Modeling Turbulent Flow Past a Hole

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Introduction

For this project, we studied the flow of cryolite (density 2075 kg/m^3, kinematic viscosity 1.35 E-6 m^2/s) through a regular tube and an irregular tube. Our main goal was to study the overall pressure drop across the entire tube for both geometries. We compared the overall pressure drop to a correlation derived by R.B. Dean. This correlation calculates pressure drop based on the tube geometry and the Reynolds number.

Method

We used Femlab 3.1 to simulate our flow. We selected the K-Epsilon turbulence module in 2-D. The density of the cryolite was set to 2075 kg/m^3 and the kinematic viscosity was set to 1.35 E-6 m^2/s. The horizontal and vertical forces were set equal to zero. Figure 1 below shows a sketch of the irregular tube that the cryolite was flowing through. The distance between the two walls was set to 0.05 meters and the length was set to 0.5 meters.

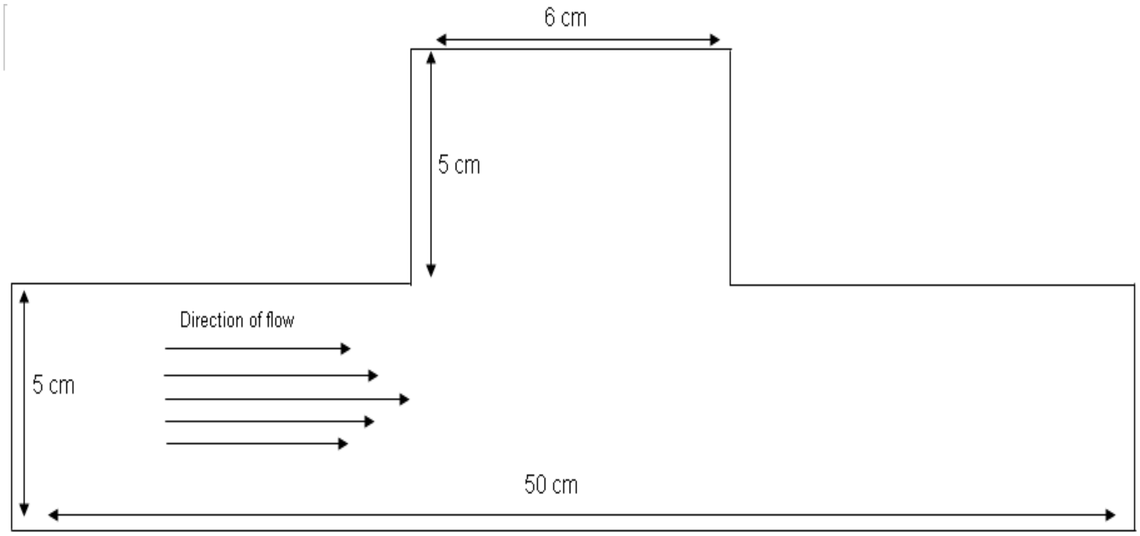

Figure 1: The diagram above shows the tube through which the cryolite is flowing. Halfway across the tube, there is a disturbance that will create pressure and velocity alterations. The fluid flow stabilizes once it goes beyond the "bump."

The regular tube had the same dimensions as the one shown in Figure 1; the only difference is that there is no disturbance in the middle.

At each of the walls of the tube, we used the logarithmic wall boundary condition and we set the layer thickness equal to the default value “h.” For the pipe inlet, we set the boundary condition to inflow and we set the y-velocity equal to a constant. This constant was changed in order to vary the Reynolds number. For the outlet of the pipe, we used the pressure boundary condition and set the pressure to 0.

The pressure drop correlation that we used was taken from R.B Dean in the Journal of Fluids Engineering (1978) [1]. The equation is as follows:

$$\frac{\Delta P}{L} = \frac{\rho < u >^2}{h} 0.073 \, \mathrm{Re}^{-0.25} \qquad \textbf{(Eq. 1`)}$$

Where ΔP is the pressure drop, L is the length of the tube, $<u>$ is the average velocity, h is twice the height of the tube (twice the diameter), and Re is the Reynolds number which is equal to the velocity times the total height of the flow duct divided by the kinematic viscosity.

The mesh for the irregular geometry consisted of 1424 elements and had 12,811 degrees of freedom. The mesh for the regular tube consisted of 1120 elements and had 10,137 degrees of freedom. Figure 2 below shows a picture of the mesh for the irregular geometry.

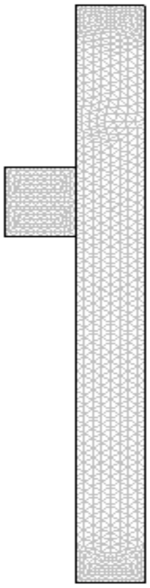

Figure 2: The diagram shown here shows the mesh refinement for the tube with the irregular 2D pattern. There were 1424 elements and 12,811 degrees of freedom.

Results

The table below shows the results from the simulation of the flow through the regular tube. It shows the initial velocity given to the fluid, the Reynolds number, the pressure drop from the simulation, and the pressure drop from the correlation (Eq. 1).

Femlab Max Velocity (m/s)	Reynolds #	Pressure Drop (Pa): Regular Tube	Pressure Drop (Pa): Correlation
0.1	3704	0.86	0.97
0.25	9259	4.5	4.83
0.5	18519	15.5	16.2
0.75	27778	32.5	33.0
1	37037	55.5	54.6
1.5	55556	117	111
2	74074	199	184
3	111111	423	373
4	148148	722	618
5	185185	1095	913
6	222222	1540	1256
7	259259	2057	1645
10	370370	4020	3070

Table 1: This table summarizes the results obtained for the flow of cryolite through a regular tube. As we can see, for Reynolds numbers of 70,000 and below, the correlation and the simulation yields similar values. For higher Reynolds numbers, the simulation starts to yield higher values than those calculated from the correlation.

This data shows that for Reynolds numbers of 70,000 and lower our simulation matches the correlation very well. For bigger Reynolds numbers we begin to see considerable deviations. The figure below shows how the overall pressure drop changes with the Reynolds number for our simulation and for the correlation.

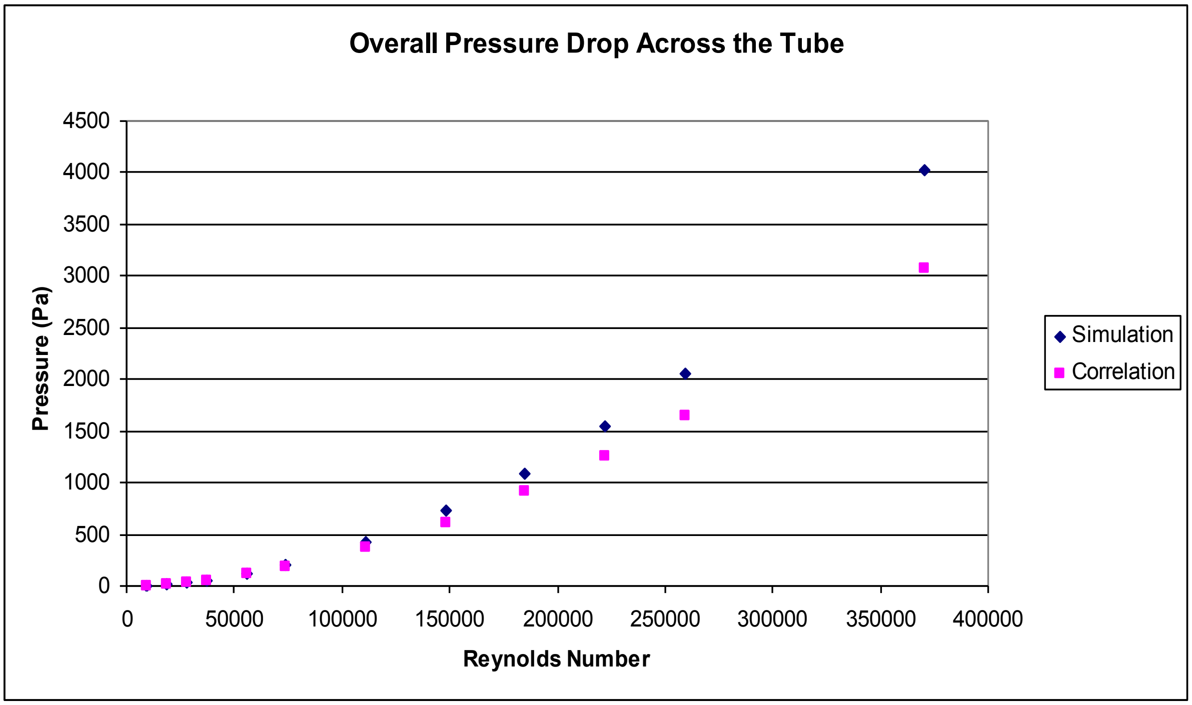

Figure 3: This graph shows how the overall pressure drop changes with the Reynolds number. We can see that as the Reynolds number gets bigger and bigger the simulation and correlation values begin to deviate considerably. For smaller Reynolds numbers the two values are very close to each other.

Figure 4 below shows the pressure profile of the fluid as it travels across the regular tube at a velocity of 1 m/s (see Table 1). From this graph we see that the pressure drop is almost perfectly linear, there is no noticeable disturbance on this curve.

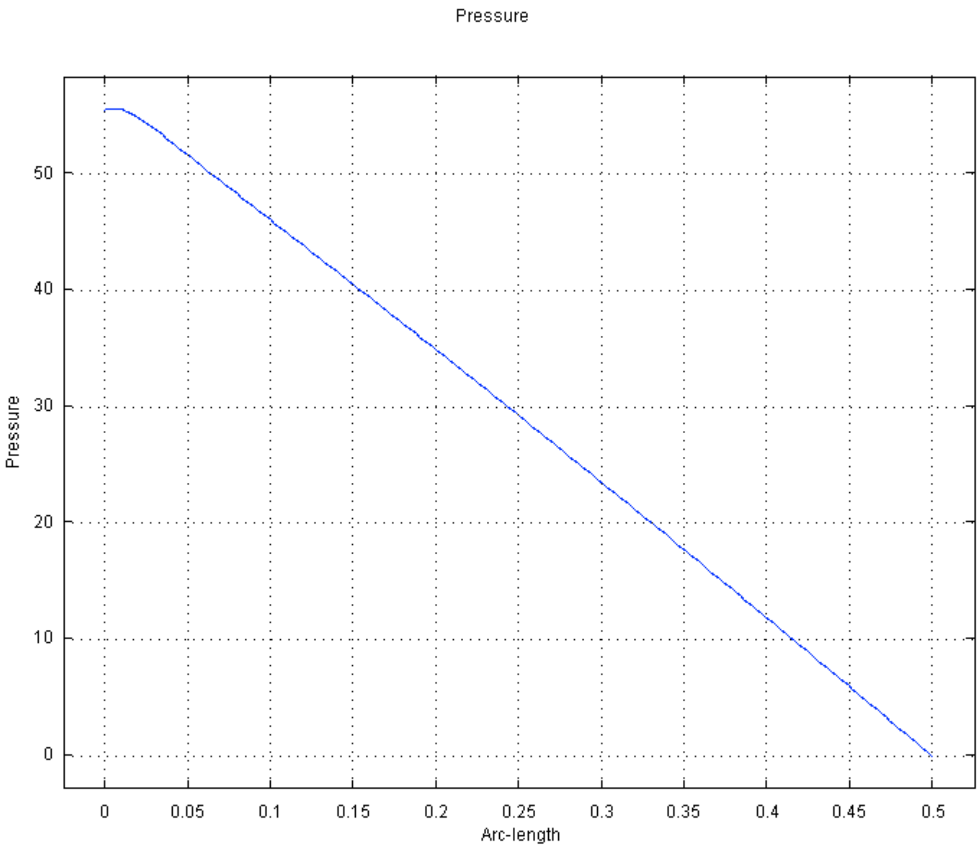

Figure 4: This graph shows the pressure drop across the tube for a fluid velocity of 1 m/s. As expected, we see a linear relationship with no disturbances whatsoever.

The pressure drop across the irregular tube was much more complicated. The figure below shows how the pressure drops as it goes across the tube for an initial velocity of 1 m/s.

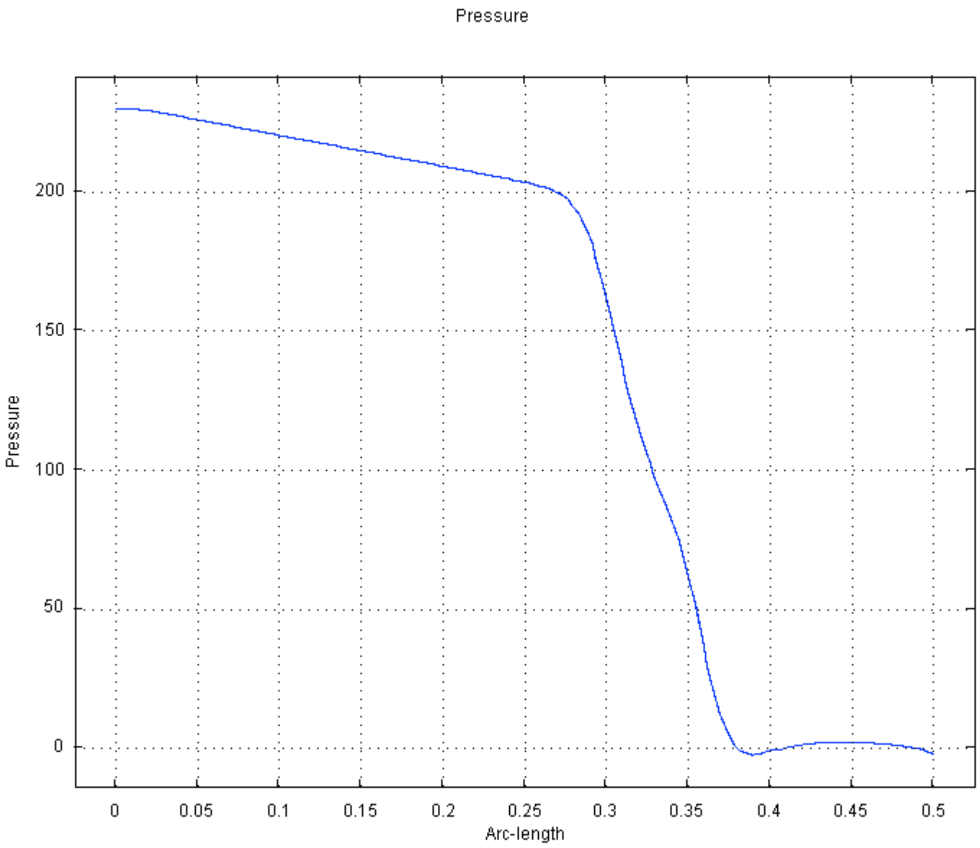

Figure 5: This figure shows how the pressure drops across the irregular tube. For the first 25 cm we see similar behavior to the regular tube, a very linear drop. When the fluid reaches the "bump," there is a huge alteration and the pressure takes a huge drop.

At first glance it might seem like there is something wrong with the simulation in Femlab (the overall pressure drop is over 200 Pascals), especially when we compare Figure 5 to Figure 4. However, a closer look reveals that this graph does make sense. If we were to fit the pressure curve over the first 25 cm to a straight line and extend it all the way to 50 cm we would end up with the following diagram:

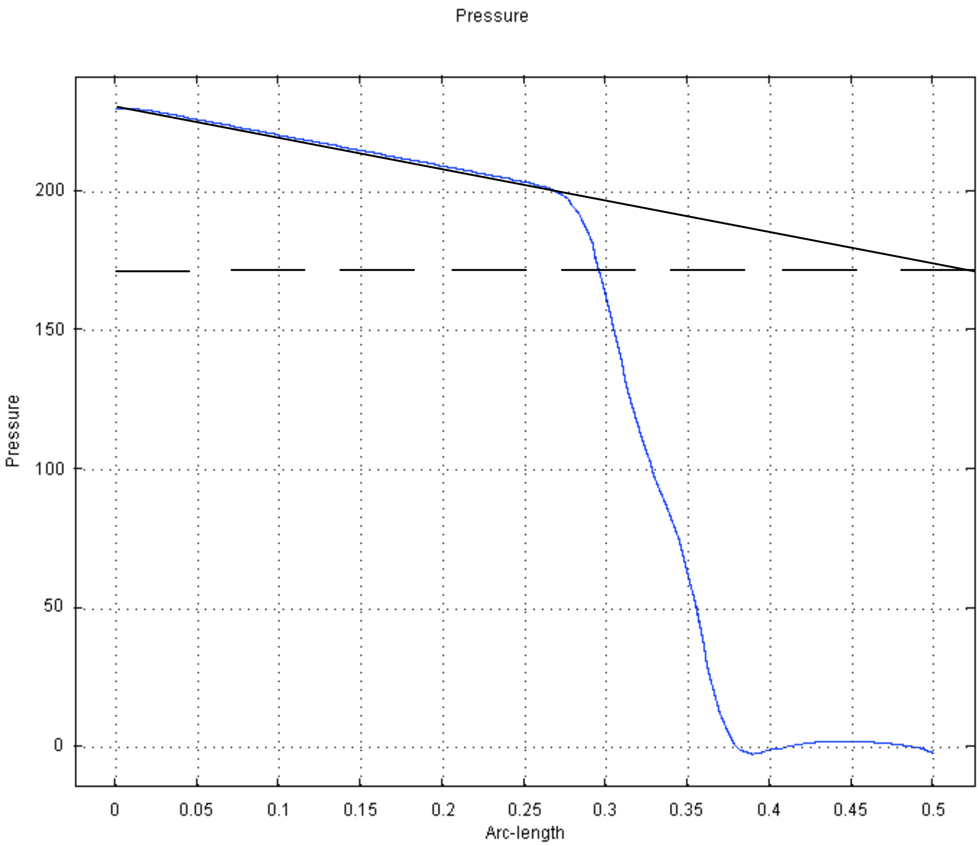

Figure 6: This diagram shows the same graph as the one in Figure 5 but with a best fit line for the first 25 cm of the tube. We can see from this sketch that if the bump were not there, the overall pressure drop would be about 55-60 Pascals which is the same as the pressure drop across the regular tube.

From this diagram, we can see that if the “bump” were not there, the overall pressure drop would be about 55-60 Pascals, which is the value determined for the regular tube (see Table 1, max velocity of 1 m/s).

Conclusions

Our simulation of flow through a regular tube seems to work very well for Reynolds numbers of up to 70,000. At bigger Reynolds numbers the values start to deviate considerably. For the irregular tube we see huge deviations as the fluid reaches the disturbance, so big that at a first glance it seems as if something is wrong. However, as was explained in the previous section, we can draw a straight line from the curve through the first 25 cm of the tube and see that the pressure drop would be the same if there was no disturbance. Therefore, we can conclude that we have a reasonably accurate simulation of turbulent cryolite flow through both regular and irregular tubes.

Works Cited

1. Dean, R. B. (1978). Reynolds Number Dependence of Skin Friction and Other Bulk Flow Variables in Two-Dimensional Rectangular Flow Duct. Journal of Fluids Engineering, 100, 215-223.