Nondimensionalized Pressure Drop for Fully Developed Laminar Flows through Square Channel

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The goal of this research is the correlation of data on various nondimensionalized pressure drops for fully developed split laminar flows through square channel. First considered correlation is for the split laminar flow through position 1 to 2 in Fig. 1.

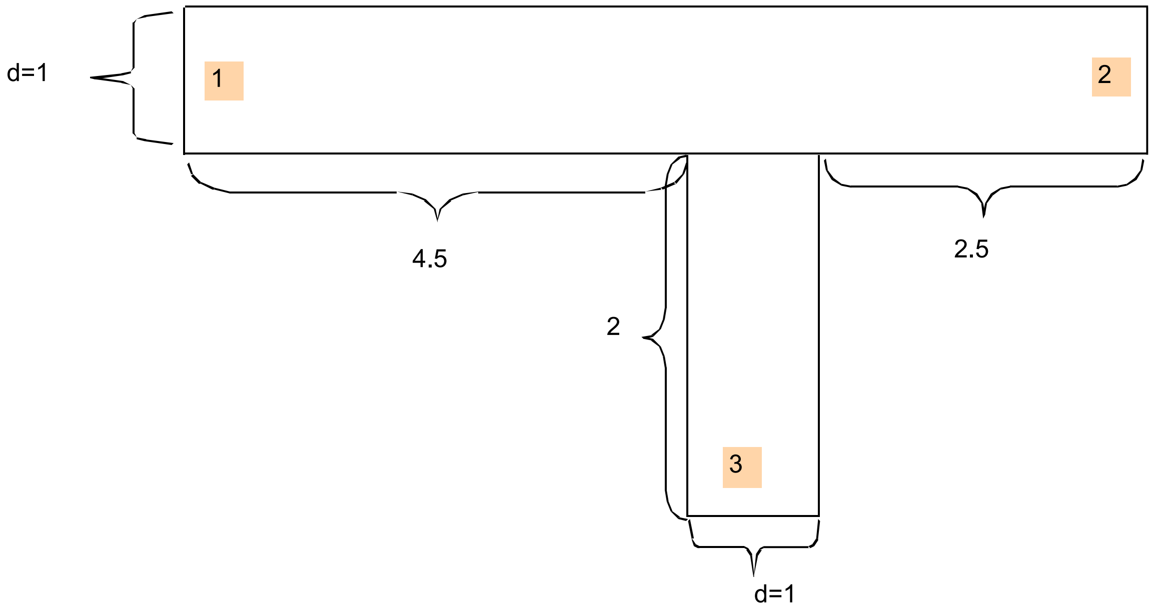


Figure 1. Scheme of square channel for case 1 and 2

By varying the pressures at the position 2 and 3, p' of each position were found. First in Comsol, an appropriate chemical engineering module was chosen in 2D when the Comsol was opened.

Step 1:

1. Space dimension: 2D
2. Application Modes
A. Chemical Engineering Module
Momentum balance
Incompressible Navier-Stokess
Steady-state analysis

Fig. 1 was drawn in 2-dimension, then it was extruded to 3-dimension.

Step 2:

1. Draw

A. Draw Object

Rectangle / Square

Draw two rectangles in Fig. 1 separately with same dimension

Select both rectangles.

B. Create Composite object

C. Extrude

Extrusion Parameters

Distance: 1

Scale x: 1

Scale y: 1

Displacement x: 0

Displacement y: 0

Twist (degrees): 0

2. Multiphysics

A. Model Navigator

Follow same as step 1, but with 3D for space dimensions.

B. Choose Geom2

The extruded image of Fig. 1 is Fig. 2.

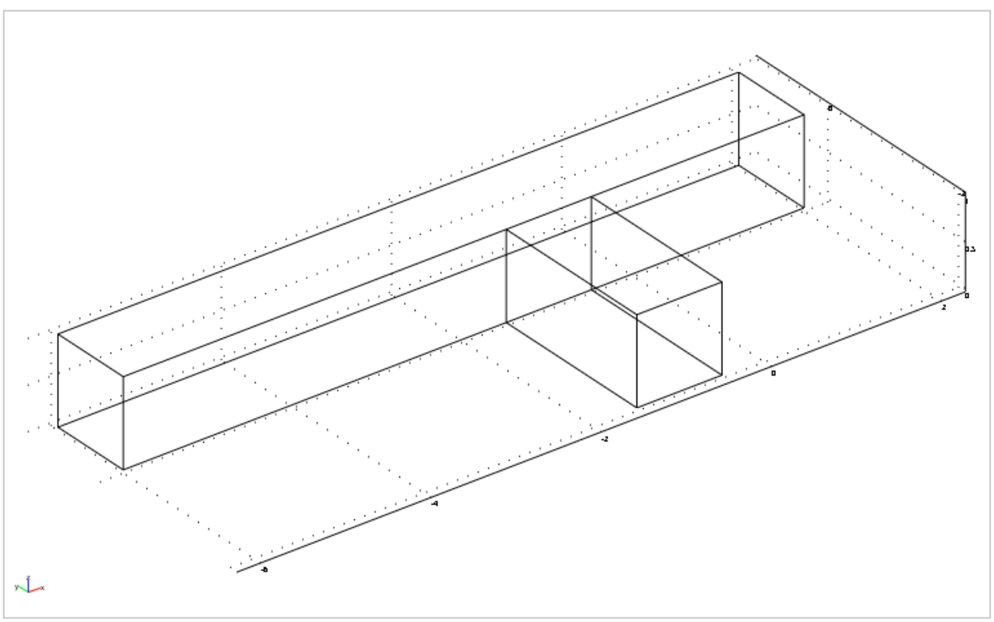


Figure 2. Extruded to 3D from 2D

Then the mesh was initialized and refined. The mesh consists of 9061 elements.

Step 3:

1. Mesh
    A. Initialize mesh
    B. Refine mesh (until the desired elements of mesh, my case = 9061)

After the mesh was refined, the subdomain and boundary setting were set up.

Step 4:

1. Physics
    A. Subdomain settings
        For both subdomains under subdomain selection,
            Every quantity equals to zero, except $\eta = 1$
    B. Boundary settings
        Boundary 1 = Laminar inflow
            Laminar inflow/outflow tab
                $u_0 = 1$
        Boundaries 7, 13 = Normal flow, pressure (the values were varied)
        Other boundaries were set to No slip

The pressure at the boundary 7 and 13 were varied. The varied values are on Fig. 3.

Finally, the pressure drop across the square channel was solved.

Step 5:

1. Solve
    A. Solve problem

The actual values of the pressure at each position were obtained from boundary integration.

Step 6:

1. Postprocessing
    A. Boundary integration
        Pressure and velocity from predefined quantities were found for each boundary.

The number of degrees of freedom solved for each calculation was 44630. The calculated values for each case are on Fig. 2.

	Input pressure to Comsol		Calculated pressure by Comsol		
Case $_i$	p'_2	p'_3	p'_1	p'_2	p'_3
1	0	0	175.9619	0.0049	0.0044
2	10	0	180.3221	10.0041	0.0049
3	20	0	184.6822	20.0034	0.0055
4	30	0	189.0424	30.0026	0.0060
5	40	0	193.4025	40.0018	0.0065
6	50	0	197.7627	50.0011	0.0070
7	60	0	202.1228	60.0003	0.0076
8	0	10	181.6018	0.0057	10.0039
9	0	20	187.2416	0.0064	20.0033
10	0	30	192.8815	0.0072	30.0028
11	0	40	198.5213	0.0079	40.0023
12	0	50	204.1612	0.0087	50.0017
13	0	60	209.8010	0.0095	60.0012
14	0	70	215.4409	0.0102	70.0007
15	0	80	221.0807	0.0110	80.0002

Figure 3. Calculated pressures by Comsol Multiphysics

From the calculated value on Fig. 3, values on Fig. 4 were calculated. Fig. 4 shows the pressure drop between the position 1 and 2, and the velocity at each position. More raw and manipulated data can be found in APPENDIX I.

Case	$p'_1 - p'_2$	u_1	u_2	u_3
1	175.9570	1	0.4360	0.5640
2	170.3180	1	0.3680	0.6320
3	164.6789	1	0.3000	0.7000
4	159.0398	1	0.2320	0.7680
5	153.4007	1	0.1640	0.8360
6	147.7616	1	0.0960	0.9040
7	142.1225	1	0.0280	0.9720
12	181.5961	1	0.5040	0.4960
13	187.2352	1	0.5720	0.4280
14	192.8743	1	0.6400	0.3600
15	198.5134	1	0.7080	0.2920
16	204.1525	1	0.7760	0.2240
17	209.7916	1	0.8440	0.1560
18	215.4307	1	0.9120	0.0880
19	221.0697	1	0.9800	0.0200

Figure 4. Pressure drops through position 1 and 2, and velocities

From the calculated values on Fig. 4, the Microsoft Excel was used for correlating the pressure drops. In Excel, the function called regression was used.

Step:

1. Tools
- ***A***. Data Analysis
  - Regression
    - Input
      - Input Y Range: Column of $p'_1 - p'_2$
      - Input X Range: Columns of velocities
    - Residuals
      - Checked residual and standardized residuals

The obtained summary of the regression can be found in APPENDIX II-A. The correlated equation is Eq. 1.

$$\Delta p'_{1-2} = 82.928\, u'_2 + 139.799 \qquad \textbf{Eq. 1}$$

Since u'_1 is constant, 1, and the sum of u'_2 and u'_3 equals to u'_1, u'_1 and u'_3 can be considered as constant or solved in terms of u'_2, $u'_3 = u'_1 - u'_2$. So the Eq. 1 only depends on u'_2. However, this pressure drop is still dimensionless. Eq. 2 can be used to find the actual pressure drop.

$$\Delta p'_{i-j} = \frac{D}{\eta < v >_1} \Delta p_{i-j}$$ **Eq. 2**

The definitions of the variables in Eq. 2 are;

D = width of the square [=] m

$<v>_i$ = velocity at position 1 [=] m/s

η = viscosity [=] Pa · s

If Eq. 1 and 2 were combined, then Eq. 3 was obtained which is actual pressure drop.

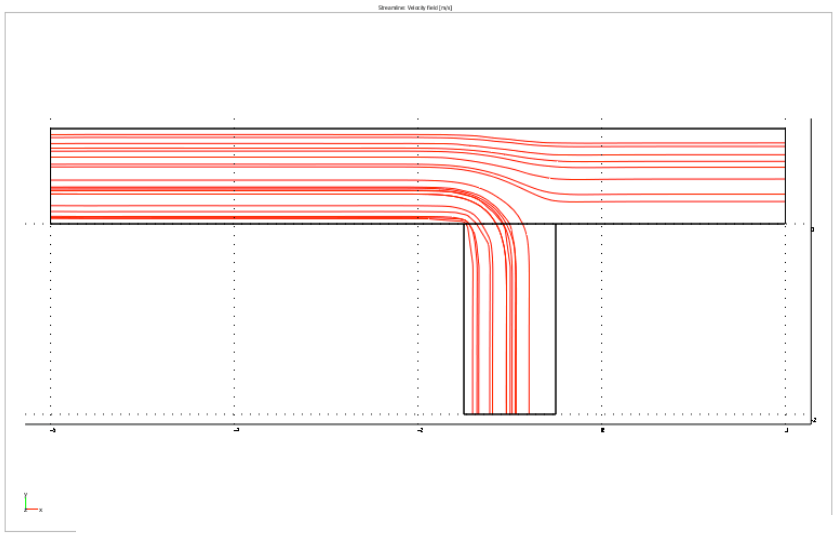
$$\Delta p_{12} = \frac{\eta < v >}{D}(82.928 u'_2 + 139.799)$$ **Eq. 3**

For checking purpose on full developed flow, the quick plot function of Comsol was used.

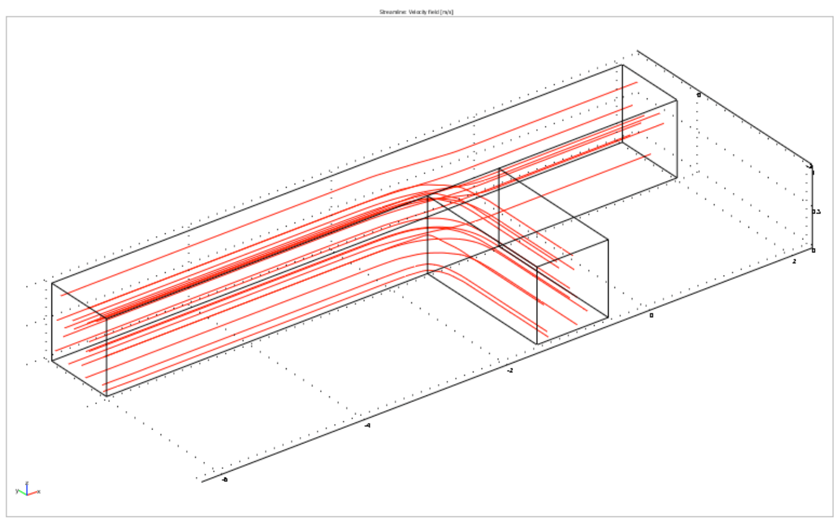
Step:

1. Postprocessing
A. Quick plot
Streamline

By checking the streamline of each result are straight at the entrance of the square channel, the full developed flow through the square channel was confirmed. Fig. 5 shows the streamline model at different viewpoints.



A) Side view



B) Angle view

Figure 5. Fully developed flow in square channel, split case 1 and 2

Another method for checking the fully developed was checking the domain surface graph on Comsol.

Step:

1. Postprocessing

A. Domain plot parameters

Surface at boundary 1 (boundary for cross-sectional area)

Fig. 6 shows the surface plot where the velocity is fastest at center and decrease as it is away from the center

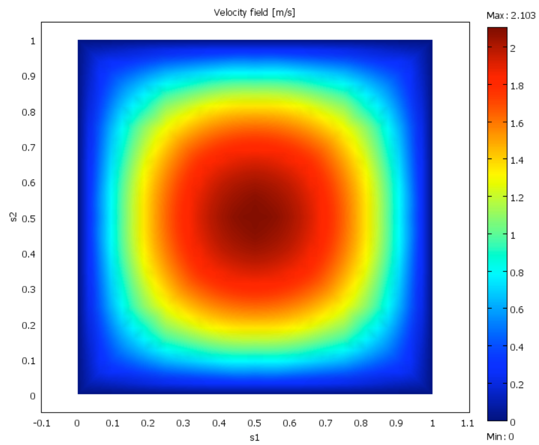


Figure 6. Surface viewpoint

Also, the laminar flow was confirmed by calculating the velocity at the inlet. Since the dimensionless inlet velocity, u'_1, equals to 1 for every cases, the flow through the channel was confirmed as laminar flow. The velocities at each position were checked by seeing the summation of the velocity of position 2 and 3 gives the velocity at position 1. Finally, the Eq. 2 was checked by substituting the u'_2 into the Eq. 2 and compared it with calculated pressure drop from Comsol. Sample calculation is in APPENDIX III.

Next considered model is the pressure drop from position 1 to 3 in Fig. 1. Same geometry and method were used. Also same data in Fig. 4 were used, except the pressure drop. Fig. 7 shows the used values. More raw and manipulated data can be found in APPENDIX I.

Case	p'_1 - p'_3	u'_1	u'_2	u'_3
1	175.9575	1	0.43602	0.56399
2	180.3172	1	0.36802	0.63199
3	184.6768	1	0.30002	0.69998
4	189.0364	1	0.23202	0.76798
5	193.3960	1	0.16402	0.83598
6	197.7556	1	0.09602	0.90398
7	202.1153	1	0.02802	0.97198
8	171.5979	1	0.50401	0.49599
9	167.2383	1	0.57201	0.42799
10	162.8787	1	0.64001	0.35999
11	158.5191	1	0.70801	0.29199
12	154.1594	1	0.77601	0.22399
13	149.7998	1	0.84401	0.15599
14	145.4402	1	0.91201	0.08799
15	141.0806	1	0.98001	0.01999

Figure 7. Pressure drops through position 1 to 3, and velocities

From the regression function in Excel, a correlation was obtained. Eq. 4 shows the correlation between the pressure drop and the velocity. Correlation data can be found in APPENDIX II-B.

$$\Delta p'_{13} = -64.113 u'_2 + 203.912 \qquad \text{Eq. 4}$$

By using Eq. 2, Eq. 5 was obtained which is actual pressure drop from position 1 to 3.

$$\Delta p_{13} = \frac{\eta < v >_1}{D}(-64.113 u'_2 + 203.912) \qquad \textbf{Eq. 5}$$

For the last case, different figure was used. However, same method was used for everything else. Fig. 8 shows the used figure.

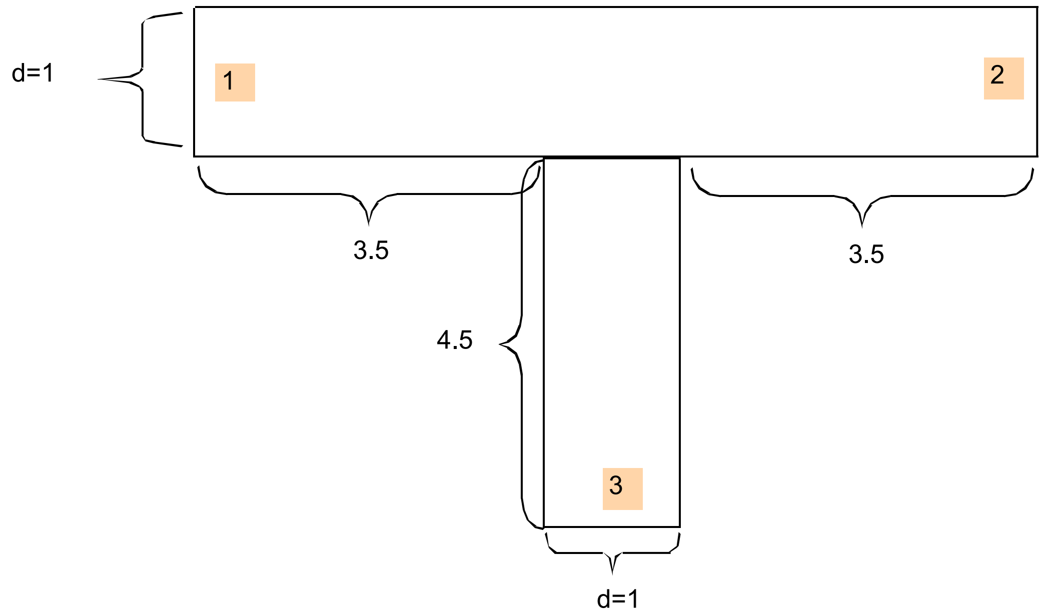


Figure 8. Scheme of square channel for case 3

For this case, pressure drop across position 3 to 1 was found. Fig. 9 shows the summary of the data. More raw and manipulated data can be found in APPENDIX IV.

Case	p'$_1$	p'$_2$	p'$_3$	u'$_1$	u'$_2$	u'$_3$	p'$_3$ - p'$_1$
1	0.00167	0.0056	188.0939	0.500054	0.49995	1.0	188.0922
2	0.00182	10.0051	193.0933	0.544907	0.45509	1.0	193.0915
3	0.00197	20.0046	198.0928	0.589761	0.41024	1.0	198.0908
4	0.00212	30.0041	203.0922	0.634614	0.36539	1.0	203.0901
5	0.00227	40.0036	208.0917	0.679467	0.32053	1.0	208.0894
6	0.00242	50.0031	213.0912	0.72432	0.27568	1.0	213.0887
7	0.00257	60.0026	218.0906	0.769174	0.23083	1.0	218.0880
8	10.00152	0.0061	193.0944	0.455201	0.54480	1.0	183.0929
9	20.00137	0.0066	198.0950	0.410348	0.58965	1.0	178.0936
10	30.00122	0.0071	203.0955	0.365494	0.63451	1.0	173.0943
11	40.00107	0.0076	208.0960	0.320641	0.67936	1.0	168.0950
12	50.00092	0.0081	213.0966	0.275788	0.72421	1.0	163.0957
13	60.00077	0.0086	218.0971	0.230934	0.76907	1.0	158.0963
14	70.00062	0.0091	223.0977	0.186081	0.81392	1.0	153.0970
15	80.00047	0.0096	228.0982	0.141228	0.85877	1.0	148.0977

Figure 9. Pressure drops through position 3 to 1, and velocities

By using the regression function in Excel, and combining the equation with Eq. 2, Eq. 6 was found. Regression summary can be found in APPENDIX V.

$$\Delta p_{31} = \frac{\eta < v >}{D}(-111.459u_2^{'} + 243.816) \qquad \textbf{Eq. 6}$$

As a checking method, flow in the square channel was observed. Fig. 10 shows the fully developed flow.

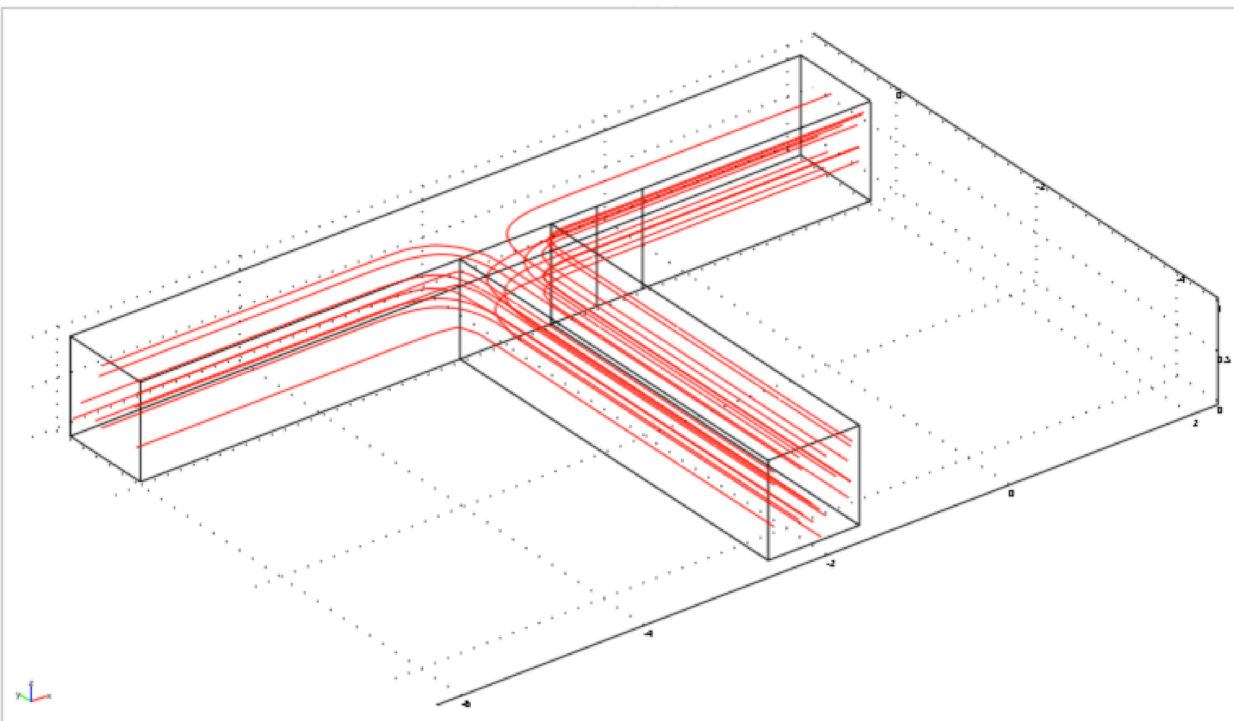


Figure 10. Fully developed flow in square channel, split case 3

Unlike varying the pressures at outlet position on the split flow, method of the join flows varied the inlet velocity from two positions. Geometry for the join flow was Fig. 8 which is same as third case of split flow. Extrude of Fig. 8 is Fig. 10 without the streamline. For the first case on the join flow, the velocity from the position 1 and 3 was varied. In order to make the outlet velocity at position 2 equals to 1, the sum of the velocity from position 1 and 3 were equal to 1. Since the velocities are nondimensionalized, they are unitless. For creating the geometry, same steps (step 1 through 3) in split flow was used. Mesh created for the join flow was 4251 elements and number of degrees of freedom solved was 22195.

For the boundary and subdomain setting, following conditions were used.

Step 4:

1. Physics

A. Subdomain settings

For both subdomains under subdomain selection,

Every quantity equals to zero, except _ = 1

B. Boundary settings

Boundary 1 = Normal flow, pressure

$p_0 = 0$

Boundaries 7, 13 = Laminar inflow/outflow

u_0 = varied

Other boundaries were set to No slip

After the step 4 was done, followed steps were same as the step in the split flow. Fig. 11 shows the varied velocity, u'_i, and the obtained pressures at each position. More raw data and manipulated data can be found in APPENDIX VII.

Case	p'$_1$	p'$_2$	p'$_3$	u'$_1$	u'$_2$	u'$_3$	Δp'$_{1-2}$	Δp'$_{3-2}$
1	212.329	0.0168	125.2844	0.9	1.00	0.10	212.3125	125.2676
2	201.183	0.0168	138.5579	0.8	1.00	0.20	201.1660	138.5411
3	190.036	0.0168	151.8314	0.7	1.00	0.30	190.0195	151.8146
4	178.890	0.0168	165.1049	0.6	1.00	0.40	178.8730	165.0881
5	167.743	0.0168	178.3784	0.5	1.00	0.50	167.7265	178.3616
6	156.597	0.0168	191.6519	0.4	1.00	0.60	156.5800	191.6351
7	145.450	0.0168	204.9254	0.3	1.00	0.70	145.4336	204.9087
8	134.304	0.0168	218.1989	0.2	1.00	0.80	134.2871	218.1822
9	123.157	0.0168	231.4724	0.1	1.00	0.90	123.1406	231.4557

Figure 11. Pressure drop across position 1 and 3 to 2, and the velocities

As shown in Fig. 11, u'_1 and u'_3 was varied from 0.9 to 0.1. Also, the sum of the two velocities equal to 1. By using Excel regression function with values in Fig. 11, Eq. 7 was obtained.

$$\Delta p'_{12} = -111.465 u'_3 + 223.459 \qquad \textbf{Eq. 7}$$

Eq. 7 is in terms of u'_3. However, it can also be expressed in terms of u'_1 by substituting $u'_3 = 1 - u'_1$. Also, Eq. 7 can be manipulated with Eq. 2 to give actual pressure drop.

$$\Delta p_{12} = \frac{\eta < v >_2}{D}(-111.465 u'_3 + 223.459) \qquad \textbf{Eq. 8}$$

In Eq. 8, the average velocity, $<v>_i$, used was the average velocity at outlet because the outlet velocity was set to reference in the join flow. Checking methods were also same as the split flow method. Fig. 12 shows the streamline view.

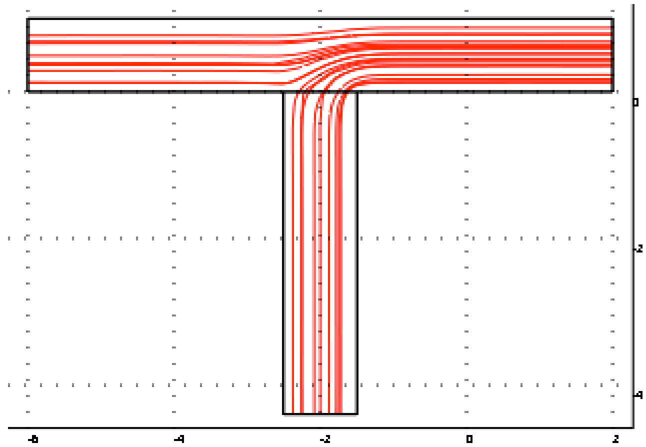


Figure 12. Fully developed flow in square channel, join case 1

From the value in Fig. 11, pressure drop across position 3 to 2 was also found, Eq.9.

$$\Delta p_{32} = \frac{\eta < v >_2}{D} (132.735 u_2^{'} + 111.994) \qquad \textbf{Eq. 9}$$

For the second case, the velocity at position 1 and 2 were varied. Same method in join flow case 1 was used, except the velocity at position 3 equals 1 in case 2. Fig. 13 shows the flows.

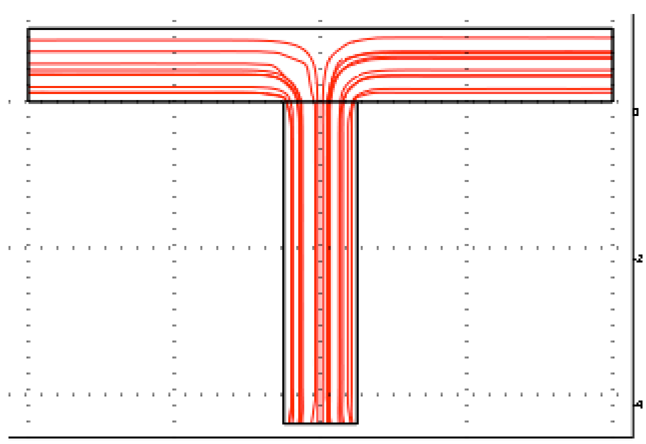


Figure 12. Fully developed flow in square channel, join case 2

The data obtained for case are on Fig. 13. More raw data and manipulated data can be found in APPENDIX VIII.

Case	p'_1	p'_2	p'_3	u'_1	u'_2	u'_3	$\Delta p'_{1-3}$	$\Delta p'_{2-3}$
1	233.522	144.3622	0.0650	0.9	0.10	1.00	233.4565	144.297184
2	222.375	155.5213	0.0650	0.8	0.20	1.00	222.3100	155.456259
3	211.229	166.6804	0.0650	0.7	0.30	1.00	211.1635	166.615334
4	200.082	177.8394	0.0650	0.6	0.40	1.00	200.0170	177.774409
5	188.936	188.9985	0.0650	0.5	0.50	1.00	188.8705	188.933484
6	177.789	200.1576	0.0650	0.4	0.60	1.00	177.7241	200.092559
7	166.643	211.3167	0.0650	0.3	0.70	1.00	166.5776	211.251634
8	155.496	222.4757	0.0650	0.2	0.80	1.00	155.4311	222.410709
9	144.350	233.6348	0.0650	0.1	0.90	1.00	144.2846	233.569784

Figure 13. Pressure drop across position 1 and 2 to 3, and the velocities

By correlating and combining the dimensionless pressure drops with Eq. 2, Eq. 10 was obtained.

$$\Delta p_{13} = \Delta p_{23} = \frac{\eta < v >_3}{D}(-111.465u'_2 + 244.603) \qquad \textbf{Eq. 11}$$

Eq. 8 and 11 have same coefficient, but different intercept constant.

For the last case in join flow is that varying the velocity from position 2 and 3. The method here used was same as the method in the join flow case 1. Fig. 14 shows the flows.

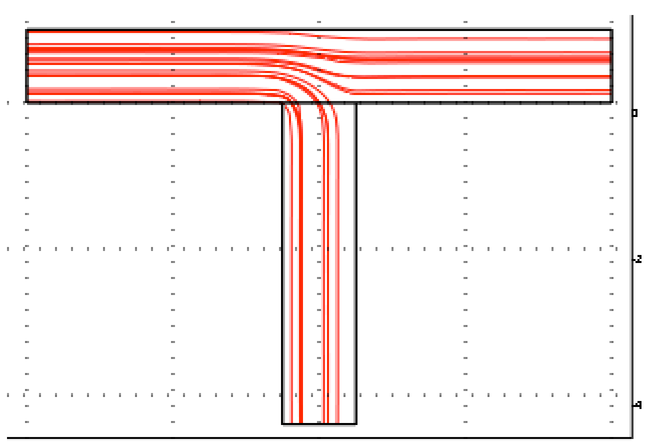


Figure 14. Fully developed flow in square channel, join case 3

The data obtained for case are on Fig. 15. More raw data and manipulated data can be found in APPENDIX IX.

Case	p'_1	p'_2	p'_3	u'_1	u'_2	u'_3	$\Delta p'_{3-1}$	$\Delta p'_{2-1}$
1	0.027	123.0773	231.3798	1	0.10	0.90	231.3532	123.0507
2	0.027	134.2364	218.1063	1	0.20	0.80	218.0796	134.2097
3	0.027	145.3955	204.8328	1	0.30	0.70	204.8061	145.3688
4	0.027	156.5545	191.5593	1	0.40	0.60	191.5326	156.5279
5	0.027	167.7136	178.2858	1	0.50	0.50	178.2591	167.6870
6	0.027	178.8727	165.0123	1	0.60	0.40	164.9856	178.8460
7	0.027	190.0318	151.7388	1	0.70	0.30	151.7121	190.0051
8	0.027	201.1908	138.4652	1	0.80	0.20	138.4386	201.1642
9	0.027	212.3499	125.1917	1	0.90	0.10	125.1651	212.3233

Figure 15. Pressure drop across position 2 and 3 to 1, and the velocities

By using the regression in Excel, and combining with Eq. 2, Eq. 12 was obtained.

$$\Delta p_{31} = \frac{\eta < v >_1}{D}(-132.735u_2' + 244.627) \qquad \textbf{Eq. 12}$$

From Fig. 15, actual pressure drop correlation from position 2 to 1 was also found, Eq. 13.

$$\Delta p_{12} = \frac{\eta < v >_1}{D}(111.591u_2' + 111.892) \qquad \textbf{Eq. 13}$$

Summary of the data regression and checking equations for join flow cases can be found in APPENDIX X and XI, respectively. Also, a list of all correlations for each case can be found in APPENDIX XII.

APPENDICES

Split flow cases

Join flow cases

Appendix I

Raw and manipulated data, split flow case 1 & 2

? = 1 u'_1 = 1 d = 1

Mesh consists of 9061 elements.
Number of degrees of freedom solved for:44630

		Raw		Manipulated			
				$(p_i')(A_{cross})/(A_{cross})$	$(u_i')(A_{cross})/(A_{cross})$		
	Position	[N], Expression: p $(p_i')(A_{cross})$	[m^3/s], Expression: u $(u_i)(A_{cross})$	[Pa] p_i'	[m/s] u_i	$p'_1 - p'_2$	$p'_1 - p'_3$
Case 1	1	175.961931	1	175.961931	1	175.957042	175.9575
$P_2 = P_3 = 0$	2	0.004889	0.436015	0.004889	0.436015		
	3	0.004396	0.563985	0.004396	0.563985		
Case 2	1	180.32208	1	180.32208	1	170.317954	180.3172
$P_2 = 10, P_3 = 0$	2	10.004126	0.368015	10.004126	0.368015		
	3	0.004926	0.631985	0.004926	0.631985		
Case 3	1	184.682229	1	184.682229	1	164.678865	184.6768
$P_2 = 20, P_3 = 0$	2	20.003364	0.300016	20.003364	0.300016		
	3	0.005455	0.699984	0.005455	0.699984		
Case 4	1	189.042378	1	189.042378	1	159.039776	189.0364
$P_2 = 30, P_3 = 0$	2	30.002602	0.232016	30.002602	0.232016		
	3	0.005985	0.767984	0.005985	0.767984		
Case 5	1	193.402528	1	193.402528	1	153.400689	193.396
$P_2 = 40, P_3 = 0$	2	40.001839	0.164017	40.001839	0.164017		
	3	0.006515	0.835983	0.006515	0.835983		
Case 6	1	197.762677	1	197.762677	1	147.7616	197.7556
$P_2 = 50, P_3 = 0$	2	50.001077	0.096017	50.001077	0.096017		
	3	0.007045	0.903983	0.007045	0.903983		
Case 7	1	202.122826	1	202.122826	1	142.122512	202.1153
$P_2 = 60, P_3 = 0$	2	60.000314	0.028018	60.000314	0.028018		
	3	0.007574	0.971982	0.007574	0.971982		
Case 8	1	181.601781	1	181.601781	1	181.59613	171.5979
$P_2 = 0, P_3 = 10$	2	0.005651	0.504014	0.005651	0.504014		
	3	10.003866	0.495986	10.003866	0.495986		
Case 9	1	187.241632	1	187.241632	1	187.235218	167.2383
$P_2 = 0, P_3 = 20$	2	0.006414	0.572014	0.006414	0.572014		
	3	20.003336	0.427986	20.003336	0.427986		
Case 10	1	192.881483	1	192.881483	1	192.874307	162.8787
$P_2 = 0, P_3 = 30$	2	0.007176	0.640013	0.007176	0.640013		
	3	30.002807	0.359987	30.002807	0.359987		
Case 11	1	198.521333	1	198.521333	1	198.513394	158.5191
$P_2 = 0, P_3 = 40$	2	0.007939	0.708013	0.007939	0.708013		
	3	40.002277	0.291987	40.002277	0.291987		
Case 12	1	204.161184	1	204.161184	1	204.152483	154.1594
$P_2 = 0, P_3 = 50$	2	0.008701	0.776012	0.008701	0.776012		
	3	50.001747	0.223988	50.001747	0.223988		
Case 13	1	209.801035	1	209.801035	1	209.791571	149.7998
$P_2 = 0, P_3 = 60$	2	0.009464	0.844012	0.009464	0.844012		
	3	60.001217	0.155988	60.001217	0.155988		
Case 14	1	215.440886	1	215.440886	1	215.43066	145.4402
$P_2 = 0, P_3 = 70$	2	0.010226	0.912011	0.010226	0.912011		
	3	70.000688	0.087989	70.000688	0.087989		
Case 15	1	221.080736	1	221.080736	1	221.069748	141.0806
$P_2 = 0, P_3 = 80$	2	0.010988	0.980011	0.010988	0.980011		
	3	80.000158	0.019989	80.000158	0.019989		

Appendix II-A

Summary of data regression, $p'_{1\text{-}2}$, split flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.846153846
Standard Error	2.22917E-05
Observations	15

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	175.957061	-1.90011E-05	-0.945636212
2	170.3179313	2.27452E-05	1.131966484
3	164.6788844	-1.94369E-05	-0.967320949
4	159.0397547	2.13095E-05	1.060514433
5	153.4007079	-1.88726E-05	-0.939238369
6	147.7615781	2.18737E-05	1.088597011
7	142.1225313	-1.93083E-05	-0.960923106
8	181.5961078	2.21809E-05	1.103883904
9	187.2352376	-1.95654E-05	-0.973718792
10	192.8742844	2.26166E-05	1.125568641
11	198.5134141	-2.01297E-05	-1.001801372
12	204.1524609	2.20523E-05	1.097486061
13	209.7915907	-1.9694E-05	-0.980116635
14	215.4306375	2.24881E-05	1.119170799
15	221.0697673	-1.92583E-05	-0.9584319

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	8903.808663	2967.936221	1.79179E+13	4.5812E-70
Residual	13	6.45999E-09	4.96922E-10		
Total	16	8903.808663			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	139.799044	1.14293E-05	12231674.96	2.75384E-86	139.7990193
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	82.92837862	1.95911E-05	4232956.57	2.69704E-80	82.9283363
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
139.7990687	139.7990193	139.7990687
0	0	0
82.92842095	82.9283363	82.92842095
0	0	0

Appendix II-B

Summary of data regression, p'_{1-3} , split flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.846153846
Standard Error	1.73152E-05
Observations	15

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	175.9575197	1.52905E-05	0.979679823
2	180.3171713	-1.73014E-05	-1.108519105
3	184.6767588	1.52191E-05	0.975103733
4	189.0364104	-1.73729E-05	-1.113095195
5	193.3959979	1.51477E-05	0.970527641
6	197.7556494	-1.74443E-05	-1.117671287
7	202.1152369	1.50763E-05	0.965951551
8	171.5979322	-1.723E-05	-1.103943013
9	167.2382806	1.5362E-05	0.984255917
10	162.8786932	-1.71586E-05	-1.099366921
11	158.5190416	1.44334E-05	0.924761049
12	154.1594541	-1.70872E-05	-1.094790831
13	149.7998025	1.55048E-05	0.993408099
14	145.440215	-1.70157E-05	-1.090214739
15	141.0805634	1.45762E-05	0.933913231

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	5321.759099	1773.9197	1.77501E+13	4.82453E-70
Residual	13	3.8976E-09	2.99816E-10		
Total	16	5321.759099			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	203.9115416	8.87772E-06	22968918.09	7.62833E-90	203.9115224
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	-64.11252341	1.52175E-05	-4213087.798	2.86715E-80	-64.11255629
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
203.9115608	203.9115224	203.9115608
0	0	0
-64.11249054	-64.11255629	-64.11249054
0	0	0

Appendix III

Sample calculation on checking Eq. 1

m =	82.92837862
intercept =	139.799044

Case	Comsol values $p'_1 - p'_2$	u_1	u_2	u_3	Eqn. values $\Delta p_{1\text{-}2}$	for Chekcing $(p'_1 - p'_2) - \Delta p_{1\text{-}2}$
1	175.9570	1	0.4360	0.5640	175.957061	-1.90011E-05
2	170.3180	1	0.3680	0.6320	170.3179313	2.27452E-05
3	164.6789	1	0.3000	0.7000	164.6788844	-1.94369E-05
4	159.0398	1	0.2320	0.7680	159.0397547	2.13095E-05
5	153.4007	1	0.1640	0.8360	153.4007079	-1.88726E-05
6	147.7616	1	0.0960	0.9040	147.7615781	2.18737E-05
7	142.1225	1	0.0280	0.9720	142.1225313	-1.93083E-05
12	181.5961	1	0.5040	0.4960	181.5961078	2.21809E-05
13	187.2352	1	0.5720	0.4280	187.2352376	-1.95654E-05
14	192.8743	1	0.6400	0.3600	192.8742844	2.26166E-05
15	198.5134	1	0.7080	0.2920	198.5134141	-2.01297E-05
16	204.1525	1	0.7760	0.2240	204.1524609	2.20523E-05
17	209.7916	1	0.8440	0.1560	209.7915907	-1.9694E-05
18	215.4307	1	0.9120	0.0880	215.4306375	2.24881E-05
19	221.0697	1	0.9800	0.0200	221.0697673	-1.92583E-05

Sample calculation on checking Eq. 4

Case	Comsol values $p'_1 - p'_3$	u'_1	u'_2	u'_3	Eqn. values $\Delta p_{1\text{-}3}$	for Chekcing $(p'_1 - p'_3) - \Delta p_{1\text{-}3}$
1	175.9575	1	0.4360	0.5640	175.9575197	1.52905E-05
2	180.3172	1	0.3680	0.6320	180.3171713	-1.73014E-05
3	184.6768	1	0.3000	0.7000	184.6767588	1.52191E-05
4	189.0364	1	0.2320	0.7680	189.0364104	-1.73729E-05
5	193.3960	1	0.1640	0.8360	193.3959979	1.51477E-05
6	197.7556	1	0.0960	0.9040	197.7556494	-1.74443E-05
7	202.1153	1	0.0280	0.9720	202.1152369	1.50763E-05
12	171.5979	1	0.5040	0.4960	171.5979322	-1.72300E-05
13	167.2383	1	0.5720	0.4280	167.2382806	1.53620E-05
14	162.8787	1	0.6400	0.3600	162.8786932	-1.71586E-05
15	158.5191	1	0.7080	0.2920	158.5190416	1.44334E-05
16	154.1594	1	0.7760	0.2240	154.1594541	-1.70872E-05
17	149.7998	1	0.8440	0.1560	149.7998025	1.55048E-05
18	145.4402	1	0.9120	0.0880	145.440215	-1.70157E-05
19	141.0806	1	0.9800	0.0200	141.0805634	1.45762E-05

Appendix IV

Raw and manipulated data, split flow case 3

? = 1 u'_1 = 1 d = 1

Mesh consists of 11504 elements.
Number of degrees of freedom solved for:56491

		Raw		Manipulated		
				$(p_i')(A_{cross})/(A_{cross})$	$(u_i')(A_{cross})/(A_{cross})$	
	Position	[N], Expression: p $(p_i')(A_{cross})$	[m^3/s], Expression: u $(u_i)(A_{cross})$	[Pa] p_i'	[m/s] u_i	$p'_3 - p'_1$
Case 1	1	0.001673	0.500054	0.001673	0.500054	188.0922
$P_1 = P_2 = 0$	2	0.005606	0.499946	0.005606	0.499946	
	3	188.093867	1.000000	188.093867	1	
Case 2	1	0.001823	0.544907	0.001823	0.544907	193.0915
$P_2 = 10, P_1 = 0$	2	10.005103	0.455093	10.005103	0.455093	
	3	193.093326	1.000000	193.093326	1	
Case 3	1	0.001973	0.589761	0.001973	0.589761	198.0908
$P_2 = 20, P_1 = 0$	2	20.0046	0.410239	20.0046	0.410239	
	3	198.092785	1.000000	198.092785	1	
Case 4	1	0.002123	0.634614	0.002123	0.634614	203.0901
$P_2 = 30, P_1 = 0$	2	30.004097	0.365386	30.004097	0.365386	
	3	203.092244	1.000000	203.092244	1	
Case 5	1	0.002274	0.679467	0.002274	0.679467	208.0894
$P_2 = 40, P_1 = 0$	2	40.003594	0.320533	40.003594	0.320533	
	3	208.091702	1.000000	208.091702	1	
Case 6	1	0.002424	0.72432	0.002424	0.72432	213.0887
$P_2 = 50, P_1 = 0$	2	50.003091	0.27568	50.003091	0.27568	
	3	213.091161	1.000000	213.091161	1	
Case 7	1	0.002574	0.769174	0.002574	0.769174	218.088
$P_2 = 60, P_1 = 0$	2	60.002588	0.230826	60.002588	0.230826	
	3	218.09062	1.000000	218.09062	1	
Case 8	1	10.001523	0.455201	10.001523	0.455201	183.0929
$P_2 = 0, P_1 = 10$	2	0.006109	0.544799	0.006109	0.544799	
	3	193.094408	1.000000	193.094408	1	
Case 9	1	20.001373	0.410348	20.001373	0.410348	178.0936
$P_2 = 0, P_1 = 20$	2	0.006612	0.589652	0.006612	0.589652	
	3	198.09495	1.000000	198.09495	1	
Case 10	1	30.001223	0.365494	30.001223	0.365494	173.0943
$P_2 = 0, P_1 = 30$	2	0.007115	0.634506	0.007115	0.634506	
	3	203.095491	1.000000	203.095491	1	
Case 11	1	40.001073	0.320641	40.001073	0.320641	168.095
$P_2 = 0, P_1 = 40$	2	0.007618	0.679359	0.007618	0.679359	
	3	208.096032	1.000000	208.096032	1	
Case 12	1	50.000923	0.275788	50.000923	0.275788	163.0957
$P_2 = 0, P_1 = 50$	2	0.008121	0.724212	0.008121	0.724212	
	3	213.096573	1.000000	213.096573	1	
Case 13	1	60.000773	0.230934	60.000773	0.230934	158.0963
$P_2 = 0, P_1 = 60$	2	0.008624	0.769066	0.008624	0.769066	
	3	218.097115	1.000000	218.097115	1	
Case 14	1	70.000623	0.186081	70.000623	0.186081	153.097
$P_2 = 0, P_1 = 70$	2	0.009127	0.813919	0.009127	0.813919	
	3	223.097656	1.000000	223.097656	1	
Case 15	1	80.000473	0.141228	80.000473	0.141228	148.0977

Appendix V

Summary of data regression, p'_{3-1} , split flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.846153846
Standard Error	3.41301E-05
Observations	15

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	188.0921856	8.38564E-06	0.272575349
2	193.0914637	3.93298E-05	1.278415176
3	198.0908532	-4.11853E-05	-1.33872735
4	203.0901312	-1.02411E-05	-0.332887524
5	208.0894093	1.8703E-05	0.607942265
6	213.0886874	4.96472E-05	1.613782092
7	218.0880769	-3.08679E-05	-1.003360435
8	183.0929076	-2.25585E-05	-0.733264476
9	178.0936295	-5.25026E-05	-1.706599284
10	173.09424	2.80124E-05	0.910543242
11	168.0949619	-2.93175E-06	-0.095296584
12	163.0956839	-3.38759E-05	-1.10113641
13	158.0962944	4.76391E-05	1.548511135
14	153.0970163	1.6695E-05	0.542671308
15	148.0977382	-1.42491E-05	-0.463168518

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	6998.064514	2332.688171	6.00762E+12	1.86724E-67
Residual	13	1.51433E-08	1.16487E-09		
Total	16	6998.064514			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	243.8157507	2.62949E-05	9272350.388	1.00876E-84	243.8156939
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	-111.4591679	4.54742E-05	-2451043.833	3.27863E-77	-111.4592661
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
243.8158076	243.8156939	243.8158076
0	0	0
-111.4590696	-111.4592661	-111.4590696
0	0	0

Appendix VI

Sample calculation on checking Eq. for p'_{3-1} , split flow

Case	$p'_3 - p'_1$	u'_1	u'_2	u'_3	Eqn. values Δp_{3-1}	for Chekcing $(p'_3 - p'_1) - \Delta p_{3-1}$
1	188.0922	0.5001	0.4999	1.0	188.0921856	8.385639E-06
2	193.0915	0.5449	0.4551	1.0	193.0914637	3.932978E-05
3	198.0908	0.5898	0.4102	1.0	198.0908532	-4.118525E-05
4	203.0901	0.6346	0.3654	1.0	203.0901312	-1.024111E-05
5	208.0894	0.6795	0.3205	1.0	208.0894093	1.870303E-05
6	213.0887	0.7243	0.2757	1.0	213.0886874	4.964717E-05
7	218.0880	0.7692	0.2308	1.0	218.0880769	-3.086786E-05
12	183.0929	0.4552	0.5448	1.0	183.0929076	-2.255850E-05
13	178.0936	0.4103	0.5897	1.0	178.0936295	-5.250264E-05
14	173.0943	0.3655	0.6345	1.0	173.09424	2.801239E-05
15	168.0950	0.3206	0.6794	1.0	168.0949619	-2.931750E-06
16	163.0957	0.2758	0.7242	1.0	163.0956839	-3.387589E-05
17	158.0963	0.2309	0.7691	1.0	158.0962944	4.763914E-05
18	153.0970	0.1861	0.8139	1.0	153.0970163	1.669500E-05
19	148.0977	0.1412	0.8588	1.0	148.0977382	-1.424914E-05

Appendix VII

Raw and manipulated data, join flow case 1

? = 1 u'_2 = 1 d = 1

Mesh consists of 4251 elements.
Number of degrees of freedom solved for:22195

		Raw		Manipulated			
				$(p_i')(A_{cross})/(A_{cross})$	$(u_i')(A_{cross})/(A_{cross})$		
	Position	[N], Expression: p $(p_i')(A_{cross})$	[m^3/s], Expression: u $(u_i)(A_{cross})$	[Pa] p_i'	[m/s] u_i	$p'_1 - p'_2$	$p'_3 - p'_2$
Case 1	1	212.329231	0.9	212.329231	0.9	212.31247	125.2676
$u'_1 = 0.9$ $u'_3 = 0.1$	2	0.016761	1	0.016761	1		
	3	125.284366	0.100000	125.284366	0.1		
Case 2	1	201.182745	0.8	201.182745	0.8	201.165984	138.5411
$u'_1 = 0.8$ $u'_3 = 0.2$	2	0.016761	1	0.016761	1		
	3	138.557874	0.200000	138.557874	0.2		
Case 3	1	190.036259	0.7	190.036259	0.7	190.019498	151.8146
$u'_1 = 0.7$ $u'_3 = 0.6$	2	0.016761	1	0.016761	1		
	3	151.831382	0.300000	151.831382	0.3		
Case 4	1	178.889773	0.6	178.889773	0.6	178.873012	165.0881
$u'_1 = 0.6$ $u'_3 = 0.4$	2	0.016761	1	0.016761	1		
	3	165.10489	0.400000	165.10489	0.4		
Case 5	1	167.743287	0.5	167.743287	0.5	167.726526	178.3616
$u'_1 = 0.5$ $u'_3 = 0.5$	2	0.016761	1	0.016761	1		
	3	178.378398	0.5	178.378398	0.5		
Case 6	1	156.596801	0.4	156.596801	0.4	156.58004	191.6351
$u'_1 = 0.4$ $u'_3 = 0.6$	2	0.016761	1	0.016761	1		
	3	191.651906	0.600000	191.651906	0.6		
Case 7	1	145.450316	0.3	145.450316	0.3	145.433555	204.9087
$u'_1 = 0.3$ $u'_3 = 0.7$	2	0.016761	1	0.016761	1		
	3	204.925413	0.7	204.925413	0.7		
Case 8	1	134.30383	0.2	134.30383	0.2	134.287069	218.1822
$u'_1 = 0.2$ $u'_3 = 0.8$	2	0.016761	1	0.016761	1		
	3	218.198921	0.800000	218.198921	0.8		
Case 9	1	123.157344	0.1	123.157344	0.1	123.140583	231.4557
$u'_1 = 0.1$ $u'_3 = 0.9$	2	0.016761	1	0.016761	1		
	3	231.472429	0.900000	231.472429	0.9		

Appendix VIII

Raw and manipulated data, join flow case 2

? = 1 u'_3 = 1 d = 1

Mesh consists of 4251 elements.
Number of degrees of freedom solved for:22195

		Raw		Manipulated			
				$(p_i')(A_{cross})/(A_{cross})$	$(u_i')(A_{cross})/(A_{cross})$		
		[N], Expression: p	[m^3/s], Expression: u	[Pa]	[m/s]		
	Position	$(p_i')(A_{cross})$	$(u_i)(A_{cross})$	p_i'	u_i	$p'_1 - p'_3$	p'_2 -
Case 1	1	233.521515	0.9	233.521515	0.9	233.456481	144.2
$u'_1 = 0.9$ $u'_2 = 0.1$	2	144.362218	0.1	144.362218	0.1		
	3	0.065034	1.000000	0.065034	1		
Case 2	1	222.375029	0.8	222.375029	0.8	222.309995	155.4
$u'_1 = 0.8$ $u'_2 = 0.2$	2	155.521293	0.2	155.521293	0.2		
	3	0.065034	1.000000	0.065034	1		
Case 3	1	211.228543	0.7	211.228543	0.7	211.163509	166.6
$u'_1 = 0.7$ $u'_2 = 0.6$	2	166.680368	0.3	166.680368	0.3		
	3	0.065034	1.000000	0.065034	1		
Case 4	1	200.082058	0.6	200.082058	0.6	200.017024	177.7
$u'_1 = 0.6$ $u'_2 = 0.4$	2	177.839443	0.4	177.839443	0.4		
	3	0.065034	1.000000	0.065034	1		
Case 5	1	188.935572	0.5	188.935572	0.5	188.870538	188.9
$u'_1 = 0.5$ $u'_2 = 0.5$	2	188.998518	0.5	188.998518	0.5		
	3	0.065034	1.000000	0.065034	1		
Case 6	1	177.789086	0.4	177.789086	0.4	177.724052	200.0
$u'_1 = 0.4$ $u'_2 = 0.6$	2	200.157593	0.6	200.157593	0.6		
	3	0.065034	1.000000	0.065034	1		
Case 7	1	166.6426	0.3	166.6426	0.3	166.577566	211.2
$u'_1 = 0.3$ $u'_2 = 0.7$	2	211.316668	0.7	211.316668	0.7		
	3	0.065034	1.000000	0.065034	1		
Case 8	1	155.496114	0.2	155.496114	0.2	155.43108	222.4
$u'_1 = 0.2$ $u'_2 = 0.8$	2	222.475743	0.8	222.475743	0.8		
	3	0.065034	1.000000	0.065034	1		
Case 9	1	144.349628	0.1	144.349628	0.1	144.284594	233.5
$u'_1 = 0.1$ $u'_2 = 0.9$	2	233.634818	0.9	233.634818	0.9		
	3	0.065034	1.000000	0.065034	1		

Appendix IX

Raw and manipulated data, join flow case 3

? = 1 u'$_1$ = 1 d = 1

Mesh consists of 4251 elements.
Number of degrees of freedom solved for:22195

	Position	Raw		Manipulated			
				$(p_i')(A_{cross})/(A_{cross})$	$(u_i')(A_{cross})/(A_{cross})$		
		[N], Expression: p	[m^3/s], Expression: u	[Pa]	[m/s]		
		$(p_i')(A_{cross})$	$(u_i)(A_{cross})$	p_i'	u_i	$p'_3 - p'_1$	$p'_2 - p'_1$
Case 1	1	0.026643	1.0	0.026643	1	231.353154	123.050
'$_3$ = 0.9 u'$_2$ = 0.1	2	123.077301	0.1	123.077301	0.1		
	3	231.379797	0.9	231.379797	0.9		
Case 2	1	0.026643	1.0	0.026643	1	218.079646	134.209
'$_3$ = 0.8 u'$_2$ = 0.2	2	134.236376	0.2	134.236376	0.2		
	3	218.106289	0.8	218.106289	0.8		
Case 3	1	0.026643	1.0	0.026643	1	204.806139	145.368
'$_3$ = 0.7 u'$_2$ = 0.6	2	145.395451	0.3	145.395451	0.3		
	3	204.832782	0.7	204.832782	0.7		
Case 4	1	0.026643	1.0	0.026643	1	191.532631	156.527
'$_3$ = 0.6 u'$_2$ = 0.4	2	156.554526	0.4	156.554526	0.4		
	3	191.559274	0.6	191.559274	0.6		
Case 5	1	0.026643	1.0	0.026643	1	178.259123	167.68
'$_3$ = 0.5 u'$_2$ = 0.5	2	167.713602	0.5	167.713602	0.5		
	3	178.285766	0.5	178.285766	0.5		
Case 6	1	0.026643	1.0	0.026643	1	164.985615	178.84
'$_3$ = 0.4 u'$_2$ = 0.6	2	178.872677	0.6	178.872677	0.6		
	3	165.012258	0.4	165.012258	0.4		
Case 7	1	0.026643	1.0	0.026643	1	151.712107	190.005
'$_3$ = 0.3 u'$_2$ = 0.7	2	190.031752	0.7	190.031752	0.7		
	3	151.73875	0.3	151.73875	0.3		
Case 8	1	0.026643	1.0	0.026643	1	138.438599	201.164
'$_3$ = 0.2 u'$_2$ = 0.8	2	201.190827	0.8	201.190827	0.8		
	3	138.465242	0.2	138.465242	0.2		
Case 9	1	0.026643	1.0	0.026643	1	125.165091	212.323
'$_3$ = 0.1 u'$_2$ = 0.9	2	212.349902	0.9	212.349902	0.9		
	3	125.191734	0.1	125.191734	0.1		

Appendix X-A

Summary of data regression, p'_{1-2} , join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	3.04725E-07
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	212.3124697	2.66667E-07	1.045952808
2	201.1659839	1.16667E-07	0.457604493
3	190.019498	-3.33333E-08	-0.130744045
4	178.8730122	-1.83333E-07	-0.71909236
5	167.7265263	-3.33333E-07	-1.307440787
6	156.5800405	-4.83333E-07	-1.895789213
7	145.4335546	3.66667E-07	1.438185055
8	134.2870688	2.16667E-07	0.84983674
9	123.1405829	6.66667E-08	0.261488313

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	7454.648808	2484.882936	8.02808E+16	4.00057E-42
Residual	7	6.5E-13	9.28571E-14		
Total	10	7454.648808			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	223.4589556	2.21377E-07	1009402926	2.47356E-61	223.4589551
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	0	0	65535	#NUM!	0
X Variable 3	-111.4648585	3.93398E-07	-283338727.3	1.8015E-57	-111.4648594

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
223.4589561	223.4589551	223.4589561
0	0	0
0	0	0
-111.4648576	-111.4648594	-111.4648576

Appendix X-B

Summary of data regression, p'_{3-2} , join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	3.04725E-07
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	125.2676053	-2.66667E-07	-1.045952843
2	138.5411131	-1.16667E-07	-0.457604372
3	151.814621	3.33333E-08	0.130744043
4	165.0881288	1.83333E-07	0.719092457
5	178.3616367	3.33333E-07	1.307440872
6	191.6351445	4.83333E-07	1.895789287
7	204.9086524	-3.66667E-07	-1.438185026
8	218.1821602	-2.16667E-07	-0.8498365
9	231.4556681	-6.66666E-08	-0.261488085

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	10571.16064	3523.720213	1.13843E+17	1.67064E-42
Residual	7	6.5E-13	9.28571E-14		
Total	10	10571.16064			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	111.9940974	2.21377E-07	505896793.9	3.11417E-59	111.9940969	111.9940979	111.9940969	111.9940979
X Variable 1	0	0	65535	#NUM!	0	0	0	0
X Variable 2	0	0	65535	#NUM!	0	0	0	0
X Variable 3	132.7350785	3.93398E-07	337406682.2	5.30517E-58	132.7350776	132.7350794	132.7350776	132.7350794

Appendix X-C

Summary of data regression, p'_{1-3} , join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	3.04725E-07
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	233.4564811	-6.66667E-08	-0.261488205
2	222.3099952	-2.16667E-07	-0.849836526
3	211.1635094	-3.66667E-07	-1.438184959
4	200.0170235	4.83333E-07	1.895789345
5	188.8705377	3.33333E-07	1.307440912
6	177.7240518	1.83333E-07	0.719092479
7	166.577566	3.33333E-08	0.130744047
8	155.4310801	-1.16667E-07	-0.457604386
9	144.2845943	-2.66667E-07	-1.045952708

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	7454.648808	2484.882936	8.02808E+16	4.00057E-42
Residual	7	6.5E-13	9.28571E-14		
Total	10	7454.648808			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	244.6029669	2.21377E-07	1104914097	1.31364E-61	244.6029664
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	-111.4648585	3.93398E-07	-283338726.9	1.8015E-57	-111.4648594
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
244.6029674	244.6029664	244.6029674
0	0	0
-111.4648576	-111.4648594	-111.4648576
0	0	0

Appendix X-D

Summary of data regression, $p'_{2\text{-}3}$, join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	1.3113E-14
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	144.297184	0	0
2	155.456259	0	0
3	166.615334	-2.84217E-14	-1.290994449
4	177.774409	-2.84217E-14	-1.290994449
5	188.933484	-2.84217E-14	-1.290994449
6	200.092559	-2.84217E-14	-1.290994449
7	211.251634	-2.84217E-14	-1.290994449
8	222.410709	0	0
9	233.569784	-2.84217E-14	-1.290994449

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	7471.497291	2490.499097	4.34511E+31	5.87016E-79
Residual	7	1.20366E-27	1.71952E-28		
Total	10	7471.497291			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	133.138109	9.5264E-15	1.39757E+16	2.5361E-111	133.138109
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	111.59075	1.69289E-14	6.59174E+15	4.884E-109	111.59075
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
133.138109	133.138109	133.138109
0	0	0
111.59075	111.59075	111.59075
0	0	0

Appendix X-E

Summary of data regression, p'_{3-1} , join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	3.24893E-07
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	231.3531543	-3.11111E-07	-1.144526848
2	218.0796464	-4.27778E-07	-1.573724338
3	204.8061385	4.55556E-07	1.675914157
4	191.5326307	3.38889E-07	1.246716562
5	178.2591228	2.22222E-07	0.817518968
6	164.9856149	1.05556E-07	0.388321583
7	151.712107	-1.11111E-08	-0.040876011
8	138.4385991	-1.27778E-07	-0.470073605
9	125.1650912	-2.44444E-07	-0.899271147

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	10571.16069	3523.720231	1.00148E+17	2.3017E-42
Residual	7	7.38889E-13	1.05556E-13		
Total	10	10571.16069			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	244.6266622	2.36029E-07	1036424528	2.05594E-61	244.6266616
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	-132.7350788	4.19435E-07	-316461425.7	8.30864E-58	-132.7350798
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
244.6266628	244.6266616	244.6266628
0	0	0
-132.7350778	-132.7350798	-132.7350778
0	0	0

Appendix X-F

Summary of data regression, $p'_{2\text{-}1}$, join flow

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	0.714285714
Standard Error	2.81718E-07
Observations	9

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	123.0506579	1.11111E-07	0.471404435
2	134.2097331	-5.55556E-08	-0.235702338
3	145.3688082	-2.22222E-07	-0.942809111
4	156.5278834	-3.88889E-07	-1.649915884
5	167.6869586	4.44444E-07	1.885617981
6	178.8460337	2.77778E-07	1.178511329
7	190.0051089	1.11111E-07	0.471404435
8	201.1641841	-5.55555E-08	-0.235702218
9	212.3232592	-2.22222E-07	-0.942809111

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	7471.497515	2490.499172	9.41409E+16	2.68662E-42
Residual	7	5.55556E-13	7.93651E-14		
Total	10	7471.497515			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	111.8915827	2.04663E-07	546710149.6	1.80915E-59	111.8915822
X Variable 1	0	0	65535	#NUM!	0
X Variable 2	111.5907517	3.63696E-07	306823837.8	1.0317E-57	111.5907508
X Variable 3	0	0	65535	#NUM!	0

Upper 95%	*Lower 95.0%*	*Upper 95.0%*
111.8915832	111.8915822	111.8915832
0	0	0
111.5907525	111.5907508	111.5907525
0	0	0

Appendix XI

Sample calculation on checking equations

Case 1

	for 1 - 2	for 3 - 2
m =	-111.4648585	132.7350785
intercept =	223.4589556	111.9940974

Case	$p'_1 - p'_2$	$p'_3 - p'_2$	Comsol values u'_1	u'_2	u'_3
1	212.3125	125.2676	0.9	1.0	0.1
2	201.1660	138.5411	0.8	1.0	0.2
3	190.0195	151.8146	0.7	1.0	0.3
4	178.8730	165.0881	0.6	1.0	0.4
5	167.7265	178.3616	0.5	1.0	0.5
6	156.5800	191.6351	0.4	1.0	0.6
7	145.4336	204.9087	0.3	1.0	0.7
8	134.2871	218.1822	0.2	1.0	0.8
9	123.1406	231.4557	0.1	1.0	0.9

Eqn. values $\Delta'p_{1-2}$	for Chekcing $(p'_1 - p'_2) - \Delta'p_{1-2}$	Eqn. values $\Delta'p_{3-2}$	for Chekcing $(p'_3 - p'_2) - \Delta'p_{3-2}$
212.3124697	0.0000	125.2676053	-2.66667E-07
201.1659839	1.16667E-07	138.5411131	-1.16667E-07
190.019498	-3.33333E-08	151.814621	3.33333E-08
178.8730122	-1.83333E-07	165.0881288	1.83333E-07
167.7265263	-3.33333E-07	178.3616367	3.33333E-07
156.5800405	-4.83333E-07	191.6351445	4.83333E-07
145.4335546	3.66667E-07	204.9086524	-3.66667E-07
134.2870688	2.16667E-07	218.1821602	-2.16667E-07
123.1405829	6.66667E-08	231.4556681	-6.66666E-08

Case 2

	for 1 - 3	for 2 - 3
m =	-111.4648585	111.59075
intercept =	244.6029669	133.138109

	Comsol values				
Case	$p'_1 - p'_3$	$p'_2 - p'_3$	u'_1	u'_2	u'_3
1	233.4565	144.2972	0.9	0.1	1.0
2	222.3100	155.4563	0.8	0.2	1.0
3	211.1635	166.6153	0.7	0.3	1.0
4	200.0170	177.7744	0.6	0.4	1.0
5	188.8705	188.9335	0.5	0.5	1.0
6	177.7241	200.0926	0.4	0.6	1.0
7	166.5776	211.2516	0.3	0.7	1.0
8	155.4311	222.4107	0.2	0.8	1.0
9	144.2846	233.5698	0.1	0.9	1.0

Eqn. values $\Delta p'_{1-3}$	for Chekcing $(p'_1 - p'_3) - \Delta' p_{1-3}$	Eqn. values $\Delta p'_{2-3}$	for Chekcing $(p'_2 - p'_3) - \Delta p'_{2-3}$
233.4564811	0.0000	144.297184	0.0000E+00
222.3099952	-2.16667E-07	155.456259	0.0000E+00
211.1635094	-3.66667E-07	166.615334	0.0000E+00
200.0170235	4.83333E-07	177.774409	0.0000E+00
188.8705377	3.33333E-07	188.933484	0.0000E+00
177.7240518	1.83333E-07	200.092559	0.0000E+00
166.577566	3.33333E-08	211.251634	0.0000E+00
155.4310801	-1.16667E-07	222.410709	0.0000E+00
144.2845943	-2.66667E-07	233.569784	0.0000E+00

Case 3

	for 3 - 1	for 2 - 1
m =	-132.7350788	111.5907517
intercept =	244.6266622	111.8915827

			Comsol values		
Case	$p'_3 - p'_1$	$p'_2 - p'_1$	u'_1	u'_2	u'_3
1	231.3532	123.0507	1	0.1	0.9
2	218.0796	134.2097	1	0.2	0.8
3	204.8061	145.3688	1	0.3	0.7
4	191.5326	156.5279	1	0.4	0.6
5	178.2591	167.6870	1	0.5	0.5
6	164.9856	178.8460	1	0.6	0.4
7	151.7121	190.0051	1	0.7	0.3
8	138.4386	201.1642	1	0.8	0.2
9	125.1651	212.3233	1	0.9	0.1

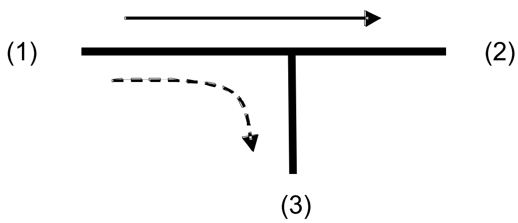
Eqn. values $\Delta p'_{3-1}$	for Chekcing $(p'_3 - p'_1) - \Delta' p_{3-1}$	Eqn. values $\Delta p'_{2-1}$	for Chekcing $(p'_2 - p'_1) - \Delta' p_{2-1}$
231.3531543	0.0000	123.0506579	1.11111E-07
218.0796464	-4.27778E-07	134.2097331	-5.55556E-08
204.8061385	4.55556E-07	145.3688082	-2.22222E-07
191.5326307	3.38889E-07	156.5278834	-3.88889E-07
178.2591228	2.22222E-07	167.6869586	4.44444E-07
164.9856149	1.05556E-07	178.8460337	2.77778E-07
151.712107	-1.11111E-08	190.0051089	1.11111E-07
138.4385991	-1.27778E-07	201.1641841	-5.55555E-08
125.1650912	-2.44444E-07	212.3232592	-2.22222E-07

Appendix XI

List of correlation for split and join flows

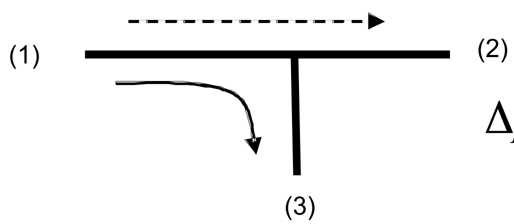
Split flow

Case1



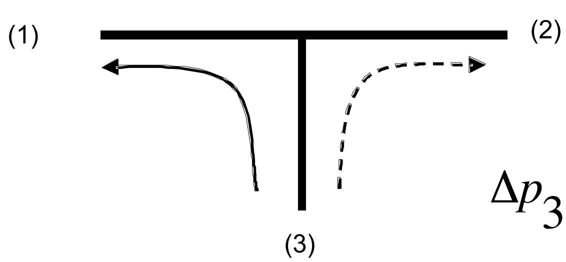
$$\Delta p_{12} = \frac{\eta < v >_1}{D}(82.928u_2' + 139.799)$$

Case 2



$$\Delta p_{13} = \frac{\eta < v >_1}{D}(-64.113u_2' + 203.912)$$

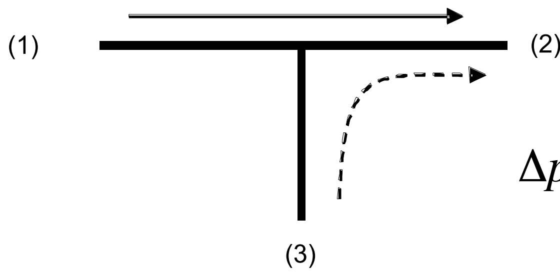
Case 3



$$\Delta p_{31} = \frac{\eta < v >_1}{D}(-111.459u_2' + 243.816)$$

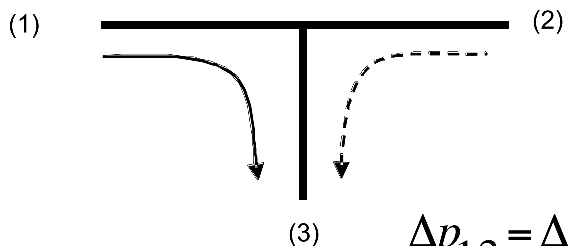
Joining flow

Case 1



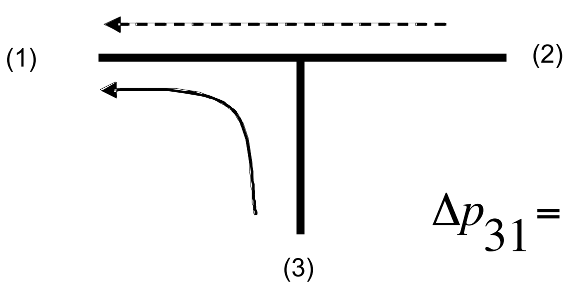
$$\Delta p_{12} = \frac{\eta \langle v \rangle_2}{D}(-111.465 u_2' + 334.459)$$

Case 2



$$\Delta p_{13} = \Delta p_{23} = \frac{\eta \langle v \rangle_3}{D}(-111.465 u_2' + 244.603)$$

Case 3



$$\Delta p_{31} = \frac{\eta \langle v \rangle_1}{D}(-132.735 u_2' + 244.627)$$