<u>3-Dimensional Cryolite Flow</u>

By:

Zudtky Wisecarver

May 17, 2005

Chem E 499

P.I:

Bruce Finlayson

Introduction

For this project, we studied the 3-dimensional flow of cryolite fluid in a thin container with a thin anode plate at the top. A circular velocity was imposed on the bottom of the container and we observed how this motion affected the fluid at the top where it did not border the anode plate, that is, at the fluid-air interface. Once we had a base solution, we varied the Reynolds number. We also varied the height of the fluid. We solved the problem for three cases, one in which the fluid just reached the level of the anode plate, another in which the fluid went above the anode plate by a length of 0.5A (where A is the depth of the container as can be seen from Figure 2) and the third where the fluid went above the anode plate by a length of A.

Method

With Femlab, we selected the incompressible Navier-Stokes module in 3-D. The equations the software solved using this module were the following:

$$\rho(u \bullet \nabla)u = \nabla \bullet [-pl + \eta(\nabla u + (\nabla u)^T)] + F \quad \textbf{(Eq. 1, 2)}$$

$$-\nabla \bullet u = 0$$

The force F was set to zero, the value entered for the viscosity was 1/Re, and the value for the density was set to 1. Ultimately, the equation that was solved was:

$$u^{'} \bullet \nabla^{'} u^{'} = -\nabla^{'} P^{'} + \frac{1}{\mathrm{Re}} \nabla^{'2} u^{'} \quad \textbf{(Eq. 3)}$$

Equation 3 above is dimensionless. The Reynolds number was varied from 1-1000 and the velocity values generated were recorded. Figure 1 below shows a diagram of the geometry in which the fluid flow was simulated. The "container" was 1.65 m * 1.5 m * 0.05 m. The anode plate at the top was 1.5 m * 1.35 m * 0.01m.

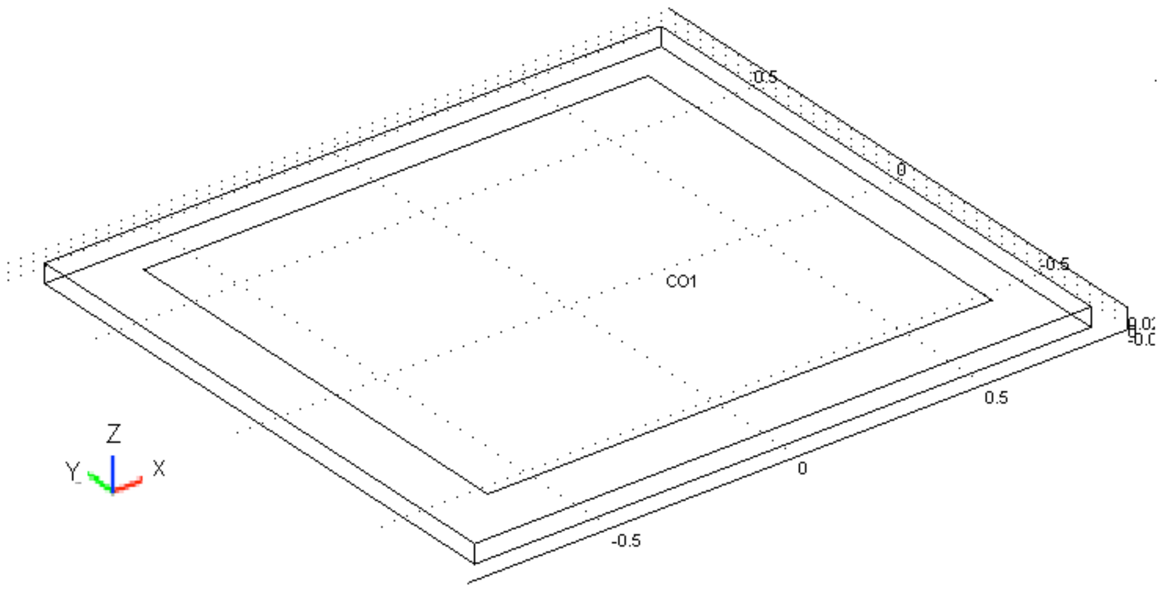


Figure 1: The figure above shows a simple 3-D schematic of the geometry through which our flow was simulated. The "container" was 1.65 m * 1.5 m *0.05 m. The plate at the top is 1.5m* 1.35m * 0.01m.

At the bottom of the container, the boundary condition was set to inflow/outflow velocity. The equations for the x and y velocity components entered were:

$$V_x = \frac{-\pi}{L_y} * \cos\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right) \quad \textbf{(Eq. 4)}$$

$$V_y = \frac{\pi}{L_x} * \sin\left(\frac{\pi x}{L_x}\right)\cos\left(\frac{\pi y}{L_y}\right) \quad \textbf{(Eq. 5)}$$

The boundary condition at the top of the container where the fluid does not border the plate was set to slip/symmetry. All other boundary conditions were set to no slip. Figure 2 on the following page shows a side view of the geometry.

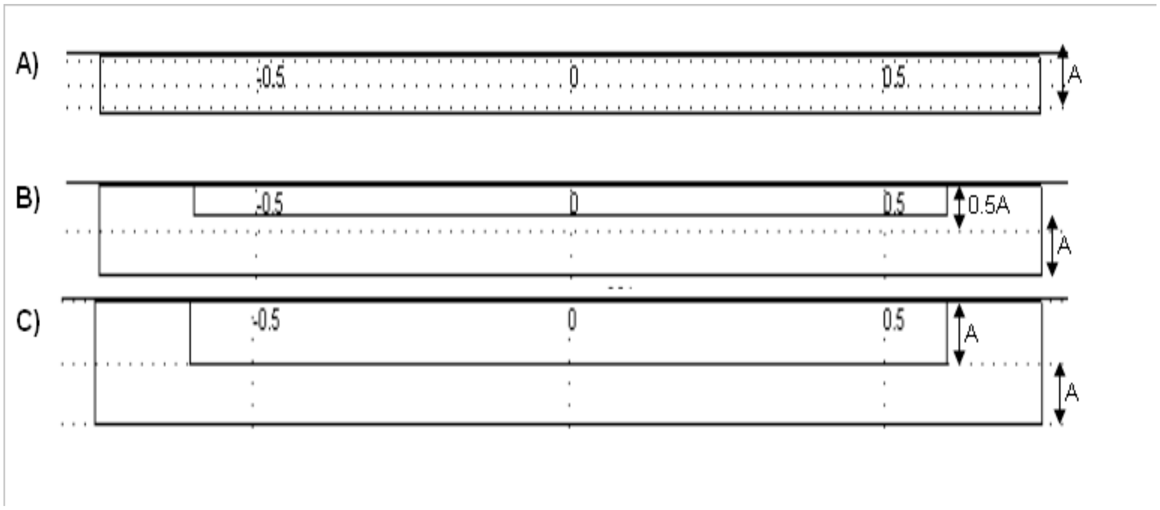


Figure 2: This figure shows the side view for each of the three cases we solved for. In case A, the anode plate just sits on top. For the other two cases, the fluid rises above the anode plate, first by 0.5 A and then by 1.0 A.

As mentioned in the introduction, the problem was solved for three different cases. In case A, the fluid reaches the anode plate at the top. In case B, the fluid goes above the anode plate by a length equal to half of the container depth, that is, 0.5A. In case C, the fluid goes above the anode plate by a length of A.

Figure 3 below shows the mesh refinement for the geometry shown in Figure 1. The mesh consisted of 5479 elements and 35589 degrees of freedom. For the second case in which the fluid reaches a level higher than the anode plate by 0.5A, there were 24164 elements and 125921 degrees of freedom, for the case in which the fluid reaches a level higher than the anode plate by A, there were 6392 elements and 39187 degrees of freedom.

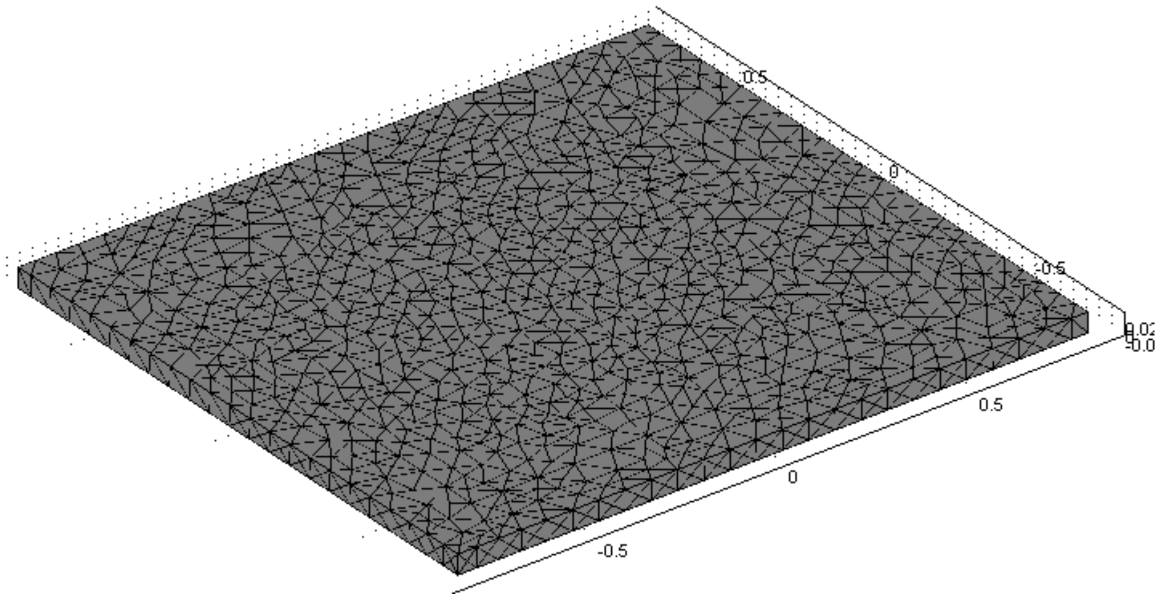


Figure 3: This figure shows the mesh refinement for our geometry. The mesh consisted of 5479 elements and 35589 degrees of freedom.

Results and Discussion

Figure 4 below shows a graph that plots the maximum velocity of the fluid versus the Reynolds number at the top surface of the container where the fluid is in contact only with air (not with the anode plate). From this graph we can see that the velocity tends to increase with an increase in Reynolds number for the cases in which the fluid reaches above the anode plate. For the case in which the anode just acts as a top, we see a decrease in the velocity rather than an increase (albeit, a very small one). We can also see from this graph that the shallower the fluid is, the greater the velocity at the top surface. This makes sense because the momentum created by the fluid flow at the bottom is transferred to the top through less fluid. In other words, there is less fluid weight to move when the fluid level is shallow.

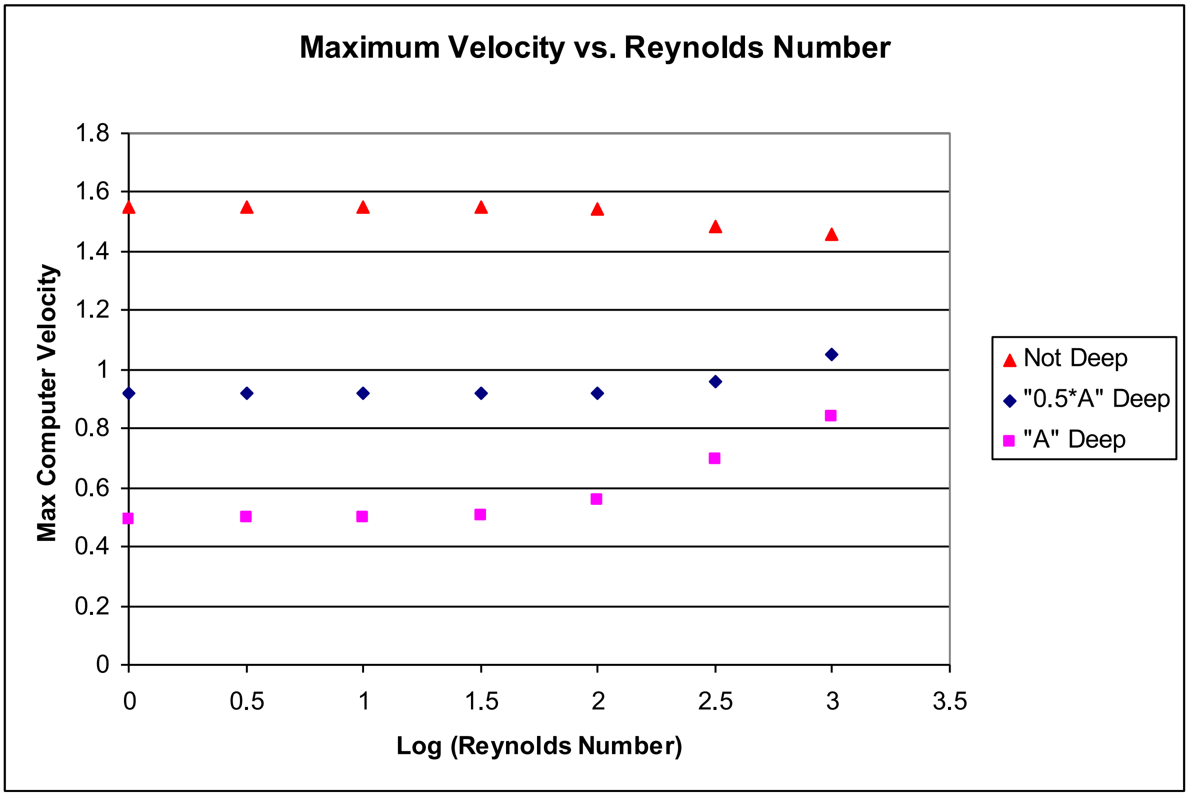
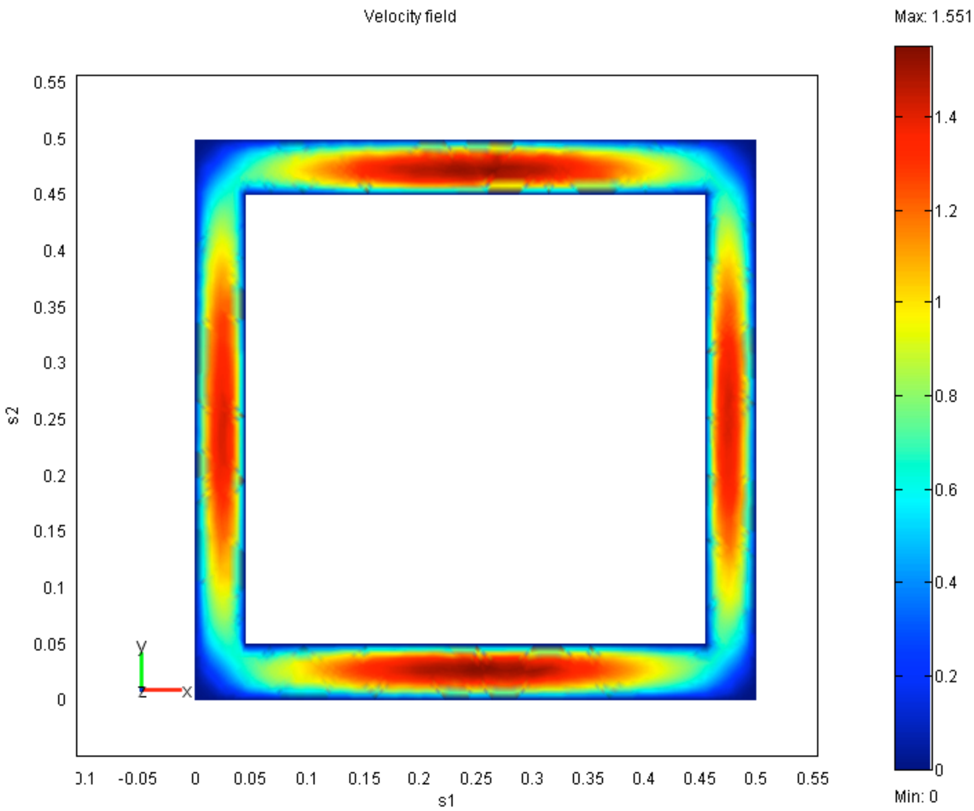
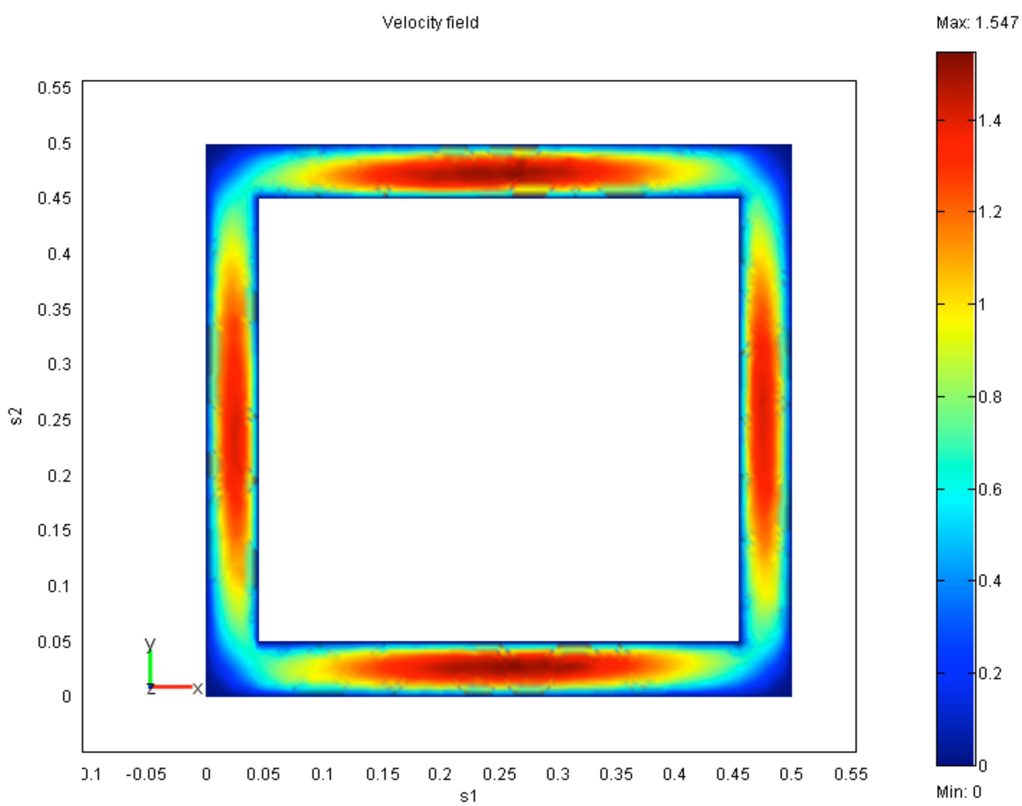
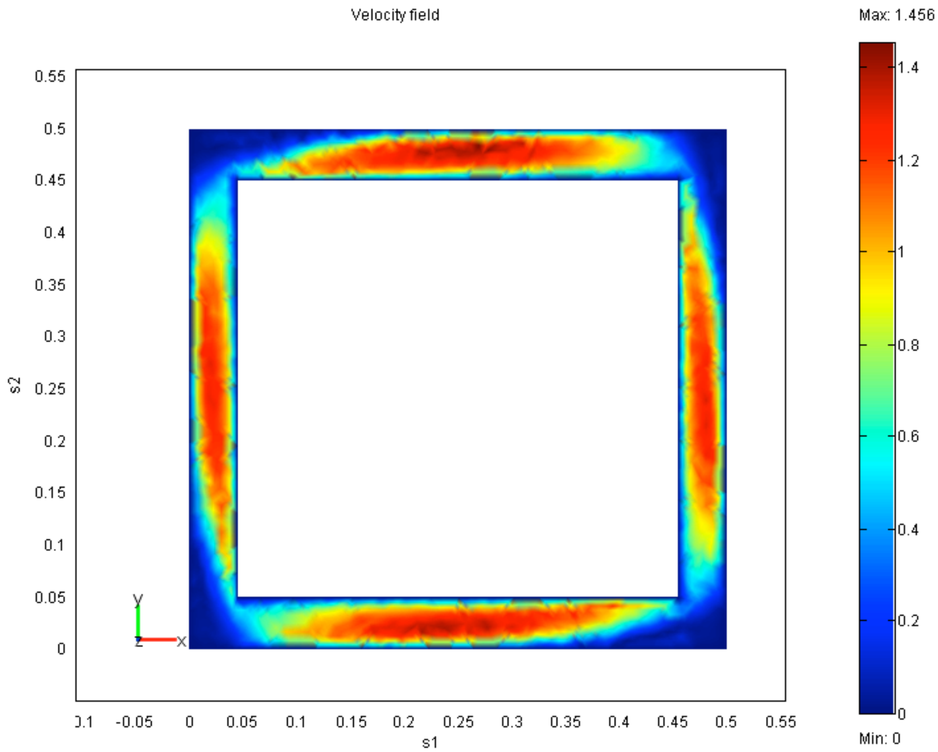


Figure 4: This graph plots the maximum computer velocity of the fluid at the top surface versus the Reynolds number. We can see here that the shallower the fluid is, the faster the velocity is at the top surface. We can also see that when the fluid is deep, the velocity increases with an increase in Reynolds number. For the first case, wee see that there is very little change in velocity with an increase in Reynolds number, in fact, we even see a small decrease.

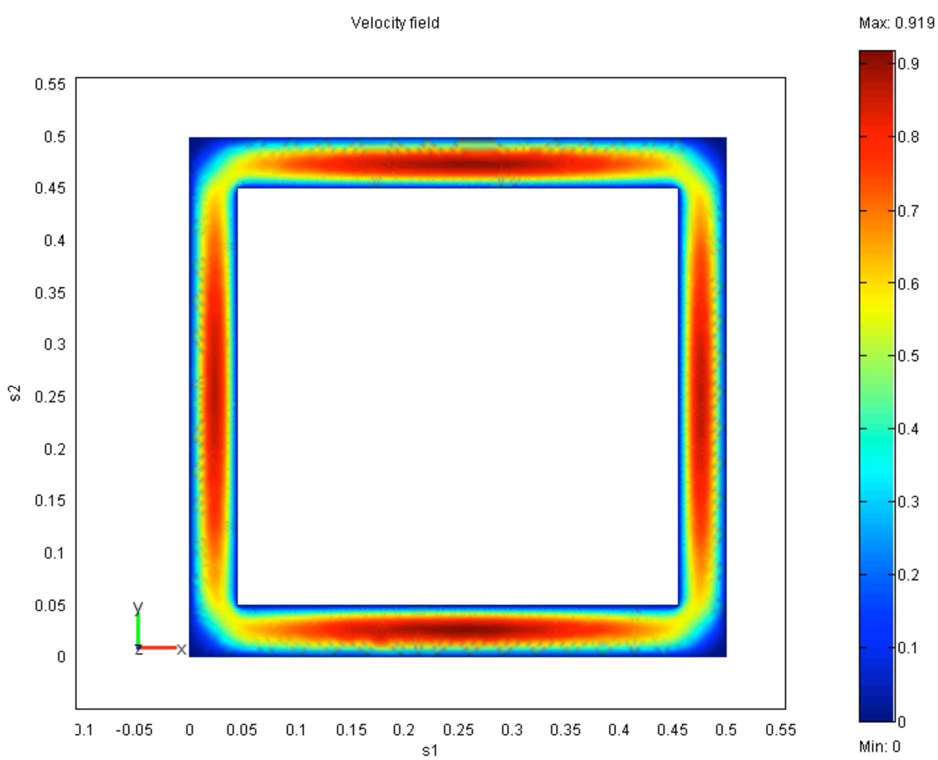
Figures 5 – 13 below show the velocity profile of the fluid at the top surface for each of the three cases. We show each case for Reynolds numbers of 10, 100, and 1000.

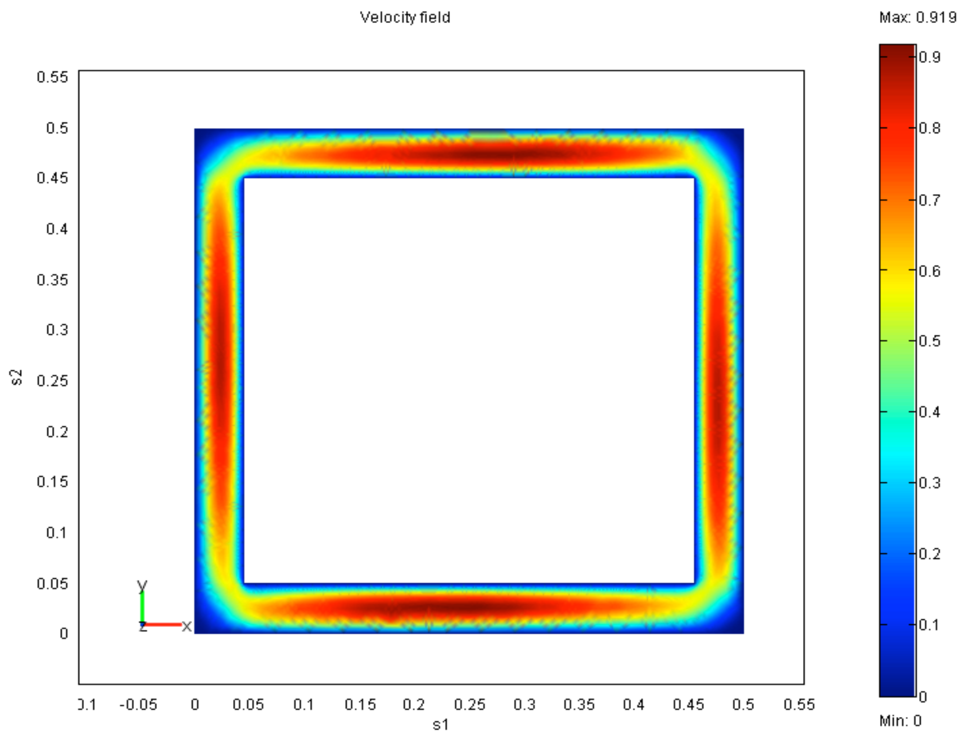

Velocity field
Max: 1.551
Min: 0
s1
s2

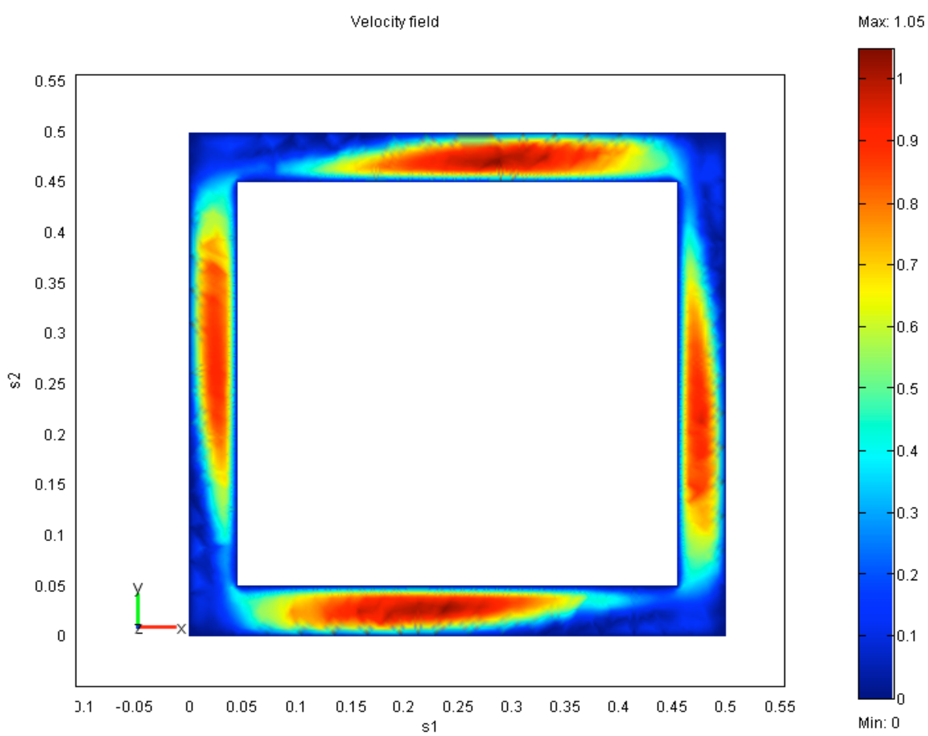

Velocity field
Max: 1.547
Min: 0
s1
s2



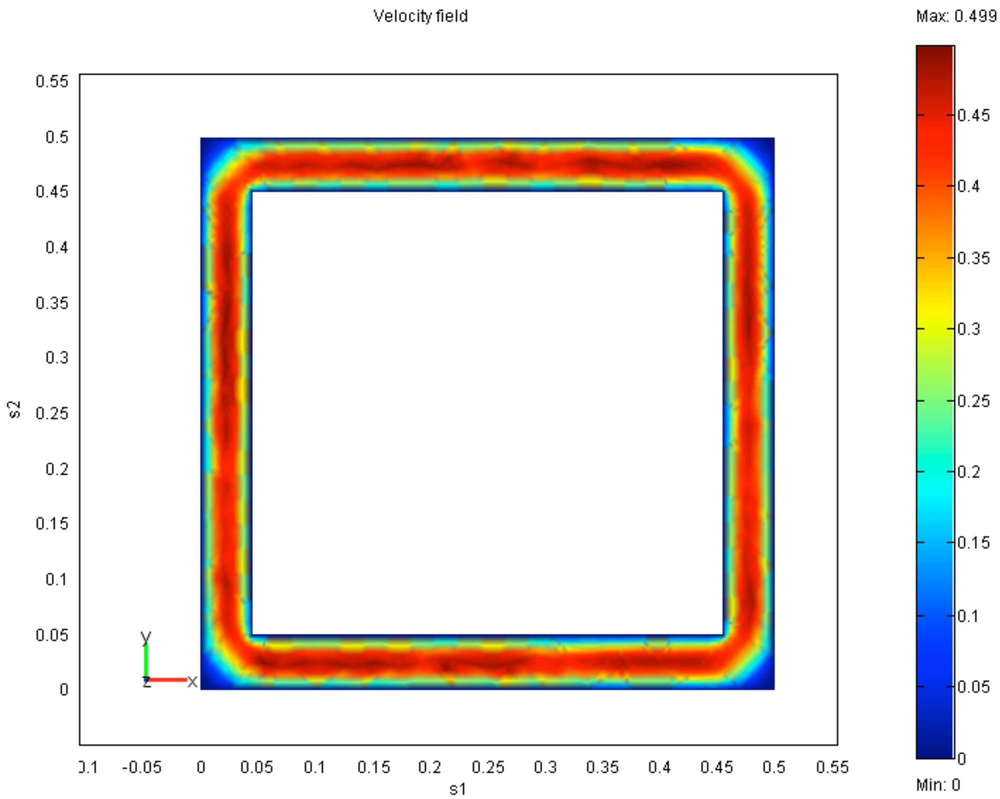
Figures 5, 6, and 7: These figures show the velocity profile of the fluid at the top surface for the case in which the fluid does not go above the flat anode plate at Reynolds numbers of 10, 100, and 1000. We can see that as the Reynolds number increases, the profile becomes less and less symmetrical due to inertia effects.

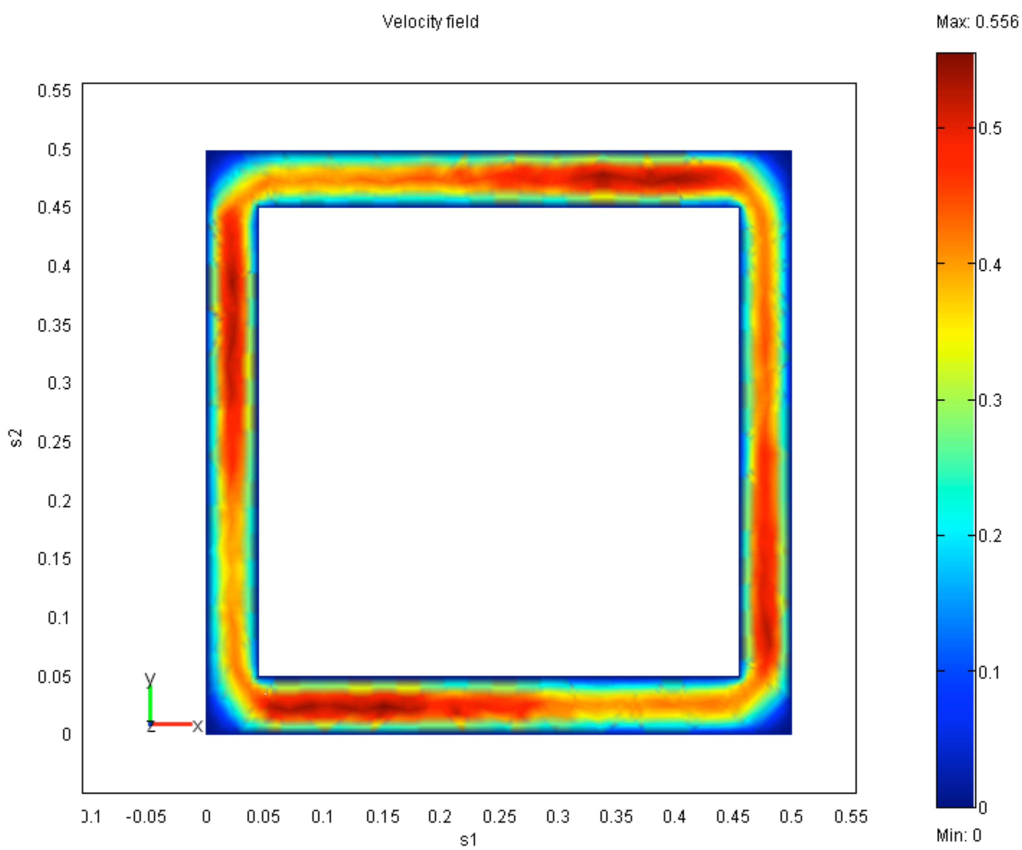


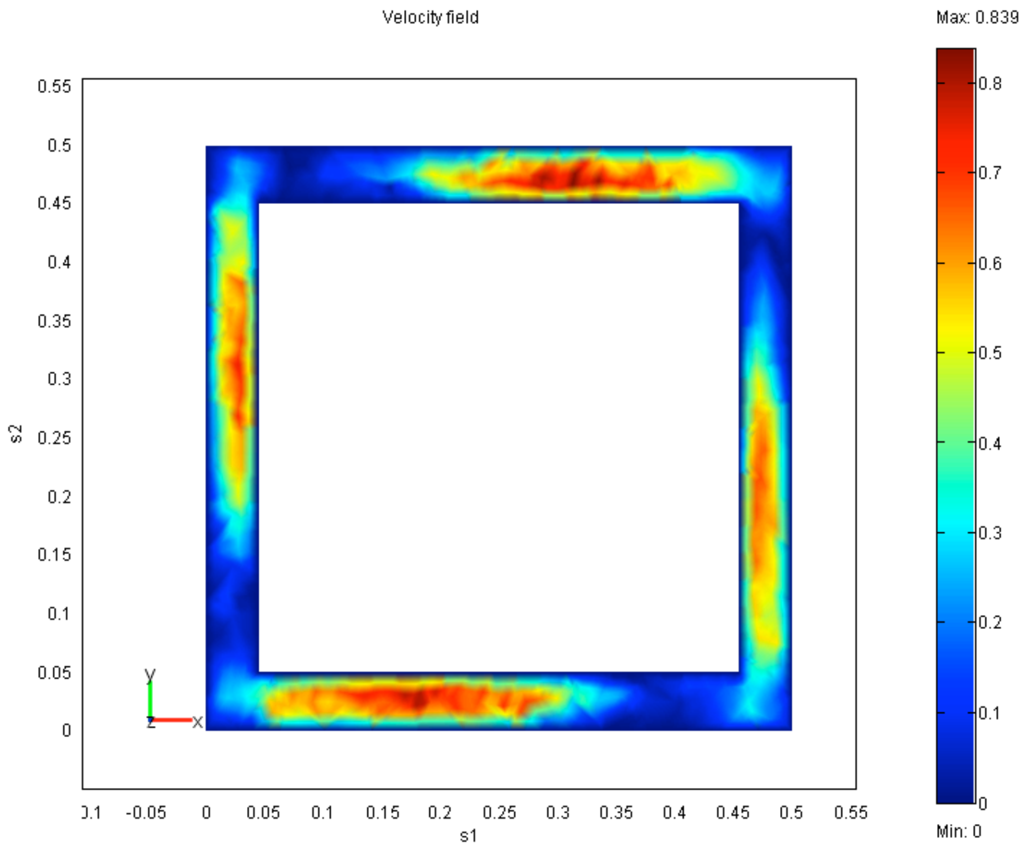




Figures 8, 9, and 10: These figures show the velocity profile of the fluid at the top surface for the case in which the fluid goes 0.5A above the plate. Again we notice that the profile becomes asymmetrical for larger Reynolds numbers. We also notice that the overall velocities are lower than they were in Figures 4-6.


Velocity field
Max: 0.499
Min: 0
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
s2
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
s1
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
y
x
z

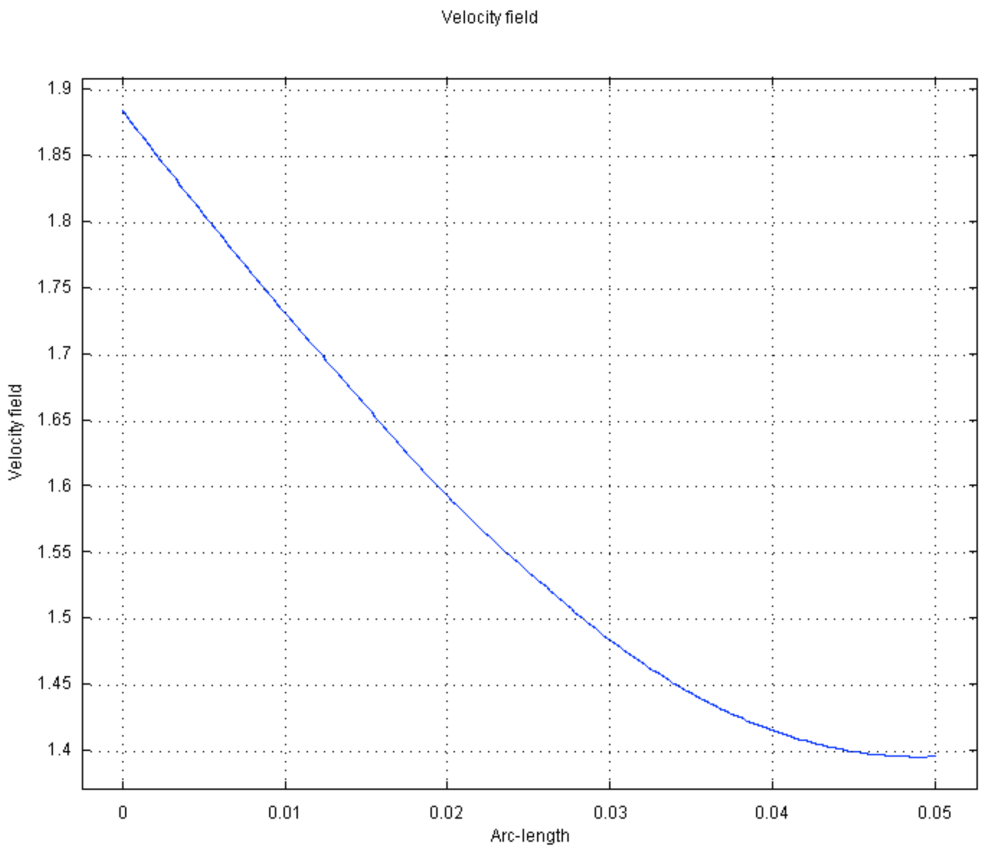

Velocity field
Max: 0.556
Min: 0
0.5
0.4
0.3
0.2
0.1
0
s2
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
s1
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
y
x
z

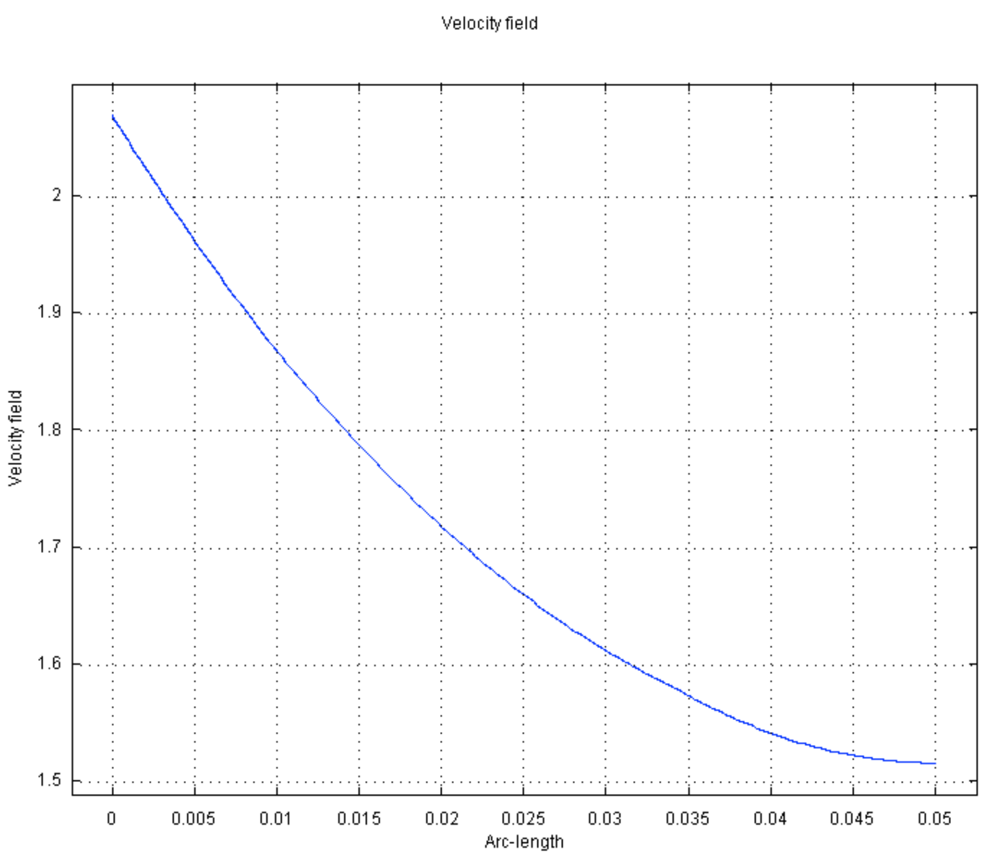


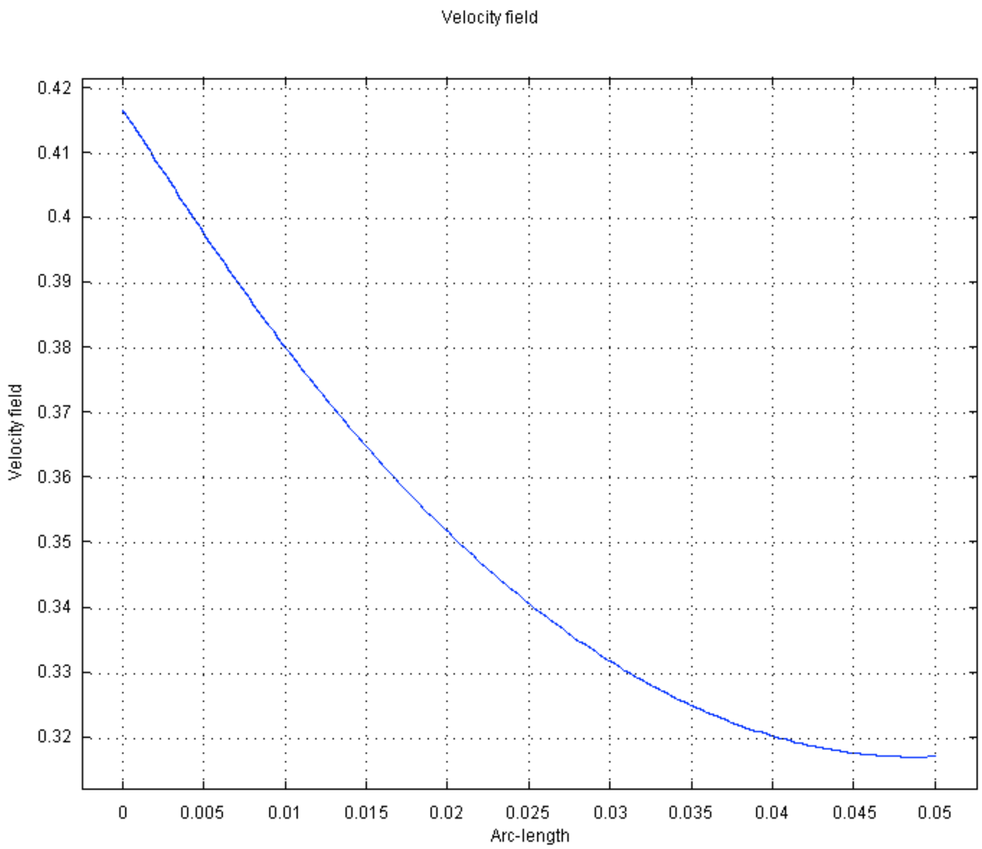
Figures 11, 12, and 13: These figures show the velocity profiles for the fluid in the case in which the fluid goes A above the plate. We can see that the profile becomes extremely asymmetrical at a Re of 1000. We can also see that the maximum values of the velocity are lower than they were in the previous two cases.

From these figures we can see that as the Reynolds number begins to increase, the flow becomes less and less symmetrical. This is because inertial effects are being felt. The fluid flow is no longer smooth. We can also see on these diagrams that as the height of the fluid increases, the velocity at the top surface decreases (which we learned from Figure 4).

Also of interest to us, was how the velocity of the fluid changes as we go from the bottom of the container to the top of the fluid. Figures 14 – 16 below show how the velocity changes from the bottom to the top for the case in which the fluid is at the same level with the anode plate (the Reynolds number was 100). Figure 14 shows how the velocity changes as we go from the bottom to the top at x = 0.75 m and y = 0 m, Figure 15 at x = 0 m and y = 0.675 m, and Figure 16 at x = 0.75 m and y = 0.675 m.


Velocity field
Velocity field
Arc-length
1.9
1.85
1.8
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
0
0.01
0.02
0.03
0.04
0.05

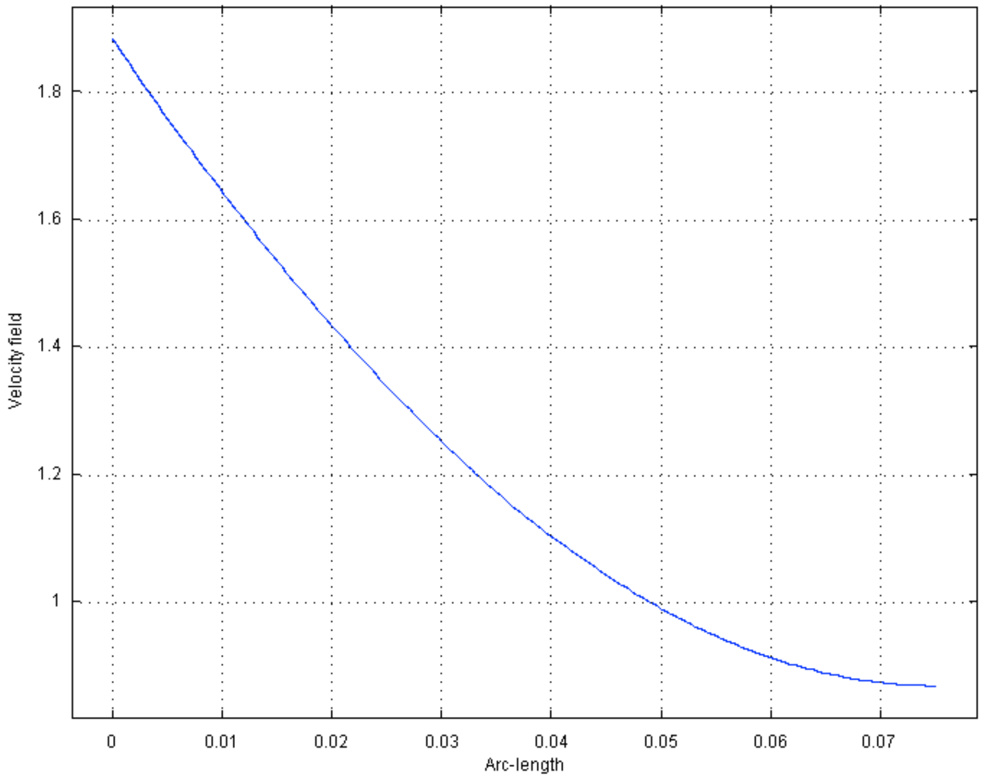

Velocity field
Velocity field
Arc-length
2
1.9
1.8
1.7
1.6
1.5
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05



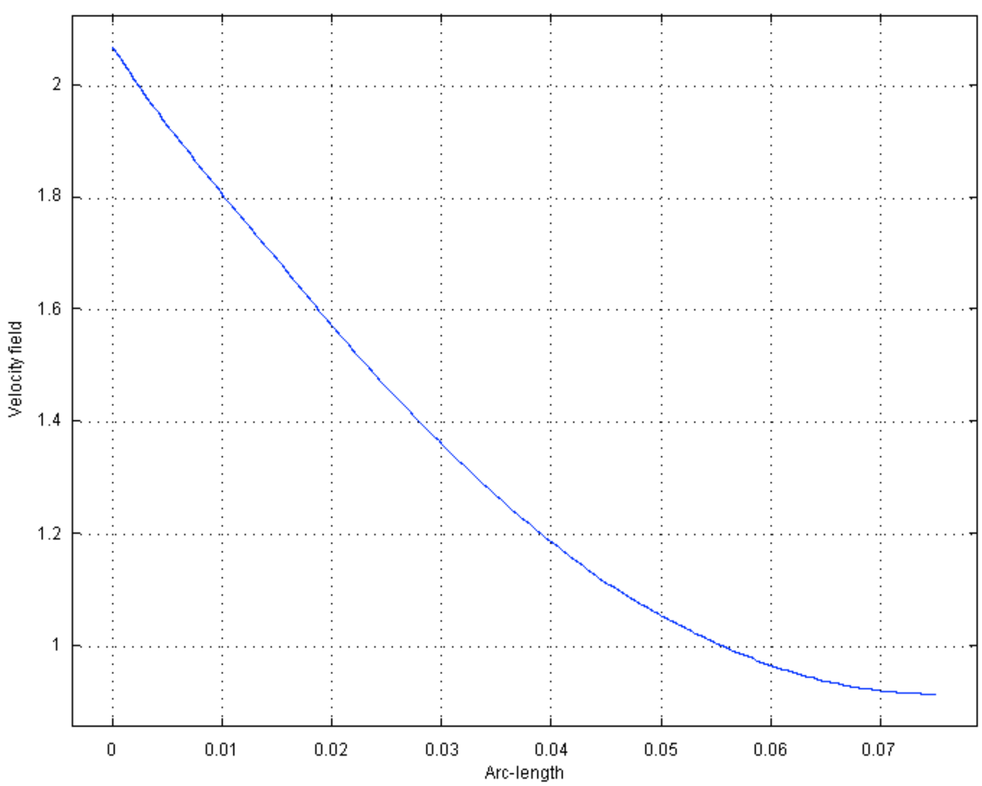
Figures 14, 15, and 16: These figures show how the velocity of the fluid changes from the bottom of the container to the top of the container where the fluid is in contact with the air. Figures 14 and 15 show the velocity towards the sides of the container while Figure 16 shows the velocity profile at a corner.

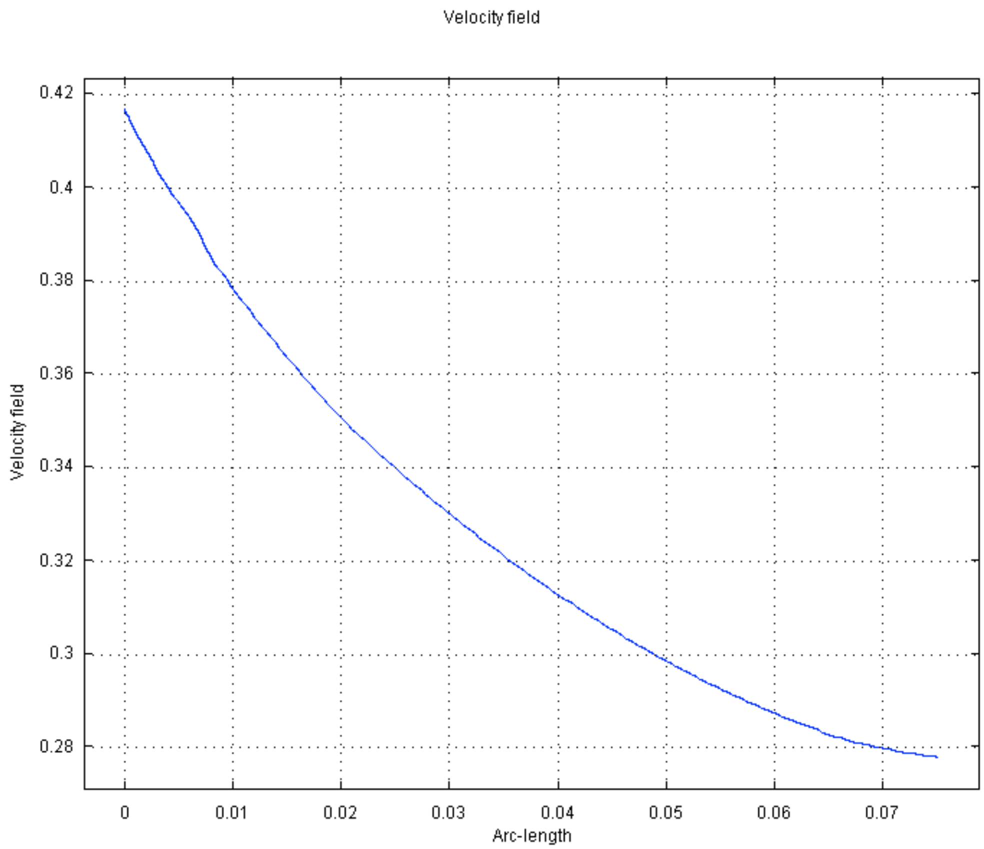
Figures 17 – 19 are similar to Figures 14 – 16 except that they are for the second case, where the fluid rises 0.5A above the anode plate (Reynolds number equal to 100).

Velocity field



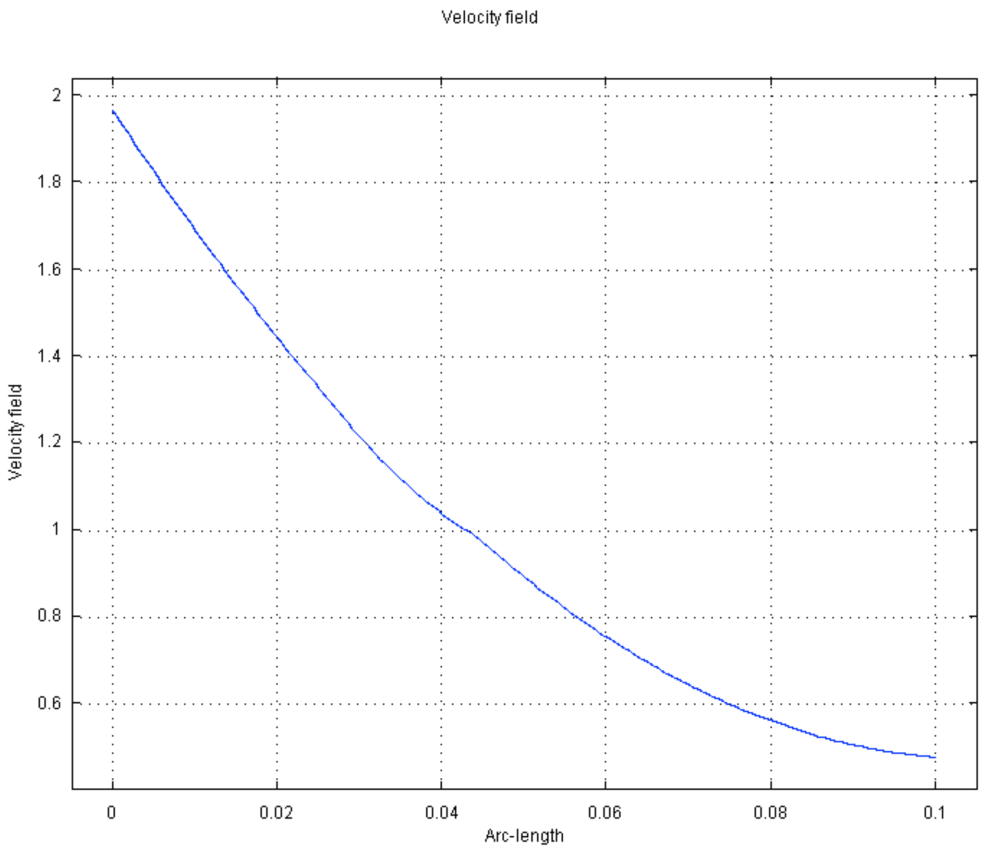
Velocity field

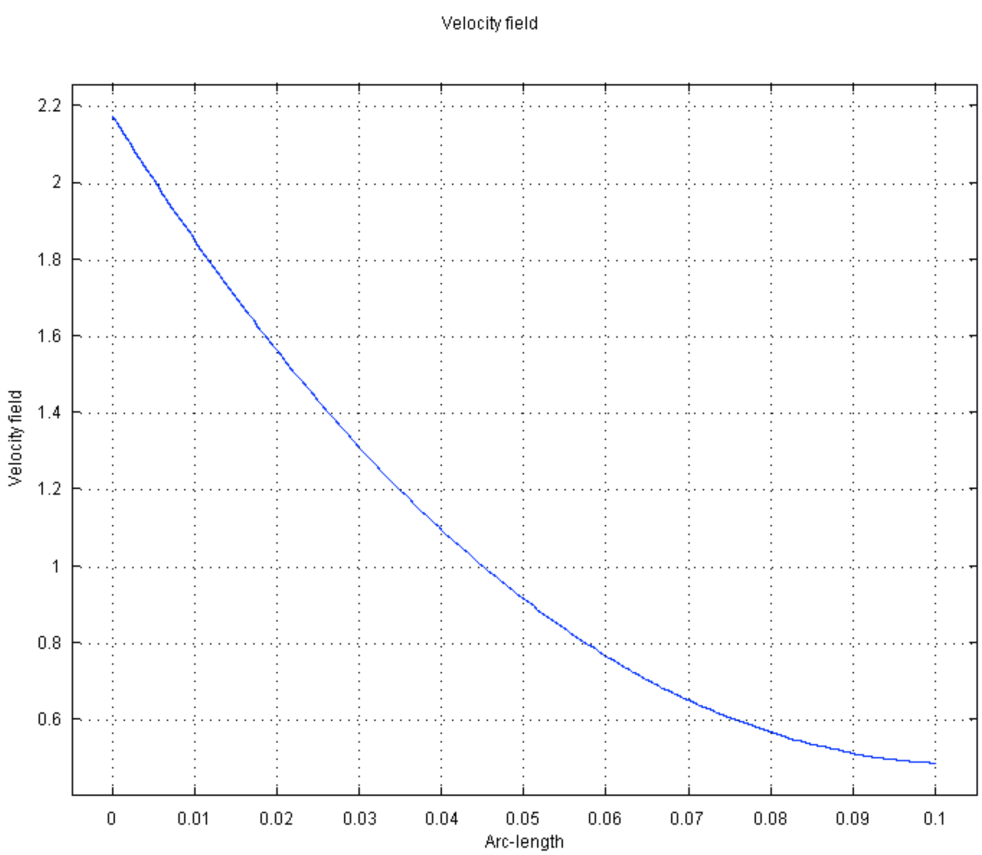


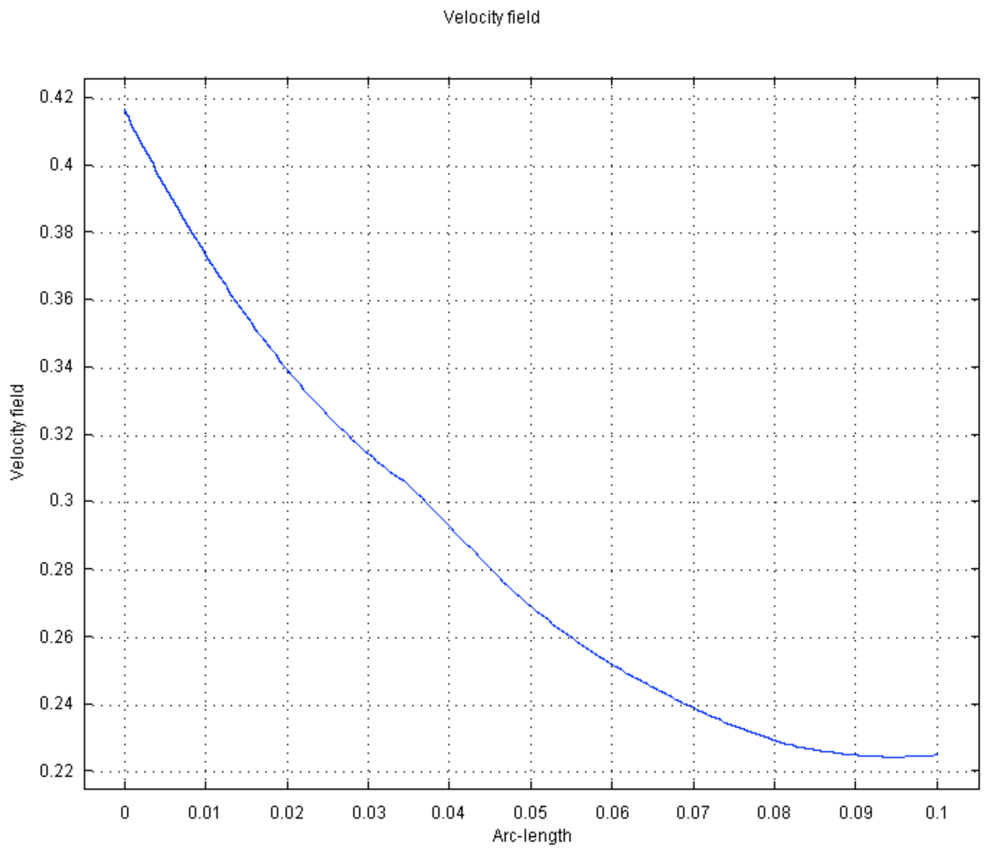


Figures 17, 18, and 19: Like figures 14-16, these figures show the velocity of the fluid from the bottom to the top of the container. We see the same pattern in these graphs that wee saw in Figures 14-16.

Figures 20 – 22 show the same plots for the third case, where the fluid rises 1.0 A above the anode plate (Reynolds number is 100).


Velocity field
Velocity field
Arc-length
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
0.02
0.04
0.06
0.08
0.1


Velocity field
Velocity field
Arc-length
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1



Figures 20, 21, and 22: These figures show the velocity profile for the case where the fluid rises to a height A above the anode plate. The patterns seen are similar to those for the previous six figures.

The overall patterns for the velocity profiles in all three cases are very similar. The peak value in each case is the same (or should be the same, any variation in this is due to some small numerical error from Femlab). We can see that the velocity decreases smoothly as we go up the container until we reach the fluid – air interface. It's also interesting to note the rates at which the velocity decreases for each of the three cases. We can compare the velocity for each case at an archlenght of 0.05. The following values are obtained from Figures, 14, 17, and 20. The velocity at an archlength 0.05 for the case where the fluid just reaches the anode level is about 1.4, the velocity at the same archlength for the case where the fluid rises 0.5 A above the anode plate is about 1, and the velocity at the same archlength where the fluid goes 1.0 A above the anode plate is about 0.9. We can see that as the height of the fluid increases, the rate at which the velocity decreases also increases. This is because of the added weight from the extra fluid. More fluid needs to be moved, so the velocity decreases more rapidly.

The fluid velocity at the fluid – anode interface is zero. The velocity profile of the fluid from the bottom of the container to the top at the fluid – anode interface is a straight line as can be seen from Figure 23 below. This is reminiscent of Couette flow.

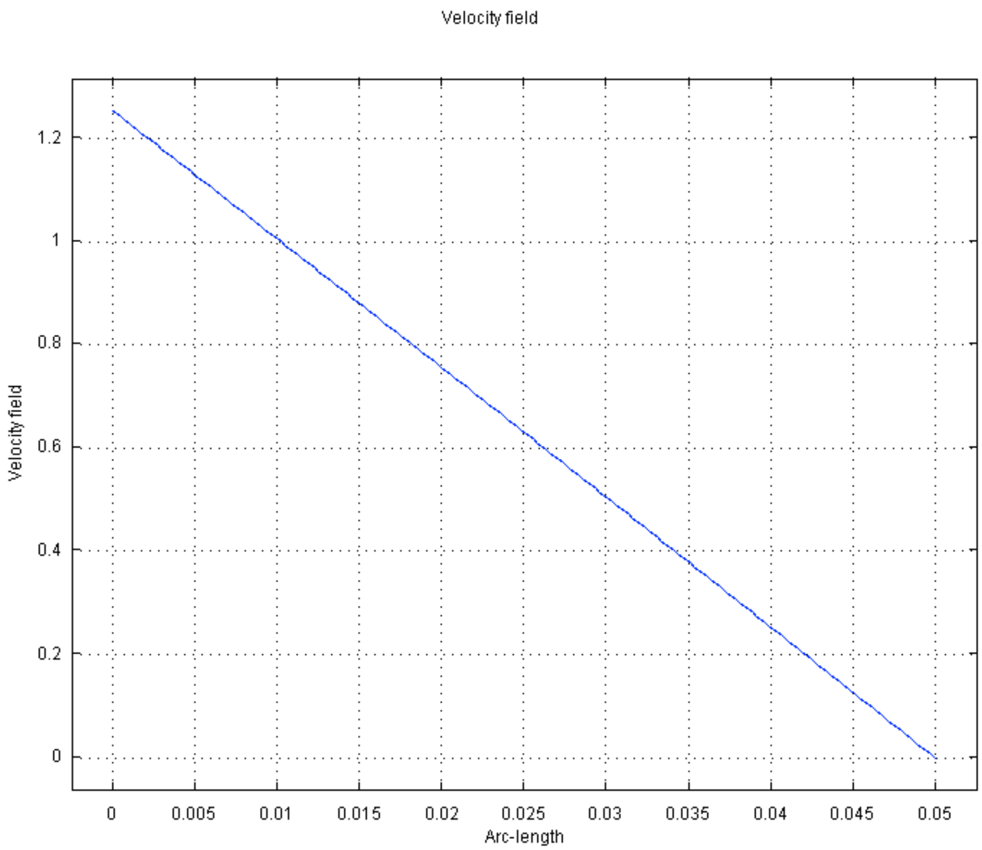


Figure 23: When we measure the velocity of the fluid from the bottom of the container to the fluid – anode interface, we get a straight line. This figure shows the velocity profile of the fluid starting at the bottom, at x = 0.3 m and y = 0.25 m, and going to the top at the fluid anode interface for the third case. The actual value of the velocity at the bottom depends on the position of the point, but we always see a linear relationship until the velocity gets to zero.

The actual value of the velocity at the bottom of the container depends on its position (see equations 4 and 5), but no matter what the peak velocity is, the velocity at the top is always zero and decays linearly. This is true for any of the three cases studied in this report. Figure 23 was obtained from the third case and its position at the bottom was x = 0.3 m and y = 0.25 m.

Conclusions and Recommendations

We have some very good working models for fluid flow in a 3-D container with a thin anode plate at the top. We can see how fluid motion at the bottom of the container affects the fluid at the top. We have been able to vary the Reynolds number as well as the height of the flow and see how these changes affect the velocity at the surface of the fluid. Our results are all sensible so we have no reason to believe that there is a problem with the way Femlab is working.

Future work involves making the container much bigger and adding more plates. The velocity at the fluid – air interface can then be studied at various positions and we can learn more about this system as a whole. Eventually, we would like to solve this problem for turbulent flow and see how those results compare with those of the laminar flow from this experiment.