Mixing Properties of a Microfluidic Device

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Introduction

This project is aimed analyzing the flow in a specific microfluidic mixing device shown in Fig. 1. The goal is to find any properties and correlations that would be useful for analyzing the mixing properties of the fluids flowing through. COMSOL Multiphysics ^{TM} will be used to assist in modeling the device in order to analyze this device and propose ways to simplify the model into more easily solvable systems.

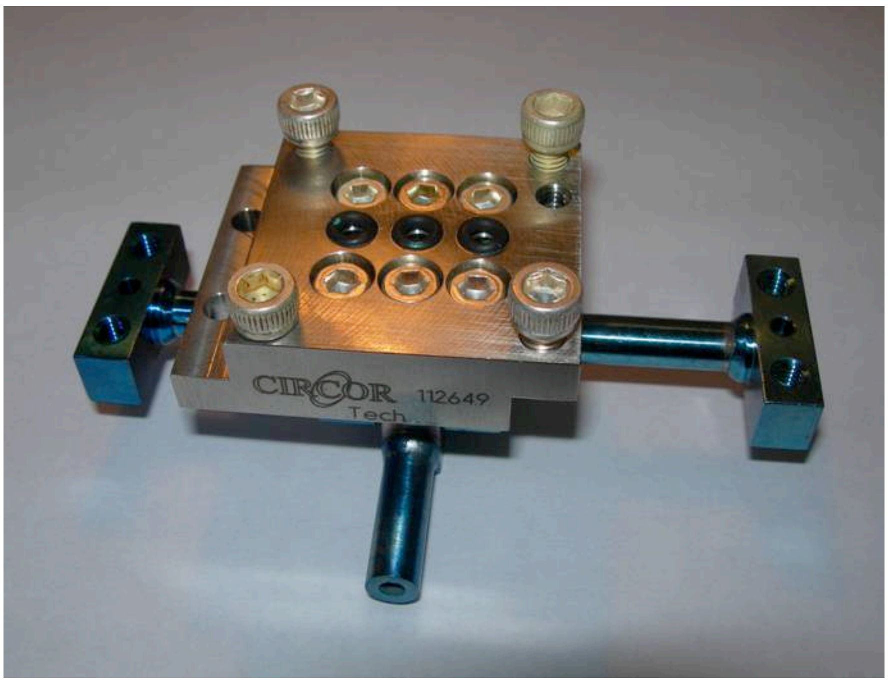

Figure 1 A photograph of the microfluidic mixing device shown without a mixing chamber attached. The two outer black rings are the input pipes and the middle ring is the outlet pipe.

To solve this problem a steady state incompressible Navier-Stokes Equation for momentum balance and a steady state convection and diffusion analysis will have to be applied to describe the flow and mixing characteristics of the system. A non-dimensional analysis will assist in simplifying the problem and allow trends based on dimensionless quantities to be found.

Objectives

The primary objective of this project is to evaluate the properties of flow through a microfluidic device and find any trends and correlations that may be associated with a device of this type. Also, it would be desirable to see if the current model geometry could be simplified in any ways to make calculation and analysis easier.

One main way of finding trends for different fluids and conditions for this problem is to use dimensionless quantities. The Reynolds number (Eq. (1)) is useful for describing the flow in the tube while the rate of diffusion can be described using the Peclet number (Eq. (2)), which can also be written in terms of the Reynolds number. A complete list of variables can be found in Appendix A. By setting a number of the variables equal to 1 (i.e. velocity (v), characteristic length (L), etc.), these non-dimensional quantities can be set to desired values in order to find correlations.

$$\mathrm{Re} = \frac{L\langle v\rangle\rho}{\eta} \tag{1}$$

$$Pe = \frac{\langle v\rangle L}{D} = \frac{\mathrm{Re}\cdot\eta}{D\cdot\rho} \tag{2}$$

The main goal in a mixing device is to get the two inlet fluids to be as well mixed as possible at the outlet. In order to do this, there must be some way to measure how well mixed the fluids are. The variance can test how far from fully mixed a cross-section of the pipe is and is given by Eq. (3).

$$c_{\mathrm{var}iance} = \frac{\int_A \left[c - c_{mixing\,cup}\right]^2 \cdot v \cdot dA}{\int_A v \cdot dA} \tag{3}$$

The mixing cup concentration ($c_{mixing\ cup}$) is the average concentration at any given point throughout a pipe, shown by Eq. (4). For this experiment the mixing cup concentration for the outlet pipe will be 0.5 because the inlets have a concentration of 0 and 1 with equal flow rates.

$$c_{mixing\,cup} = \frac{\int_A c \cdot v \cdot dA}{\int_A v \cdot dA} \tag{4}$$

In addition, other properties of the mixing device will be examined such as different inlet flow rates and its effect on the concentration. This situation might arise when it would be desirable to have an average outlet concentration different than 0.5. For this it would be helpful to know how much the inlet pressures can vary while still getting flow into both inlet pipes. In other words, with defined pressures into one inlet pipe and the outlet pipe, what does the pressure have to be to get no flow through the other inlet pipe?

Methods

This problem will first be analyzed in two dimensions, with flow trends and correlations being examined. The model will then be reexamined in three dimensions to see if the two dimensional analysis holds. Along the way, the model should be simplified if a way to reduce the size of the model is found without changing the results. This way a more in depth analysis can be done on the portion of the model where the significant events are occurring.

It is important to note that many simplifications are made to this model before the project is done. The device itself does not have a mixing chamber attached to it so it is approximated as a T-sensor shape. Also, the flow in the mixer is assumed to have Laminar flow. Mixing would clearly be different if there is turbulent flow in the pipes. The fluid must also be assumed to be Newtonian and incompressible.

Results and Discussion

The model is first approximated as flat plates in two dimensions with equal flow rates at each side. The inlet velocity can be specified by Eq. (5) where the fluid comes in fully developed. The average velocity will be 2/3 of the coefficient in front; it is set to 1 in this case.

$$v = 1.5\left(1 - \left(\frac{y}{0.5}\right)^2\right) \tag{5}$$

For the analysis of different inlet flow rates, a stop pressure ratio must be found for different Reynolds numbers. Figure 2 shows the ratio of inlet pressures where the lower pressure is the inlet pipe with no flow. For this experiment the high inlet pressure is constant at 1000 and the outlet pressure at 0.

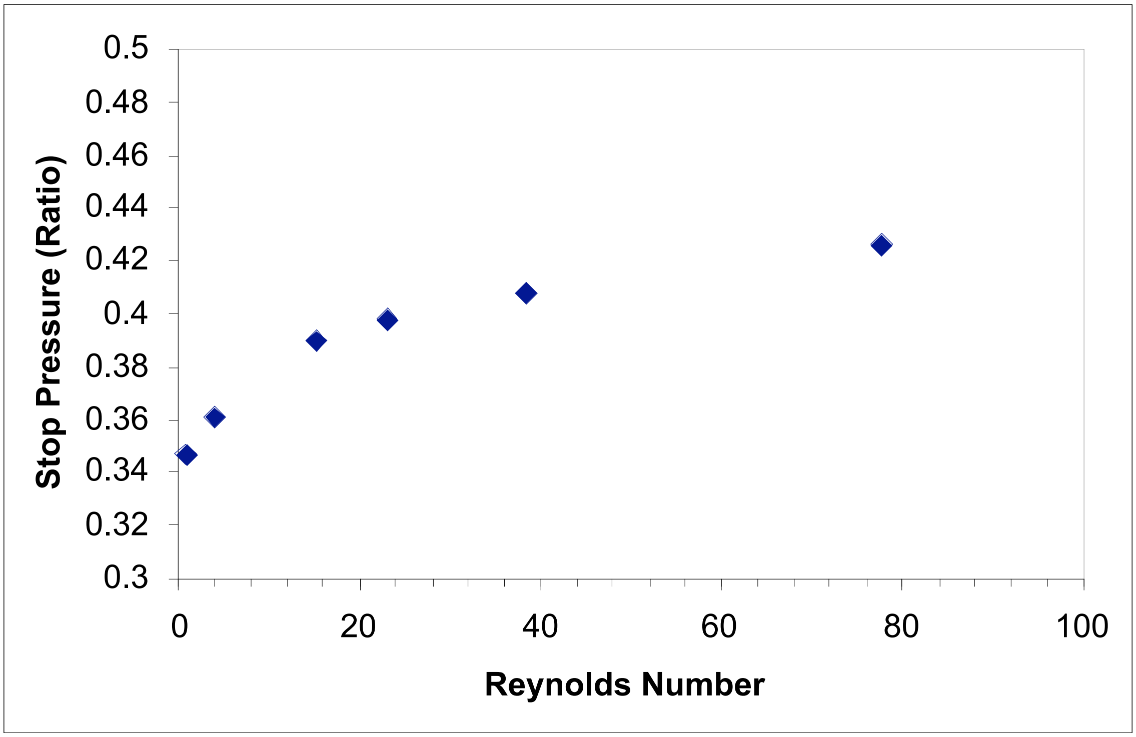

Figure 2 This graph shows the ratio of inlet pressures for a T-sensor where the inlet pipe with the lower pressure has no flow. The high inlet pressure and outlet pressure remain constant at p=1000 and p=0, respectively.

This graph clearly shows that there is a distinct correlation between the stop pressure and the Reynolds number, but that it lies in between 0.45 and 0.35. This means that as long as this ratio is kept high enough it will be applicable for all Reynolds numbers.

A model for convection can diffusion is then applied once the momentum balance has been adequately solved for. With a Peclet number of 50, the model is represented by Fig. (3).

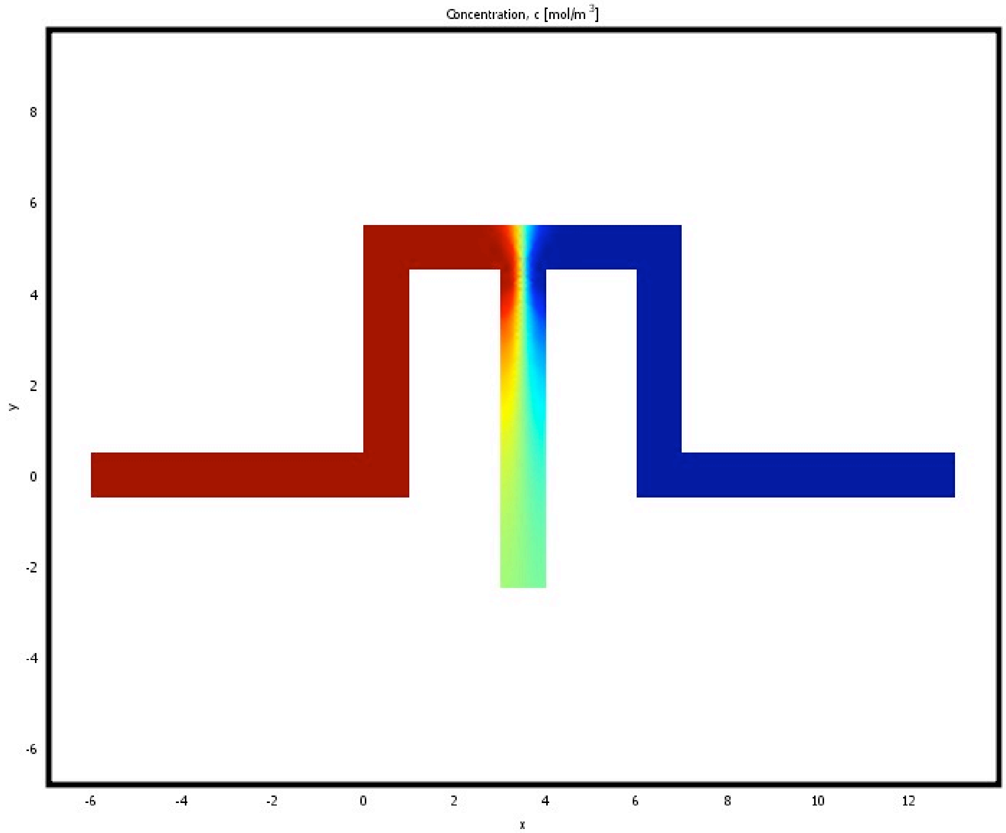

Figure 3 A plot of concentration for a Peclet number of 50 in two dimensions. The inlet concrentration are 1 (left) and 0 (right). The inlet flows are equal for each side.

At this stage it is apparent that the model could be approximated as a T-sensor only, without the extremities. This is because there are no important mixing events further out in the inlet pipes and that the model can be shortened. This will make the three dimensional model much easier to model and solve.

In calculating the variance in the outlet pipe for a wide range of Peclet numbers, a boundary integration is performed to evaluate Eq. (3) for different distances along the outlet pipe. This variance is then plotted against the distance from the joint over the Peclet number to see any trends (Fig. 4). The reason for plotting (z/Pe) on the x axis is that there is a clear correlation between variance at a given point and the Peclet number, and without this correction the plot at each Peclet number would clearly have different variance trends.

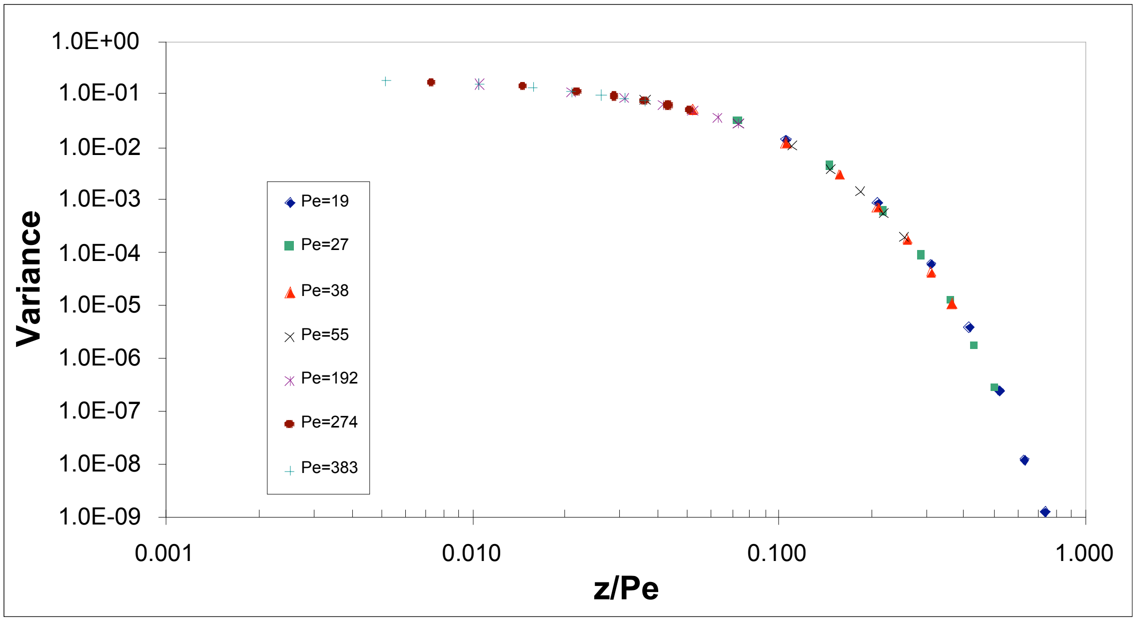

Figure 4 A plot of the variance versus the distance from the joint over the Peclet number for two dimensions. The shape of this plot is independent of Peclet number.

The next step is to update the model into three dimensional circular pipes. The new model must be analyzed for changes from the two dimensional model. Figure 5 shows that the primary difference when the same problem is resolved in three dimensions is that the edge effects where the pipe is rounded has more diffusion than the center, yielding a rounded concentration cross section.

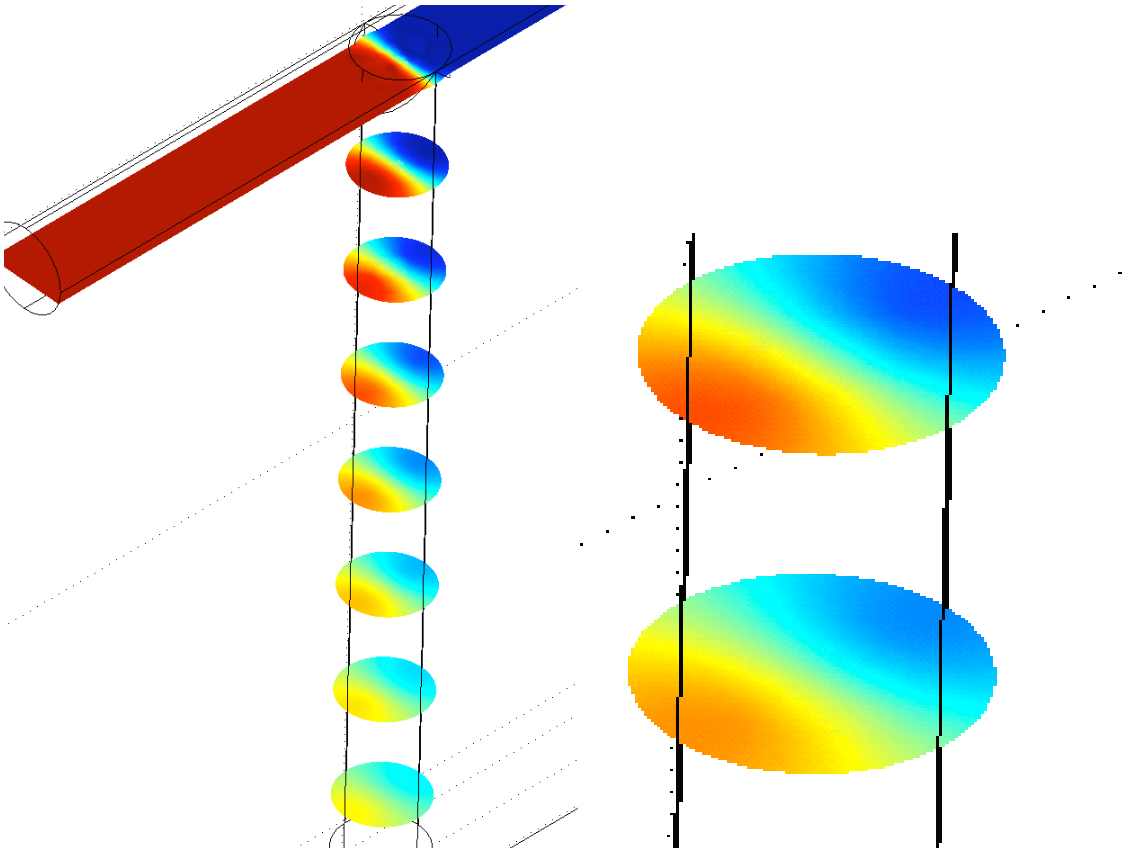

Figure 5 These cross sections of the 3D mixing model show that the amount of diffusion is greater at the sides of the pipe due to the slower flow rate. The Peclet number in this case is 50.

Despite the identified differences the only way to see if the same correlation from two dimensions (Fig. 4) holds for three dimensions is to repeat the variance calculation and compare them. The variance is again plotted against the distance over the Peclet number and shown in Fig. 6. This plot appears to have the same shape as the two dimensional case, indicating that the same correlation would hold for both cases.

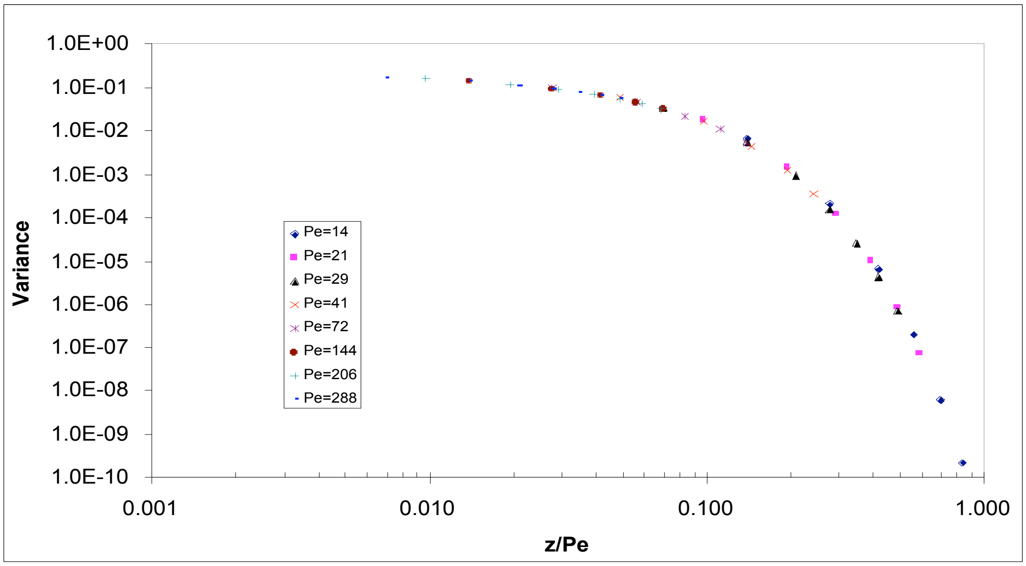

Figure 6 A plot of the variance versus the distance from the joint over the Peclet number for three dimesnions. It is clear that the shape of this plot is similar in shape as the two dimensional plot.

Further comparison is done by plotting both Fig. 4 and Fig. 6 on the same graph (Fig. 7). This new plot shows undoubtedly that both the two dimensional and three dimensional cases are identical in the way they mix and implies that the differences in geometry between the two are inconsequential.

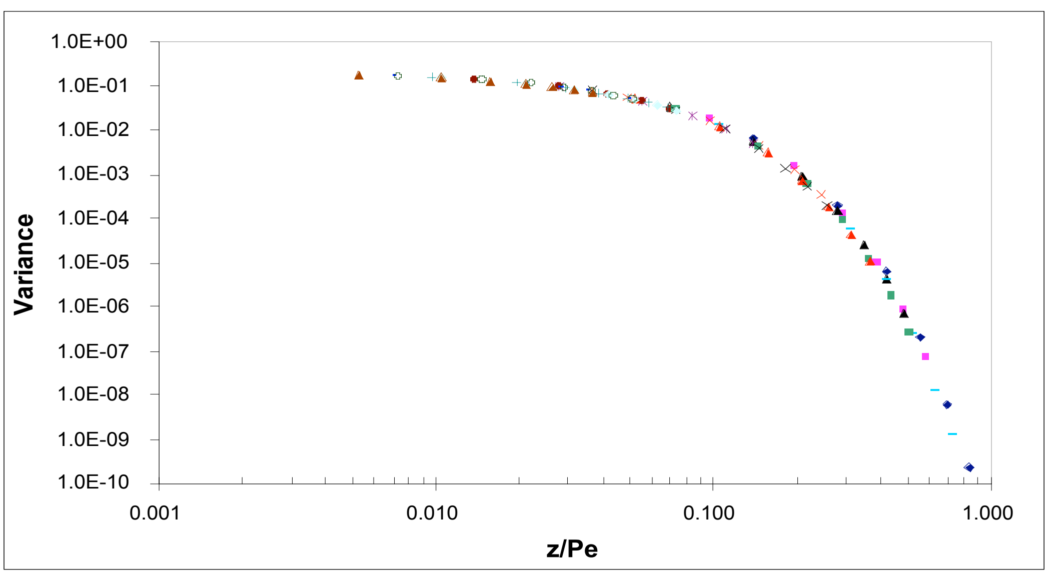

Figure 7 Both the two and three dimensional correlations of Fig. 4 and Fig. 6. The overlap shows that the correlation is holds for both dimensions and the problem can be studied in either.

Conclusions and Recommendations

The main conclusion that can be made is that the trend for variance in a T-sensor in three dimensional circular pipes can be successfully approximated as two dimensional flat plates. This is a valuable conclusion because it will allow this problem to be studied in an easier two dimensional geometry and be applied into a more complicated three dimensional geometry.

It can also be concluded that the variance for this experiment is only a function of the distance (z) and the Peclet number. The Peclet number is a function of many parameters but this correlation will allow the amount of mixing to be found at a given distance from the joining of the inlet pipes when the flow conditions and fluid properties are known.

This trend can be confirmed by a non-dimensional mass balance on the system for the diffusion as given by Eq. (6) shows that the concentration varies with z'/Pe.

$$v' \cdot \left(\frac{\partial c'}{\partial (z'/Pe)} \right) = \left(\frac{\partial^2 c'}{\partial x'^2} + \frac{\partial^2 c'}{\partial y'^2} \right) \tag{6}$$

The next step for this problem might be to model the mixing device being analyzed in more dimensional units and choose a different shape for the mixing compartment that would yield a more completely mixed outlet product.