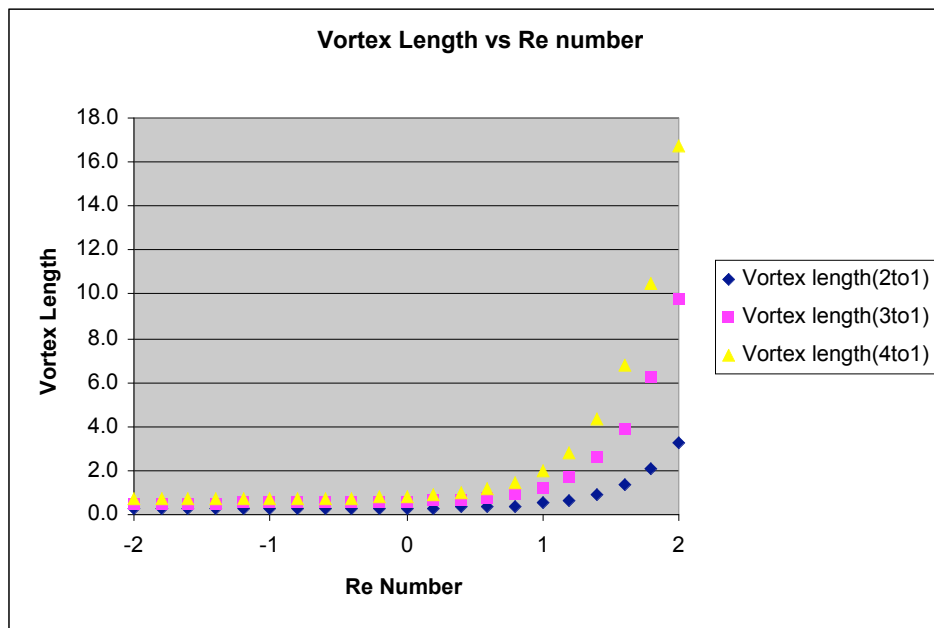

Figure 8

For the recovery length similar behavior is noted only that the initial constant at which each expansio starts is different getting slightly larger for increasing expansion ratios (Figure 9),
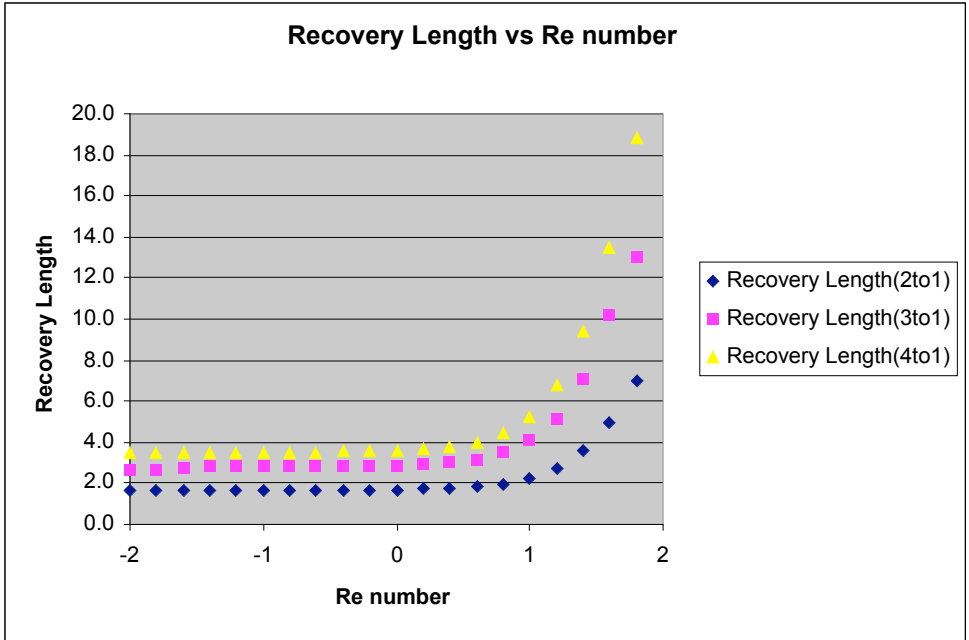
Recovery Length vs Re number

Figure 9

What is interesting to note is that for small Reynolds numbers the ratio between the vortex and re-circulation length is almost constant which can be deduced because they are both constant at small Reynolds numbers (Figure10).

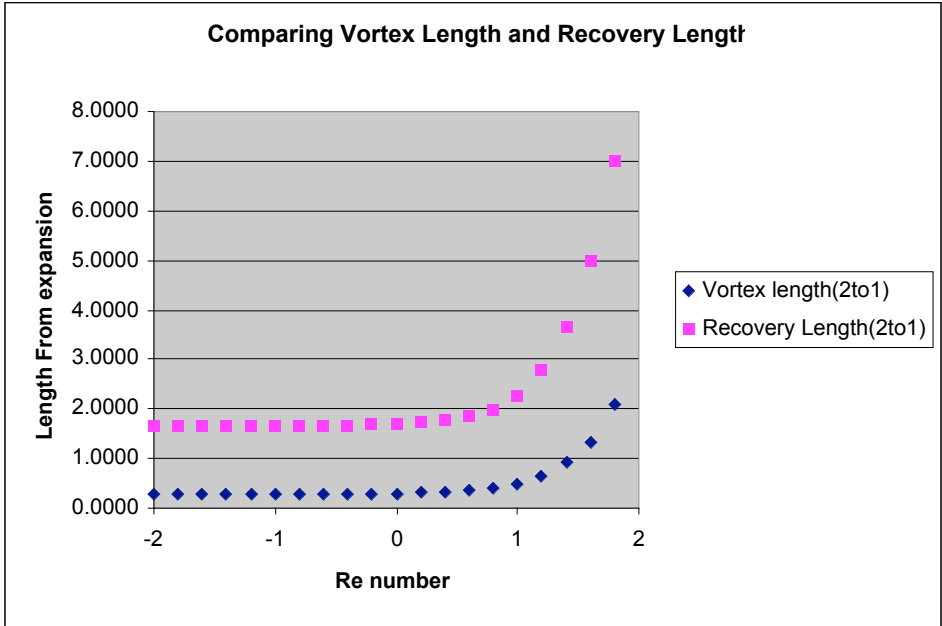**Comparing Vortex Length and Recovery Length**

Figure 10

While the results for the 3-D case were similar to those for the 2-D case there were some differences. In the 3-D vortex length examination it can be seen that again there is an

initially constant value but at about 10^.6 the length starts to increase linearly instead of exponentially (Figure 11).
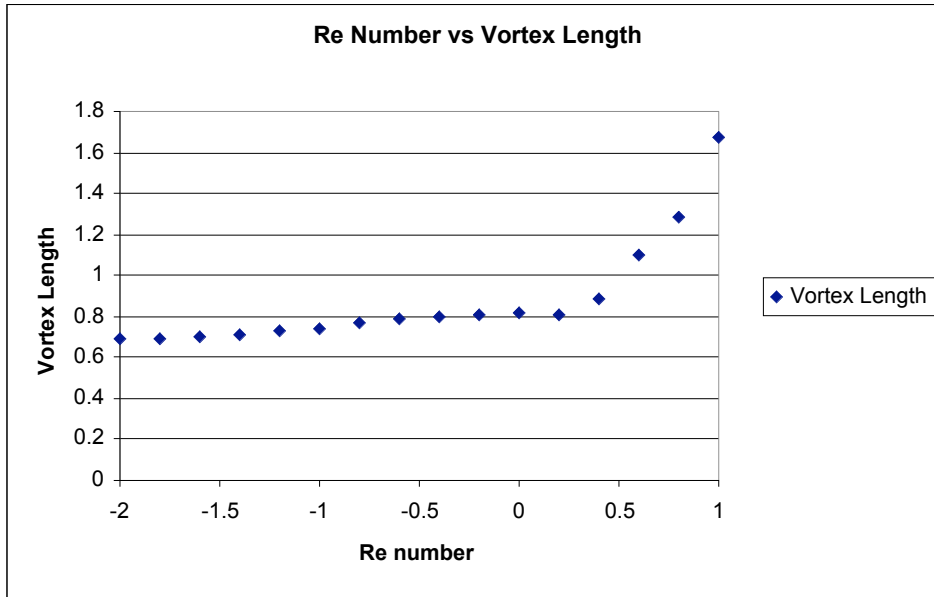
Re Number vs Vortex Length

Figure 11

In the recovery let situation it exhibited the same kind of behavior as in that it started out at some constant and the went up linearly upon reaching 10^.6. (Figure12).

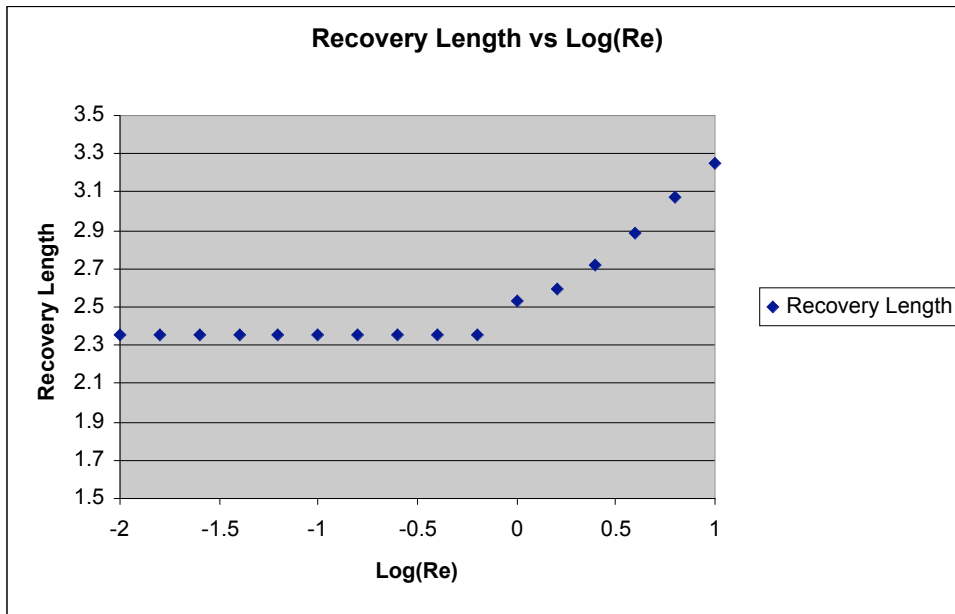**Recovery Length vs Log(Re)**

Figure 12

The 3:1 cylindrical case the vortex length/small diameter was 0.49 whereas in the 3:1 square case the ratio was 0.69, i.e. longer. For the recovery length, though, in the 3:1 cylindrical case the revovery length/small diameter was 2.63 whereas in the 3:1 square case the ratio was 2.35, i.e. slightly shorter. These numbers refer to the case of vanishingly small Reynolds number.

Conclusions and Recommendations

The results from the study closely follow what can be expected from an intuitive study of the situation. What's most interesting is the exponential increase in the 2-D and the linear increase in the 3-D. I believe it would be beneficial to do a more rigorous study using a more quantitative method for determining the re-circulation and recovery lengths. After this is done the differences that exhibit themselves between the two dimensions can be examined and the reason for this difference can be found and to see if in fact some mistake was made in this research. Once these things are accounted for a general method for predicting the effects of an expansion can be further explored.

Appendix A

(Equation Derivations)

2-D plane geometry

Parabolic velocity profile

$$\langle u' \rangle = \frac{2}{3} \cdot u'_{max}$$

$$u'(y) = u'_{max} \cdot \left(1 - \left(\frac{y}{H}\right)^2\right)$$

$$u'(y) = \frac{3}{2} \cdot \langle u' \rangle \cdot \left(1 - \left(\frac{y}{H}\right)^2\right)$$

choose $\langle u' \rangle = 1$

$$u_{out} = \frac{3}{2} \cdot \left(1 - \left(\frac{y}{H}\right)^2\right)$$

3-D Geometry

Hydraulic Radius

$$D_H = \frac{4A}{C} = \frac{4 \cdot 4 \cdot H^2}{8H} = 2H$$

Outlet size is therefore .5 making the inlet 1.5

Make inlet shape parabolic

$$\langle u' \rangle_{in} = u'_0 \cdot \left(1 - \left(\frac{y}{1.5}\right)^2\right) \cdot \left(1 - \left(\frac{z}{1.5}\right)^2\right)$$

Integrate from 0 to 1.5

$$\langle u' \rangle_{in} \frac{4}{9} u'_0$$

Need to make $\langle u' \rangle_{out} = 1$

$$\langle u' \rangle_{out} \cdot D_{H,out}^2 = \langle u' \rangle_{in} \cdot D_{H,in}^2$$

$$\langle u' \rangle_{in}\cdot = \langle u' \rangle_{out} \cdot \frac{D_{H,out}^2}{D_{H,in}^2} = 1$$

$$\langle u' \rangle_{in} = \frac{1}{9}$$

Choose $u_0' = .25$

Therefore the velocity profile in becomes

$$\langle u' \rangle_{in} = .25 \cdot \left(1 - \left(\frac{y}{1.5} \right)^2 \right) \cdot \left(1 - \left(\frac{z}{1.5} \right)^2 \right)$$