Mesh refinement analysis for a standard 3:1 contraction

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Introduction

The purpose of this project is to investigate the effects of mesh refinement for a standard 3:1 contraction. More specifically how increasing mesh refinement (or decrease in mesh element size) correlates to error in excess pressure drop. Figure 1, to the right shows visually how increased mesh refinement looks for a 3:1 contraction case. The 9 refinements shown to the right reflect 9 preset mesh refinement settings within COSMOL Multiphysics software package.

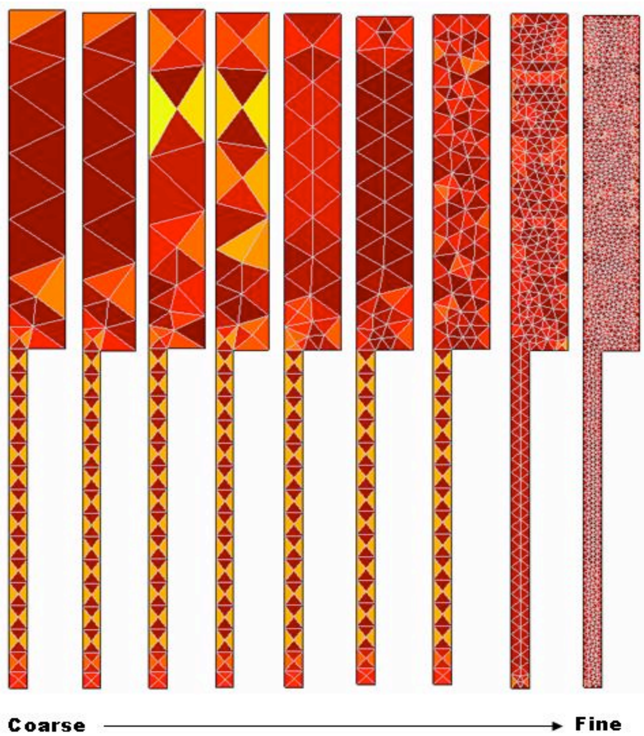

Figure 1. *Spectrum of mesh refinement presets in COSMOL Multiphysics software.*

These 9 presets will be used at different Reynolds numbers for a 3:1 contraction as a base case for this mesh refinement analysis. The hope is that the trends that are discovered for this simple base case will be replicated in more complex geometries and problems.

Materials and Methods

As mentioned before the COSMOL Multiphysics (formerly FemLAB) software package is used for the computer simulations for this research. There are some specific settings used for this research: The length of the small channel and the large channel is 10 units while the width of the large channel is 1.5 units and the width of the small channel is 0.5 units. The boundary conditions used are as follows (look at figure 1 for reference): the boundaries on the left-hand side (on the symmetry line) are slip symmetry, the boundaries on the right-hand side are no-slip conditions, and the small channel outlet is normal flow/pressure while the big channel inlet is inflow/outflow velocity.

The specific settings used for the 9 preset refinement methods are shown in the table below:

	Max element size scaling factor	Element growth rate	Mesh curve factor	Mesh curvature cut off
Extremely Coarse	5.00	2.00	1.00	0.0500
Extra Coarse	3.00	1.80	0.80	0.0200
Coarser	1.90	1.50	0.60	0.0100
Coarse	1.50	1.40	0.40	0.0050
Normal	1.00	1.30	0.30	0.0010
Fine	0.80	1.30	0.30	0.0010
Finer	0.55	1.25	0.25	0.0005
Extra Fine	0.30	1.20	0.25	0.0003
Extremely Fine	0.15	1.20	0.20	0.0001

Table 1. M*esh refinement statistics for the nine presets in COSMOL Multiphysics.*

The variable that will be used for most of the analysis in this research is the "Max element size scaling factor". Note that this factor decreases with increasing mesh refinement. Some other setting that were used, mainly to assist in the convergence of the solution are as follows: for low Reynolds numbers η = 1, ρ = Re and for high Reynolds numbers η = 1/10^x , ρ = 1 and where x = log Re.

Upon convergence of a solution in the Navier-Stokes equation for incompressible fluids, using the boundary integration can be done within COSMOL Multiphysics (go to "Post-processing" > "Boundary integration"). Integration is done on boundary 2 at the outlet to get the value 0.785398. Integration is done again with the input "p/ 0.785398" to determine the pressure at the outlet. This procedure is also done at the inlet, where on boundary 4 we get the value 7.068583. Integration is done with the following input "p/ 7.068583" to determine the pressure at the inlet. The difference of pressure at the inlet and outlet is the total pressure drop.

The procedure above is done for each of the nine presets for the following Reynolds numbers: 0, 1, 5, 10, 50, 70 and 100. After finding the pressure at the small outlet and the large inlet we can use the following equations to determine the excess pressure drop for a 3:1 contraction via MS Excel.

$$\Delta P = \frac{32 L \eta v}{D^2}$$

The equation above is used to determine the pressure drop for fully developed flow in the small and large regions. The following equation was used to determine the excess pressure drop:

$$\Delta P_{excess} = \Delta P_{total} - \Delta P_{small} - \Delta P_{l\arg e}$$

Note that the excess pressure drop is equivalent to K_L. After calculating the corresponding K_L values for each of the nine refinement presets we graphed K_L vs. Max element size scaling factor. We then we did a linear fit on the last three data points representing Finer, Extra Fine and Extremely Fine refinement settings and found the corresponding linear equation. The y-intercept of the linear equation is the “true” value or the value when the max element size scaling factor approaches zero. With this “true value” we were able to calculate the error associated with each of the K_L values calculated for each preset.

Results and Discussion

This section will highlight the results obtained for a Reynolds number of zero. To see the calculations and analysis of other Reynolds numbers please refer to the accompanying MS Excel spreadsheet. Below is the table highlighting different variables and setting for different presets for Re=0.

	Elements	DOF	Time	P_2	P_4	P_T	P_L	P_S	K_L
Extremely Coarse	81	485	0.703	-3.437E-11	331.599	331.599	3.951	320.00	7.6483
Extra Coarse	81	485	0.672	-3.437E-11	331.597	331.597	3.951	320.00	7.6460
Coarser	88	514	0.703	-3.321E-11	331.457	331.457	3.951	320.00	7.5066
Coarse	90	530	0.672	-3.405E-11	331.551	331.551	3.951	320.00	7.6007
Normal	103	594	0.688	-2.572E-11	331.594	331.594	3.951	320.00	7.6433
Fine	120	683	0.688	-2.520E-11	331.615	331.615	3.951	320.00	7.6647
Finer	191	1020	0.765	-2.678E-11	331.588	331.588	3.951	320.00	7.6371
Extra Fine	503	2539	0.985	-2.263E-15	332.037	332.037	3.951	320.00	8.0861
Extremely Fine	2134	10146	2.563	-9.363E-15	332.360	332.360	3.951	320.00	8.4095

Table 2. Table of results for different presets for a Reynolds number of zero.

The max element size factor is graphed with the excess pressure drop in figure 2 below. The linear trend line (in red) was taken and the y-intercept is 8.68329 which is the “true value” of K_L for Re=0.

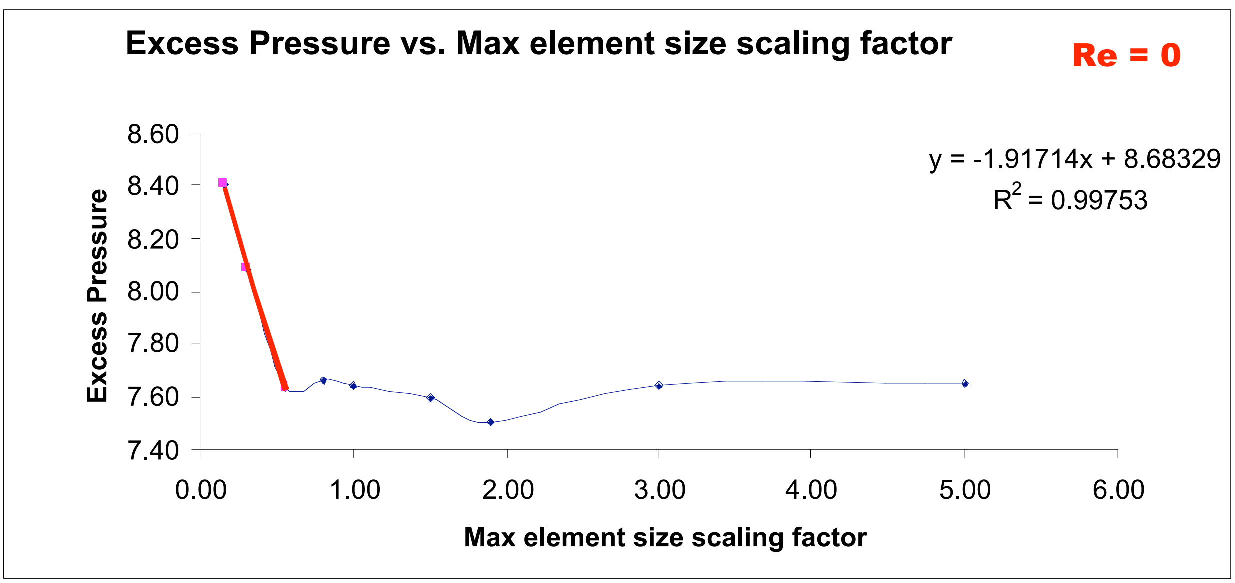

Figure 2. *Excess pressure drop vs. max element size scaling factor for different mesh refinements for Re=0.*

The K_L values for all the presets were calculated and there respective errors were calculated. The distribution of these errors is shown below for each Reynolds number.

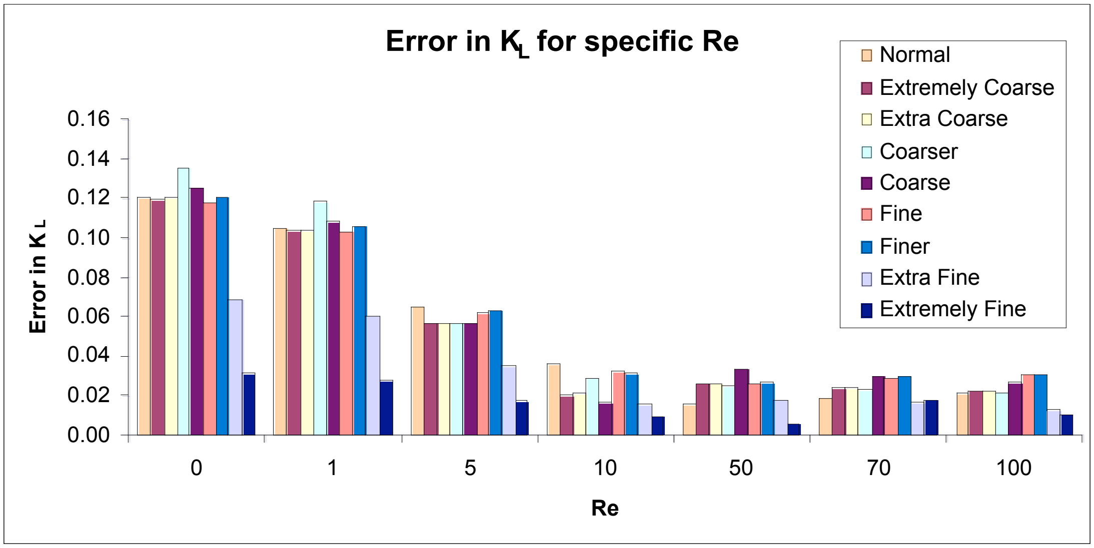

Figure 3. *K_L error distribution for different Reynolds numbers and refinement presets.*

Notice the distributions of the error for low Reynolds number and high Reynolds numbers. For low Re there is a significant difference in the errors between the normal and with the Extremely Fine refinements while for a high Re there is little difference in error between these mesh refinement settings. Figure 4 below also shows this trend when graphing the CPU Time vs. Error in K_L. So with Figure 3 and 4 it seems that when one utilizes small Re values, at least for a 3:1 contraction case, using a finer refinement is advisable since the error distribution is significant. For higher Re it is better to use finer but it is not necessarily needed since the difference in error is so small between normal and finer refinements. Using normal for more complex geometries with higher Re numbers would be advisable to save significant time in running simulations (assuming this trend is seen in other geometries).

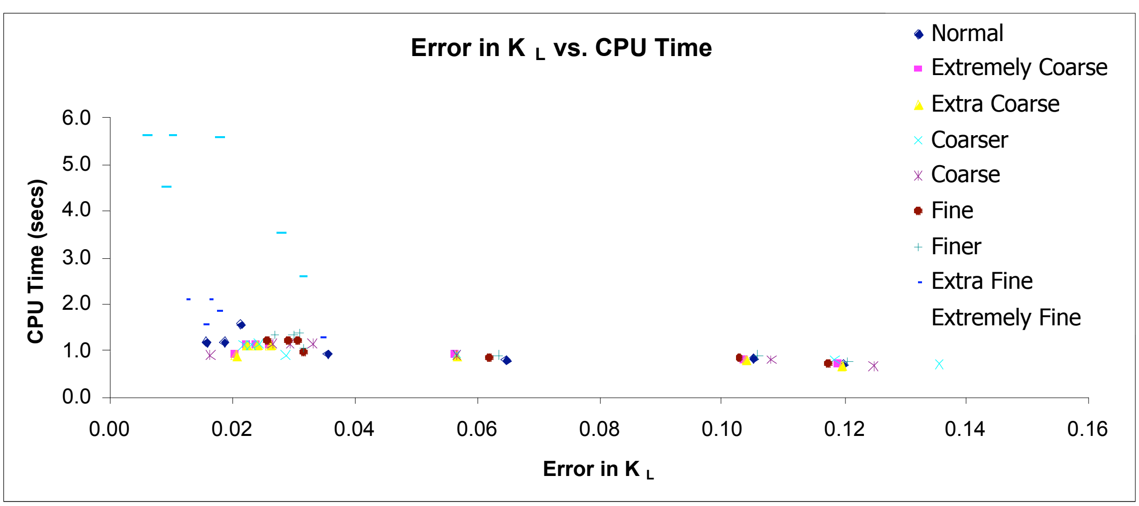

Figure 4. *K_L error vs. CPU time for different mesh refinement presets.*

Figure 5, below is a log-log plot of the error in K_L with the mesh size. Notice the spacing between the lower Reynolds numbers and the grouping of the higher Reynolds numbers 50, 70 and 100. The grouping of the higher Reynolds numbers may be described by viewing the error distribution in figure 3. As discussed earlier, the higher the Reynolds number the smaller the error is between the mesh refinement presets. So in Figure 5, it makes sense that when you get to higher Reynolds numbers that grouping would start to occur.

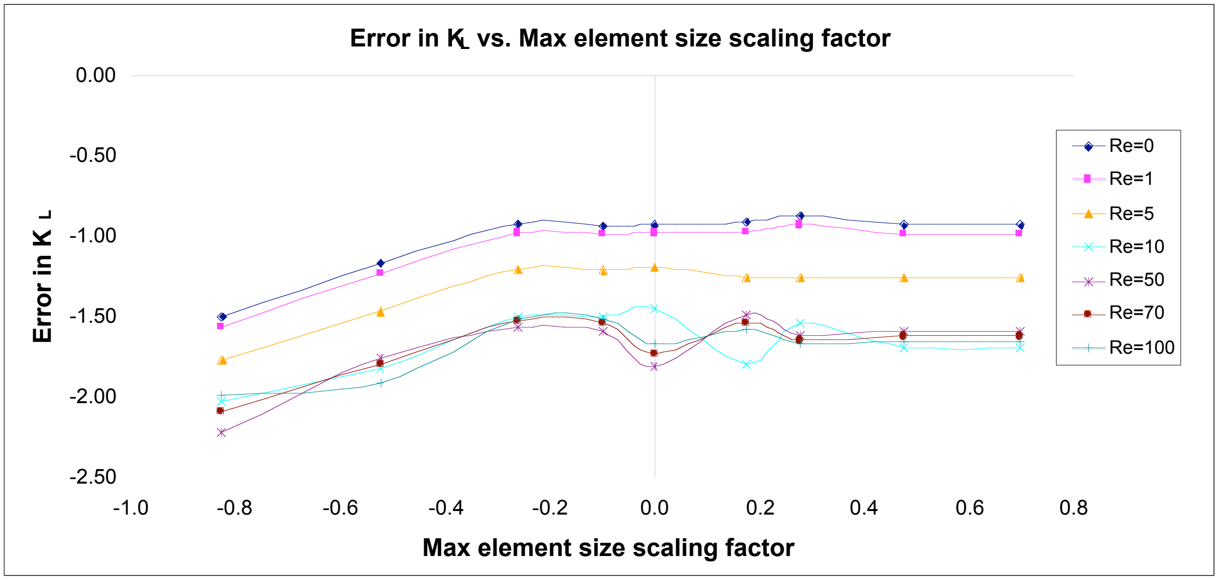

Figure 5. *Log-log plot of K_L error vs. mesh size for different Re values.*

Taking the slope and intercepts of a linear fit of the first three points, finer, extra fine and extremely fine mesh refinements gives you the following:

Re	Slope	Intercept
0	**1.03396**	-0.64065
1	**1.02883**	-0.70007
5	**1.00080**	-0.93724
10	**0.92572**	-1.28414
50	**1.15316**	-1.22730
70	**0.99488**	-1.26726
100	**0.83030**	-1.35675

Table 3. *Figure 5 slope and intercepts for different Re values.*

As one can notice the slopes for this region for all Reynolds numbers are nearly the same. The slope on the log-log graph gives the power: error = A (mesh size)^n. The rate of convergence depends upon the polynomial used for velocity and pressure. If the polynomial is a second order for velocity then the corresponding power in the error formula is two. If the velocity uses quadratic polynomials, then the pressure must use linear ones, and the error pressure then goes as mesh size to the first power [Gimzunger, 1989]. As the polynomial in the finite element goes up or down by one, the power of the (mesh size)^power goes up or down by one. In this case pressure profiles were modeled by linear polynomials so we expect the power of the error to be one.

Conclusions and Recommendations

Ultimately, the results show that the error for the excess pressure drop for different mesh refinement presets at low Reynolds numbers are pretty significant while for higher Reynolds numbers the errors were very small between mesh refinement presets. This means for higher Reynolds numbers it is just as good to do a normal refinement as it is to do a finer mesh refinement. Considering CPU time it also better to do this for more complex geometries since the difference in errors are not that significant for large Reynolds numbers.

It is important to note that this research was done for a simple base case, a 3:1 standard contraction. Other more complex geometries should be investigated to see if these trends hold. Also it would be good to investigate more Reynolds numbers between 10 and 50 to see the transition between the spacing of error plots for low Reynolds numbers to the grouping of high Reynolds numbers pictured in figure 5.

References

Gunzburger, M. D., *Finite Element Methods for Viscous Incompressible Flows*, Academic Press, 1989, p. 32.

Acknowledgments

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- UW Chemical Engineering Dept.
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