Pressure Drop in Microdevice

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Goal

To correlate the pressure coefficient in micro devices when the Reynolds number is small and flow is laminar

There are microchemical reactors and micro sensors.

The real devices are 3D.

As a start , do some 2D cases in FEMLAB

At high Re : $\Delta P = K \rho \langle u \rangle^2/2$

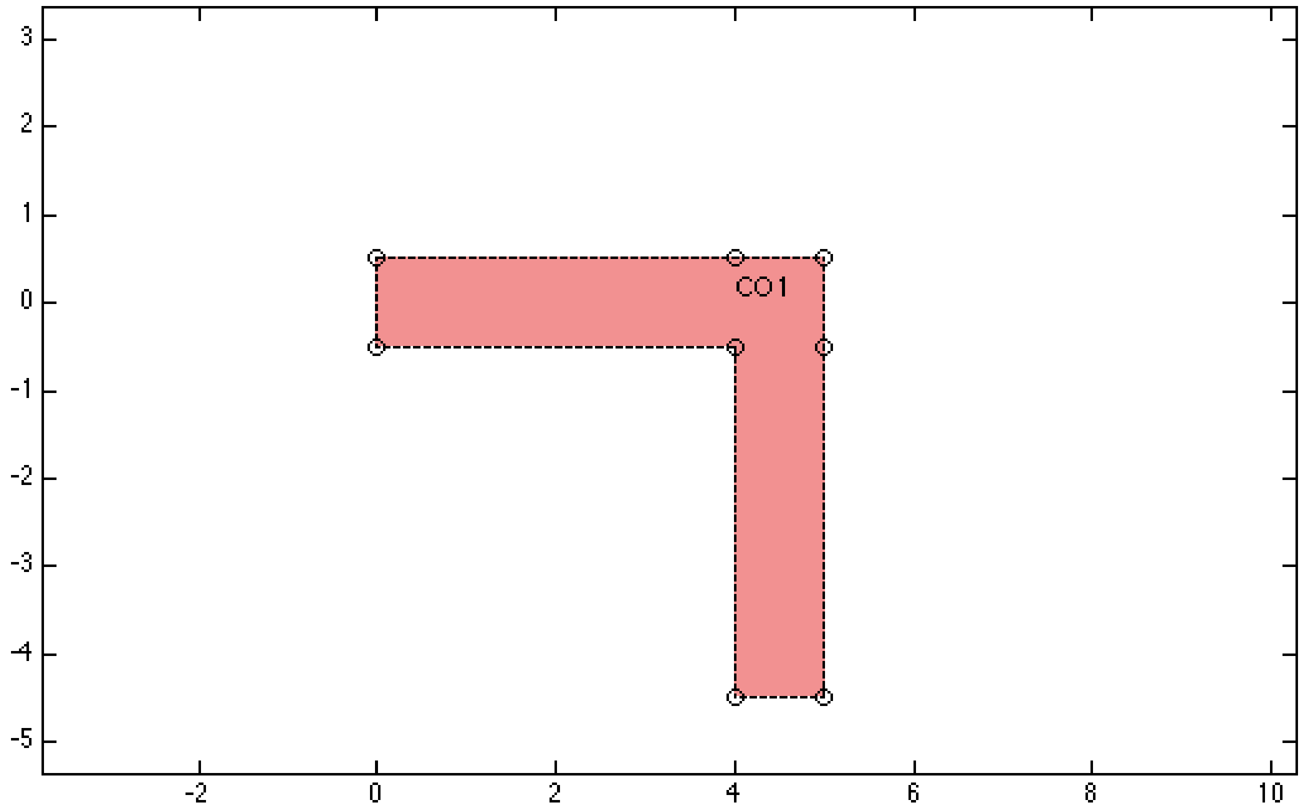
Ex : for 90^o elbows $K = 0.75$

At low Re : $\Delta P = K \mu \langle u \rangle / x_s$

non-dimensionlize: $\Delta P' p_s = K \mu / x_s \, u_s \langle u \rangle'$

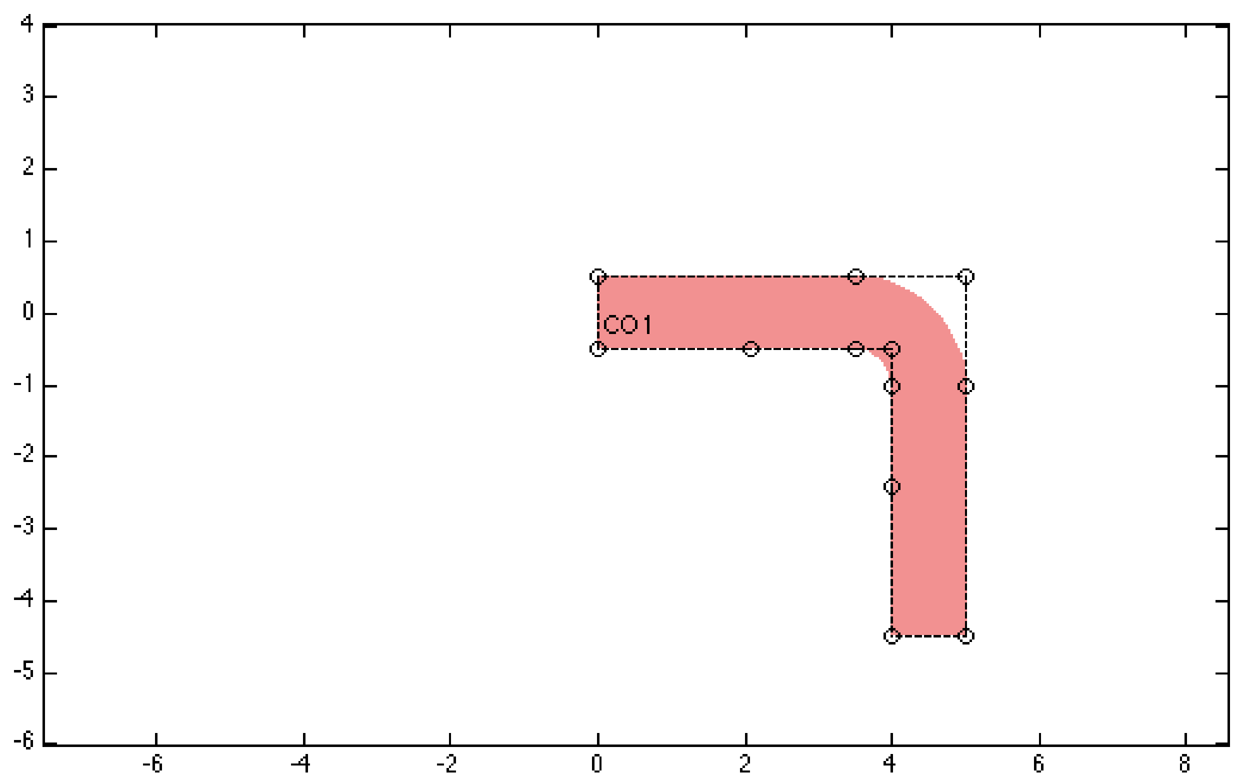
choose $p_s = \mu u_s / x_s$: $\boxed{\Delta P' = K \langle u \rangle'}$

Bend 90^o



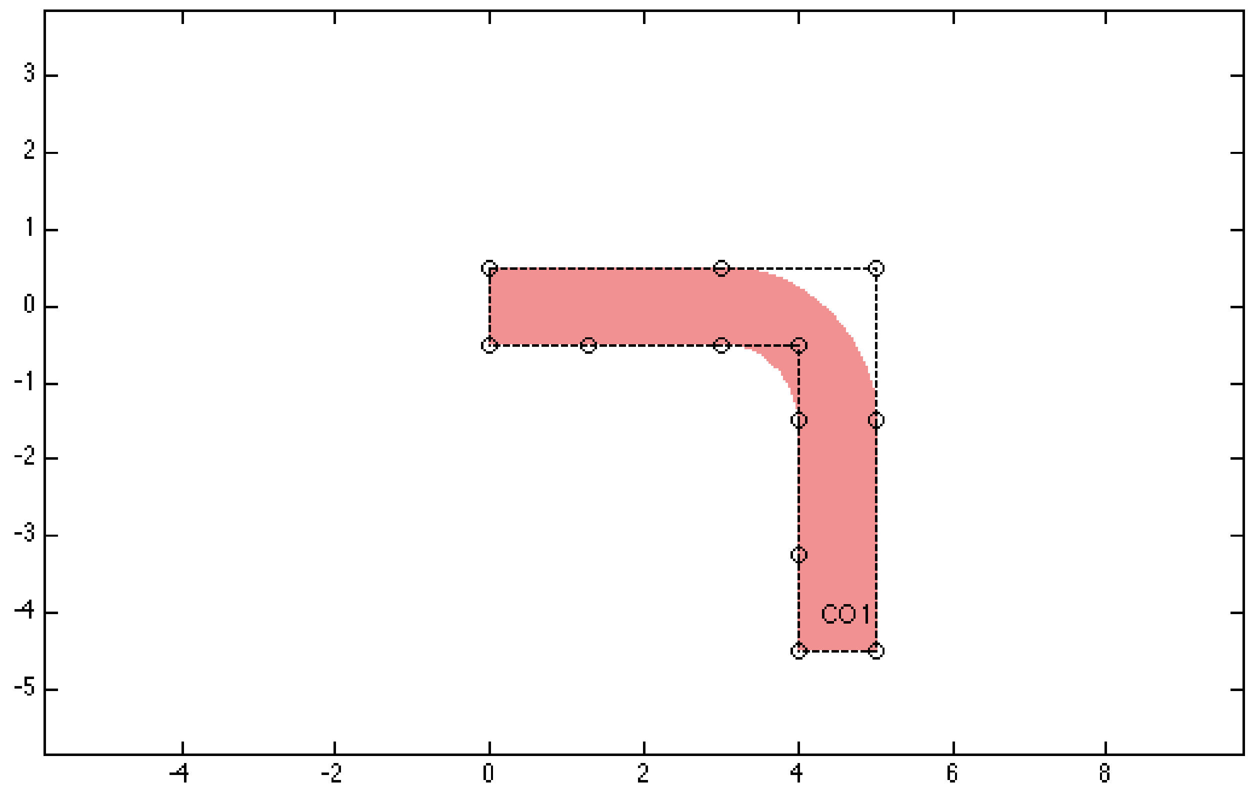
Bend 90° with smooth corner

(short radius)



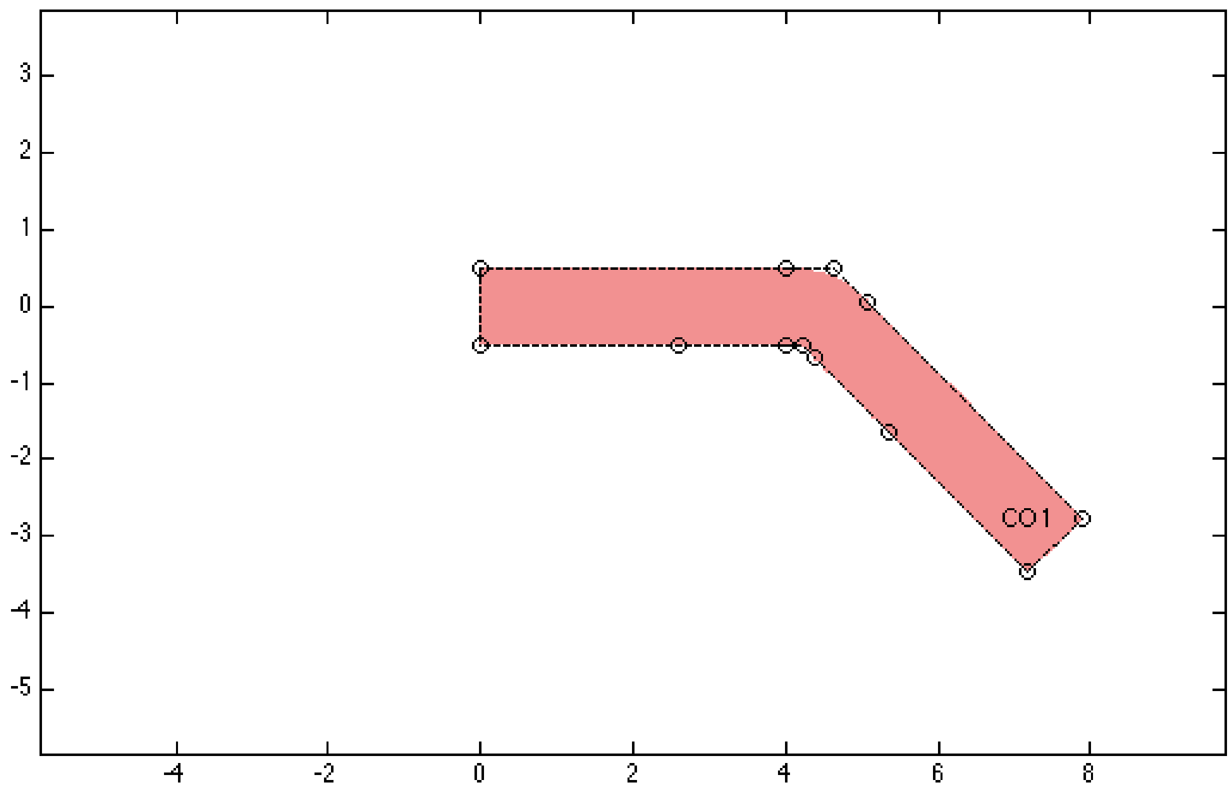
Bend 90^o with smooth corner

(long radius)



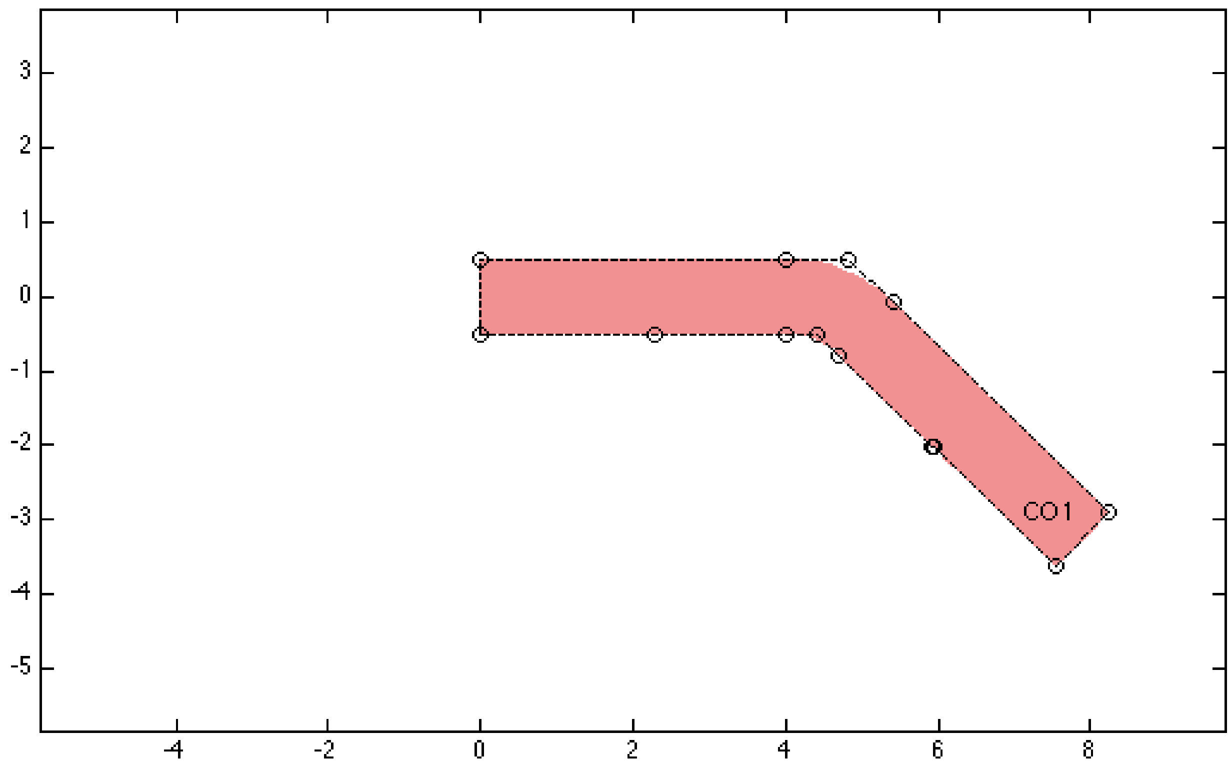
Bend 45º with smooth corner

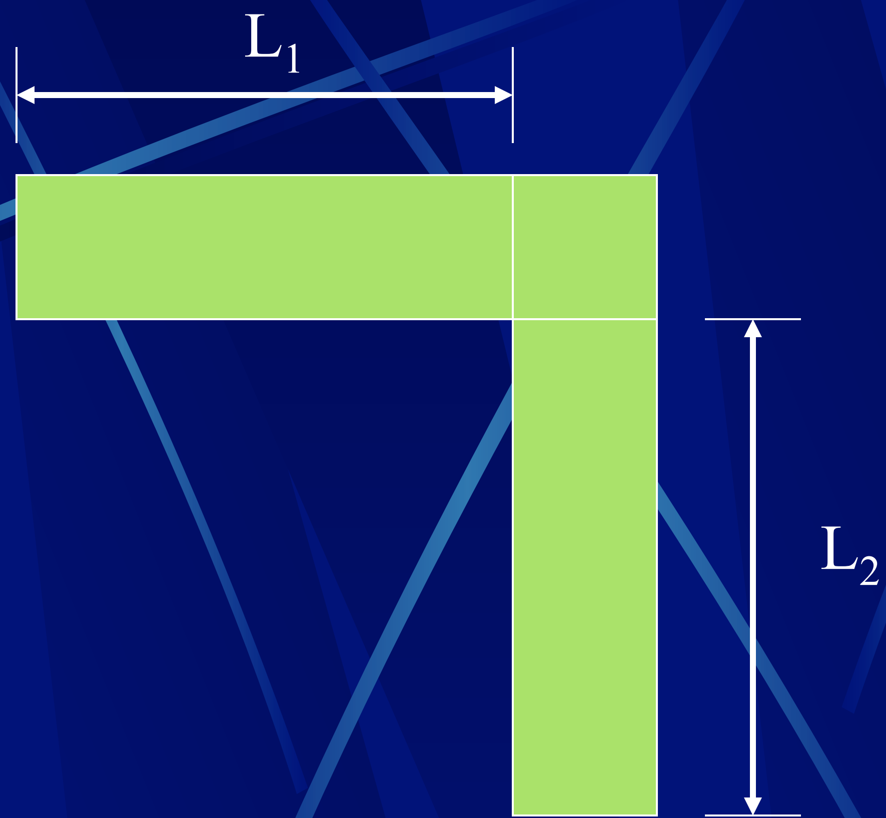
(short radius)



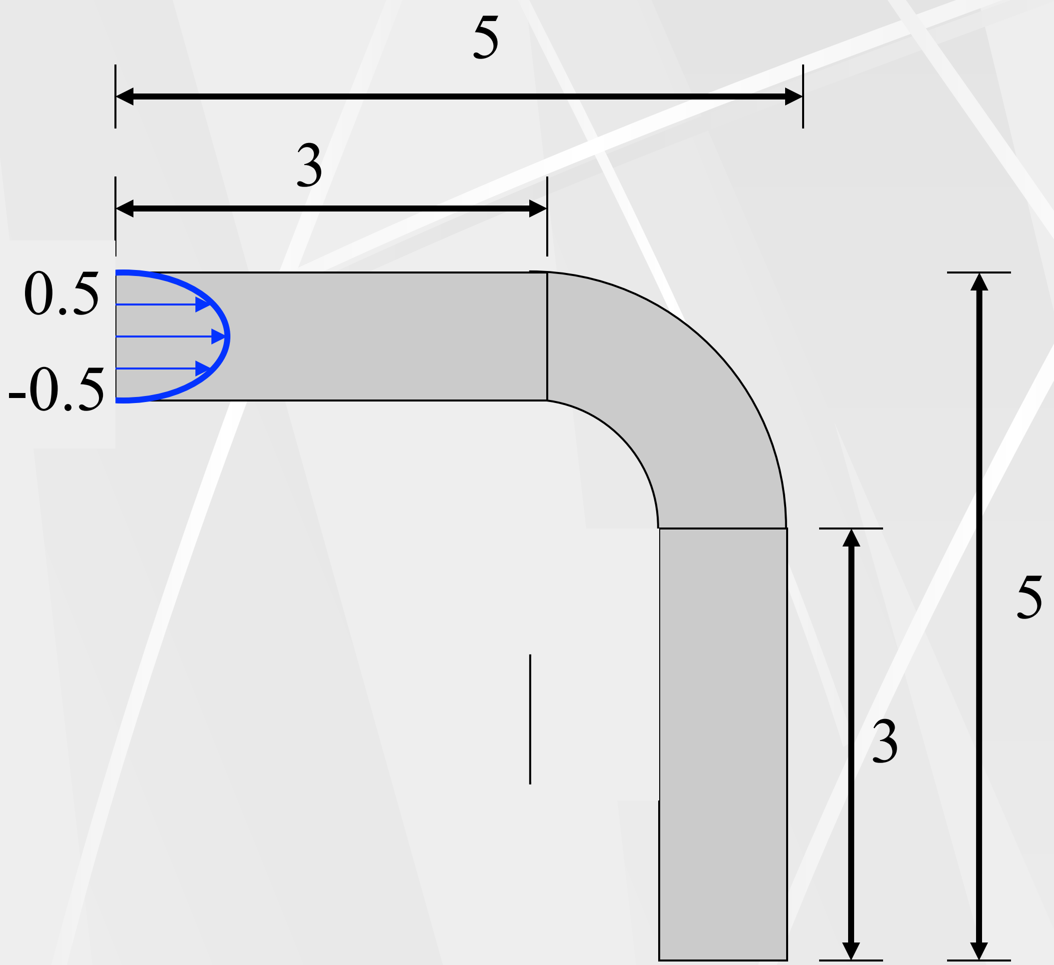
Bend 45° with smooth corner

(long radius)





$\Delta P'_{total} = \Delta P'_{L1} + \Delta P'_{L2} + \Delta P'_{corner}$



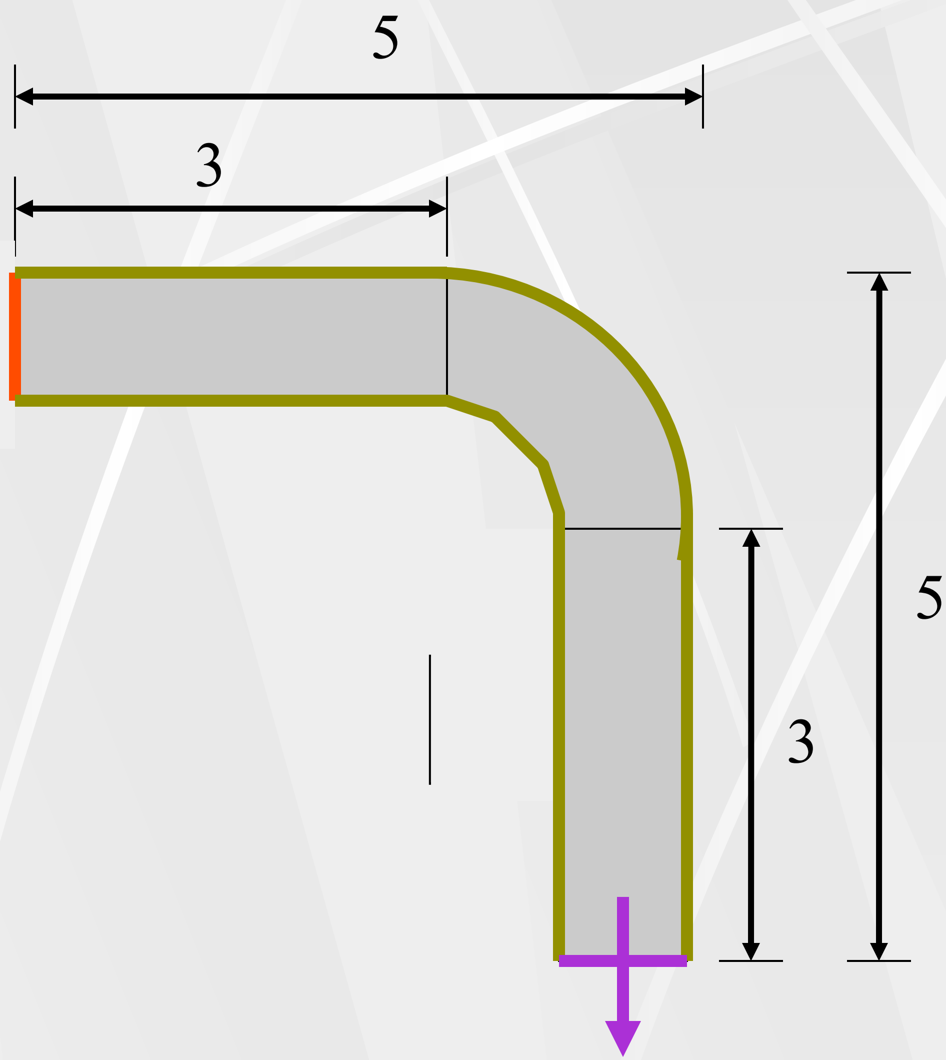
Boundary conditions

1) $u(y) = \mathbf{C}\,[1-(y/0.5)^2]$

$$\langle u\rangle = \frac{\int_{-0.5}^{0.5} u(y)\,dy}{\int_{-0.5}^{0.5} dy}$$

$$1 = \mathbf{C}\left(1 - \frac{y^3}{3(0.5)^2}\,\Bigg|_{-0.5}^{0.5}\right)$$

$$\boxed{\mathbf{C} \;=\; \mathbf{1.5}}$$



Boundary conditions

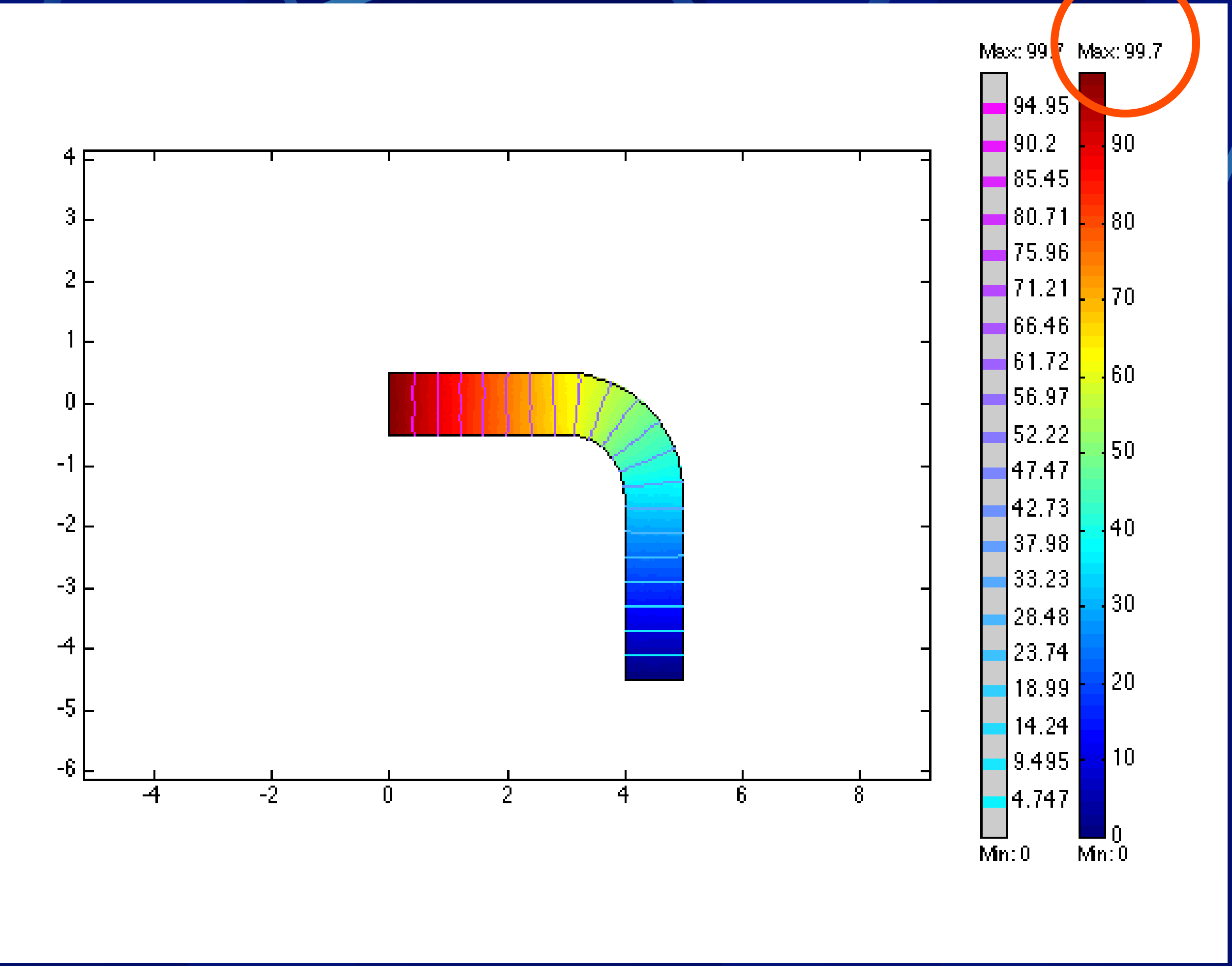
1) $u(y) = 1.5\,[1-(y/0.5)^2]$

2) $u = 0$, $v = 0$

3) Straight out , $p = 0$

Plot Pressure Contours

(Solved with Re = 0 , ρ = 1)



mesh refinement # 1 (533 nodes , 984 triangles)

P (0.00462,-0.0216) = 99.6034

Use Hagen-Poiseuille Equation to find $\Delta P'_{L1}$ and $\Delta P'_{L2}$

$$\boxed{\frac{-\Delta P'_{L1}}{L_1} = \mu \frac{d^2u'}{dy^2}}$$

where $u' = 1.5\,[1 - (y/0.5)^2\,]$

$$du'/dy' = 1.5\,(\,-\,4*2y)$$

$$d^2u'/dy'^2 = -\,12$$

$$\Delta P'_{L1} = (\,-\,L_1) \times (\,-\,12) = 3 \times 12 = 36$$

$\Delta P'_{corner} = \Delta P'_{total} - \Delta P'_{L1} - \Delta P'_{L2}$

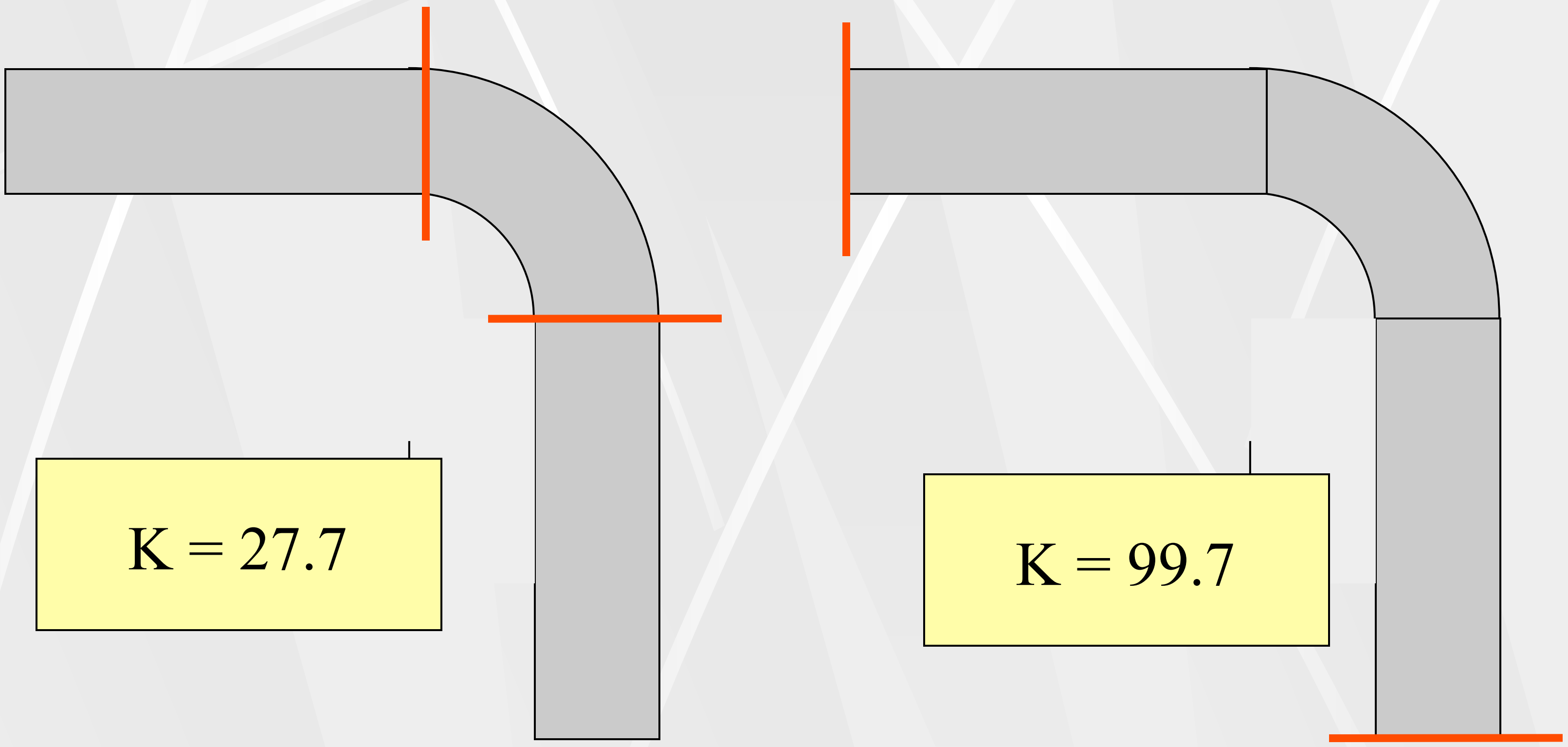
$$= 99.7 - 36 - 36$$

$$= 27.7$$

$$\boxed{\Delta P' = K \langle u \rangle'}$$

$$\boxed{K = 27.7}$$

Two ways to report K values

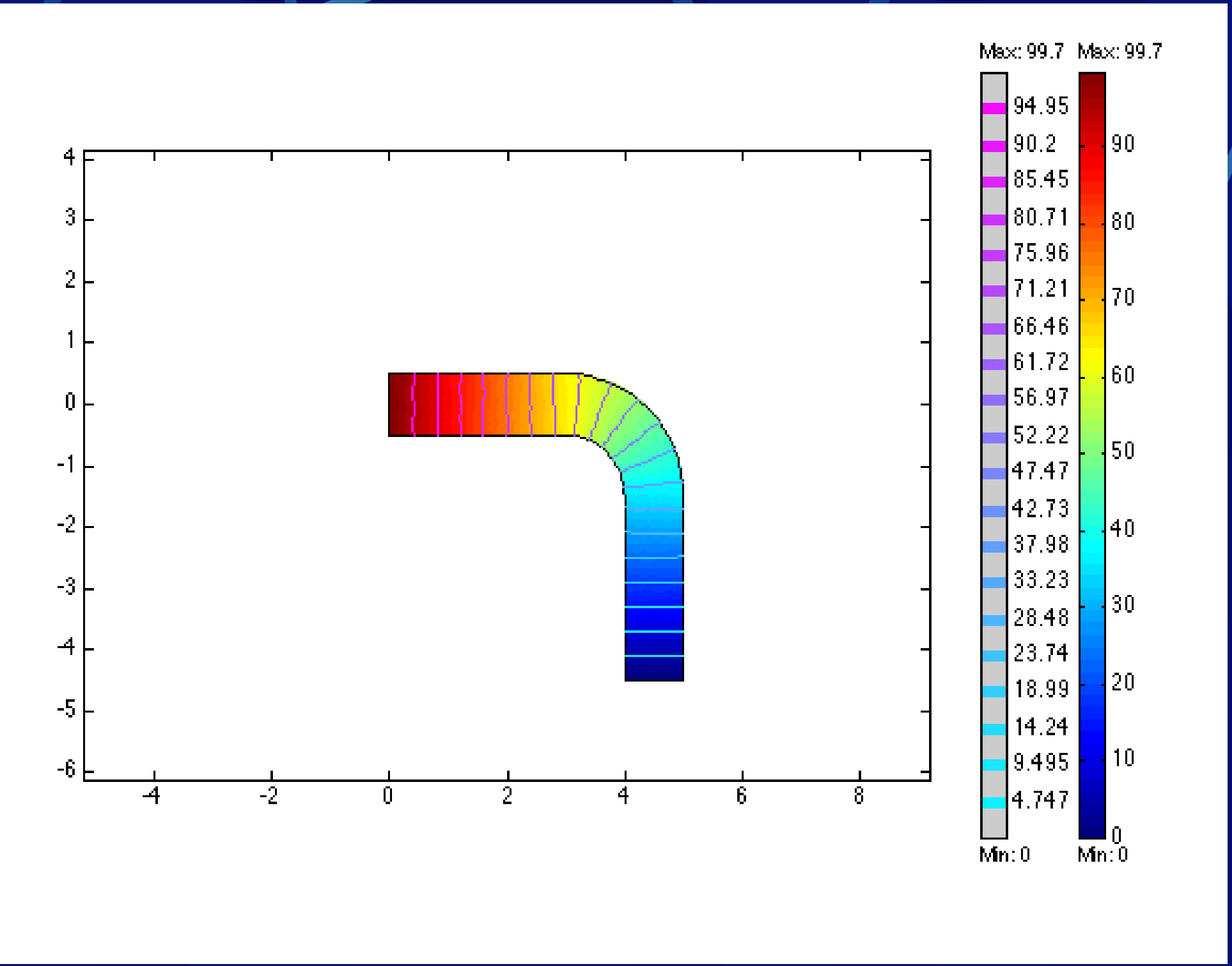


K for various 2D planar geometries

geom. / If use	bend90°	bend90° smooth (short)	bend90° smooth (long)	bend45° smooth (short)	bend45° smooth (long)
K_{corner}	9	18	27.7	6	11
K_{total}	105	102	99.7	102	107

Plot Pressure Contours

(Solved with Re = 0 , ρ = 1)

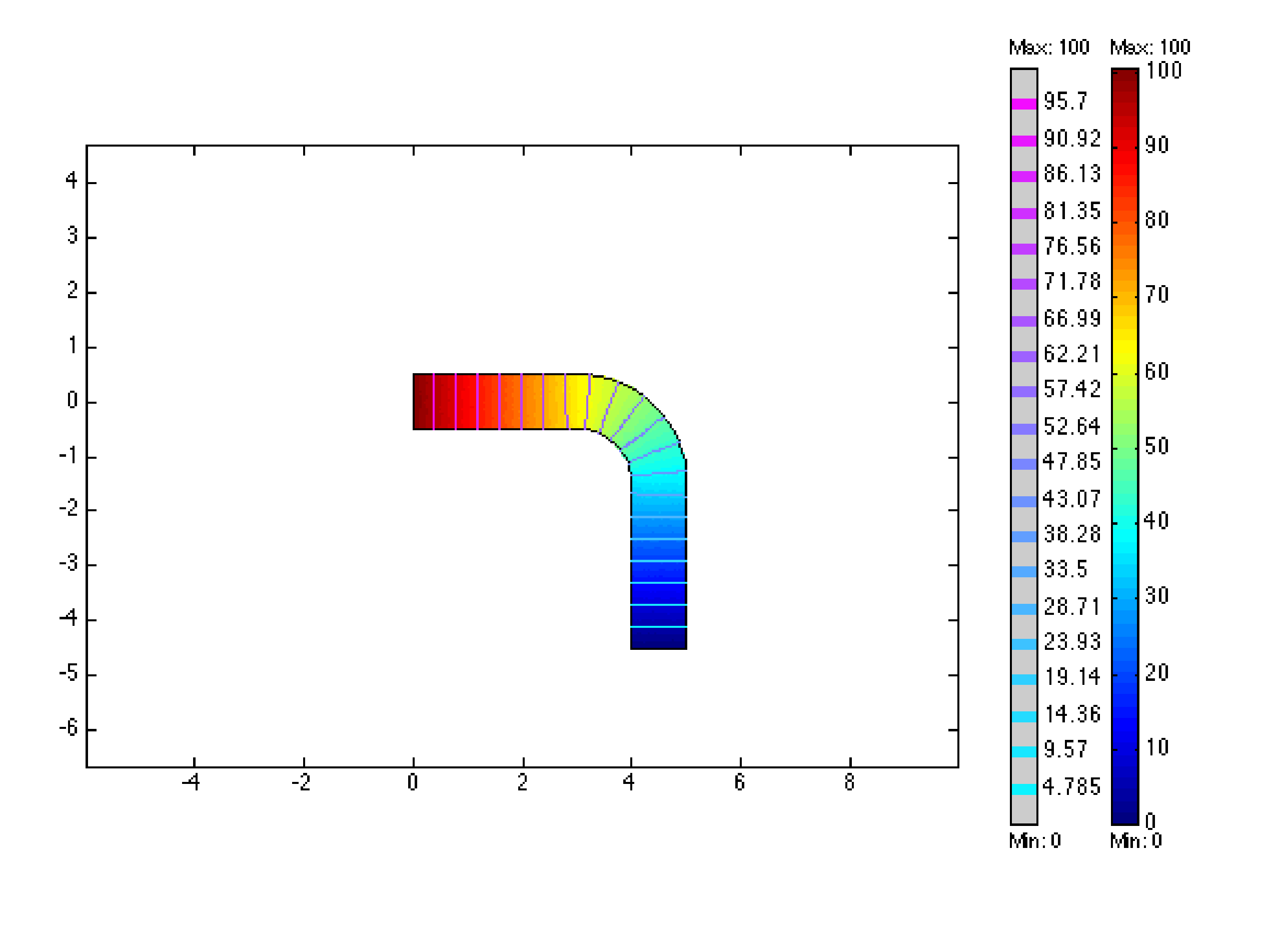


mesh refinement # 1 (533 nodes , 984 triangles)

P (0.00462,-0.0216) = 99.6034

Plot Pressure Contours

(Solved with Re = 0 , ρ = 1)

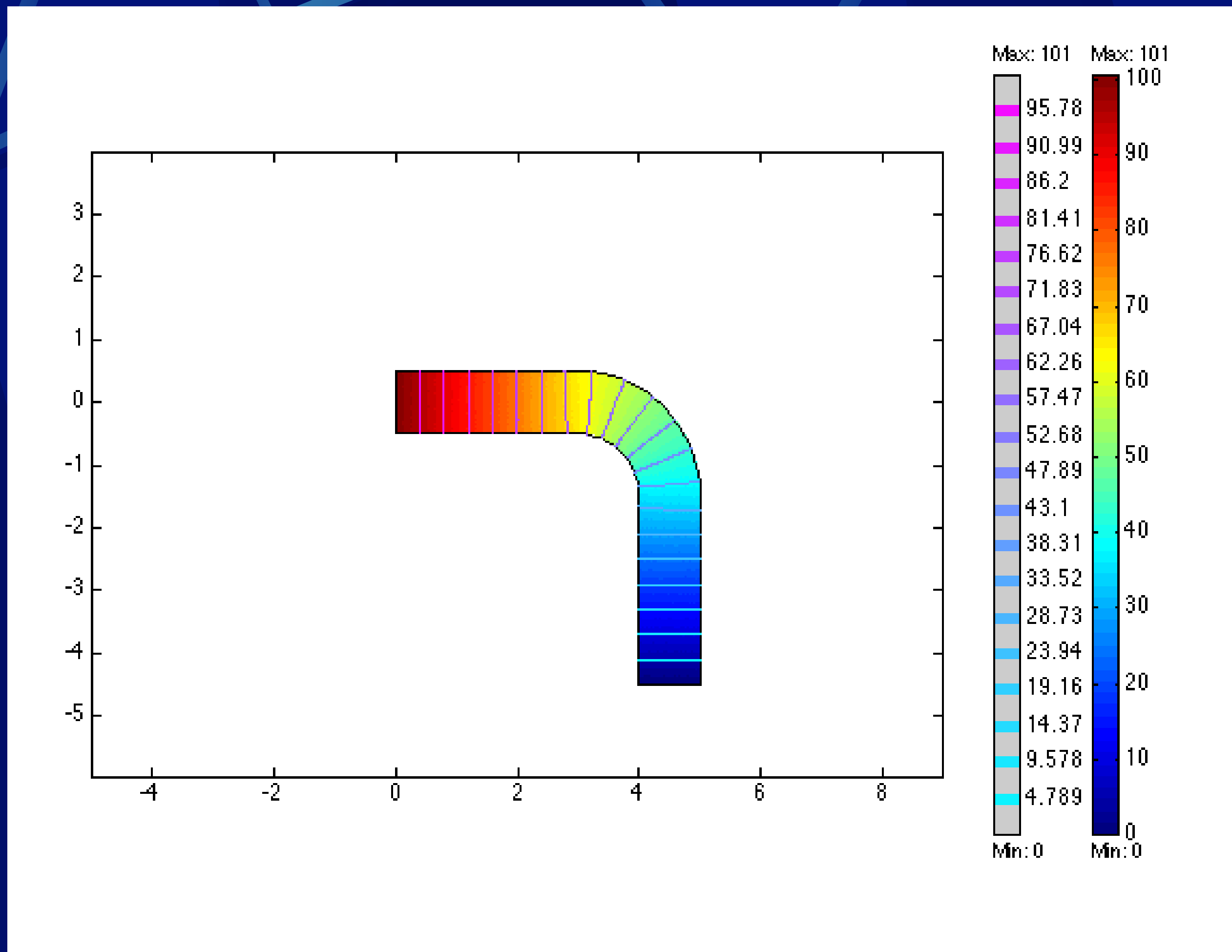


mesh refinement # 2 (2089 nodes , 3936 triangles)

P (0.00462,-0.0216) = 100.382

Plot Pressure Contours

(Solved with Re = 0 , ρ = 1)

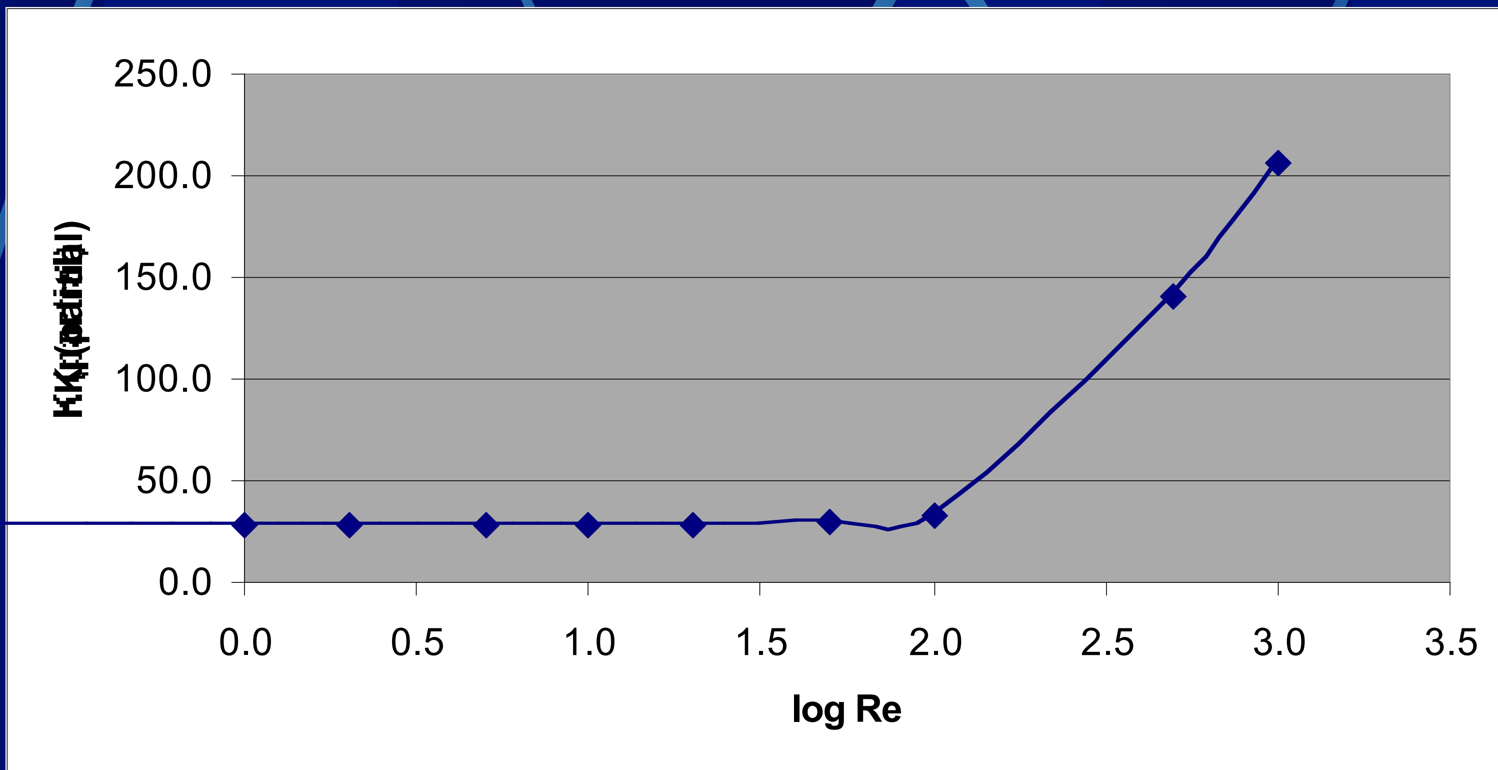


mesh refinement # 3 (8113 nodes , 15744 triangles)

P (0.00462,-0.0216) = 100.488

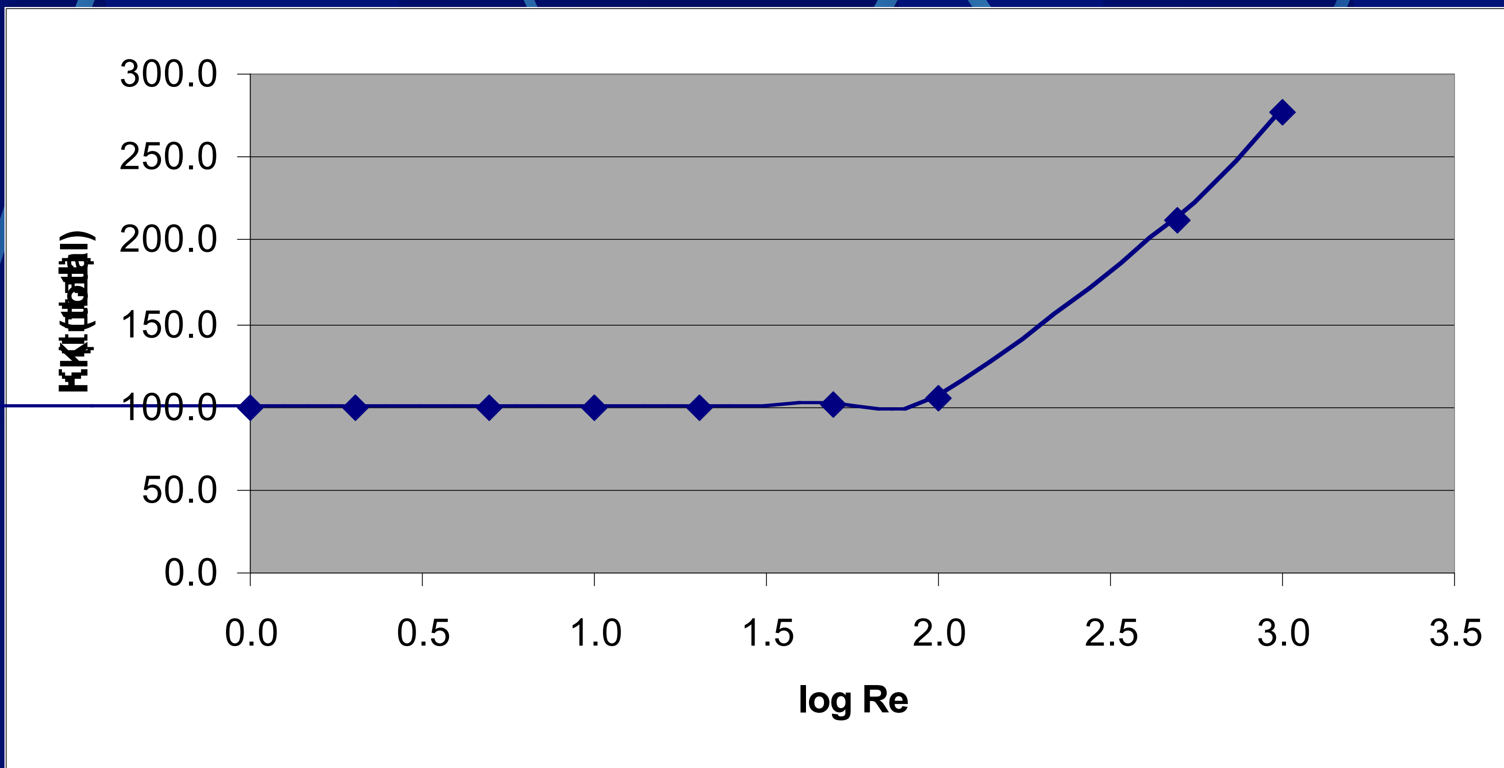
The effect of Reynolds number on K

(from $\Delta P'_{corner}$)

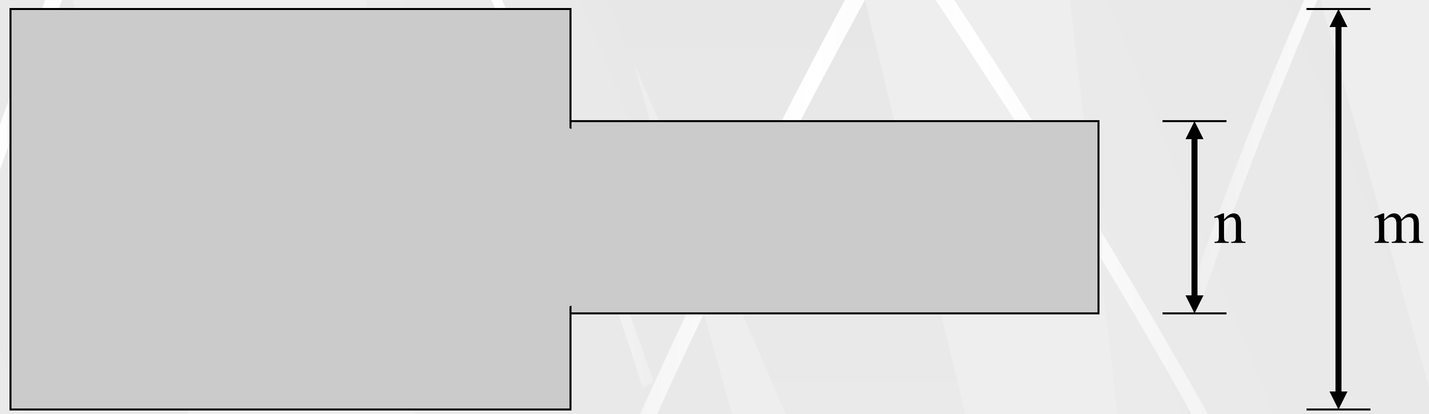


The effect of Reynolds number on K

(from $\Delta P'_{total}$)



Sudden Contraction



Boundary conditions

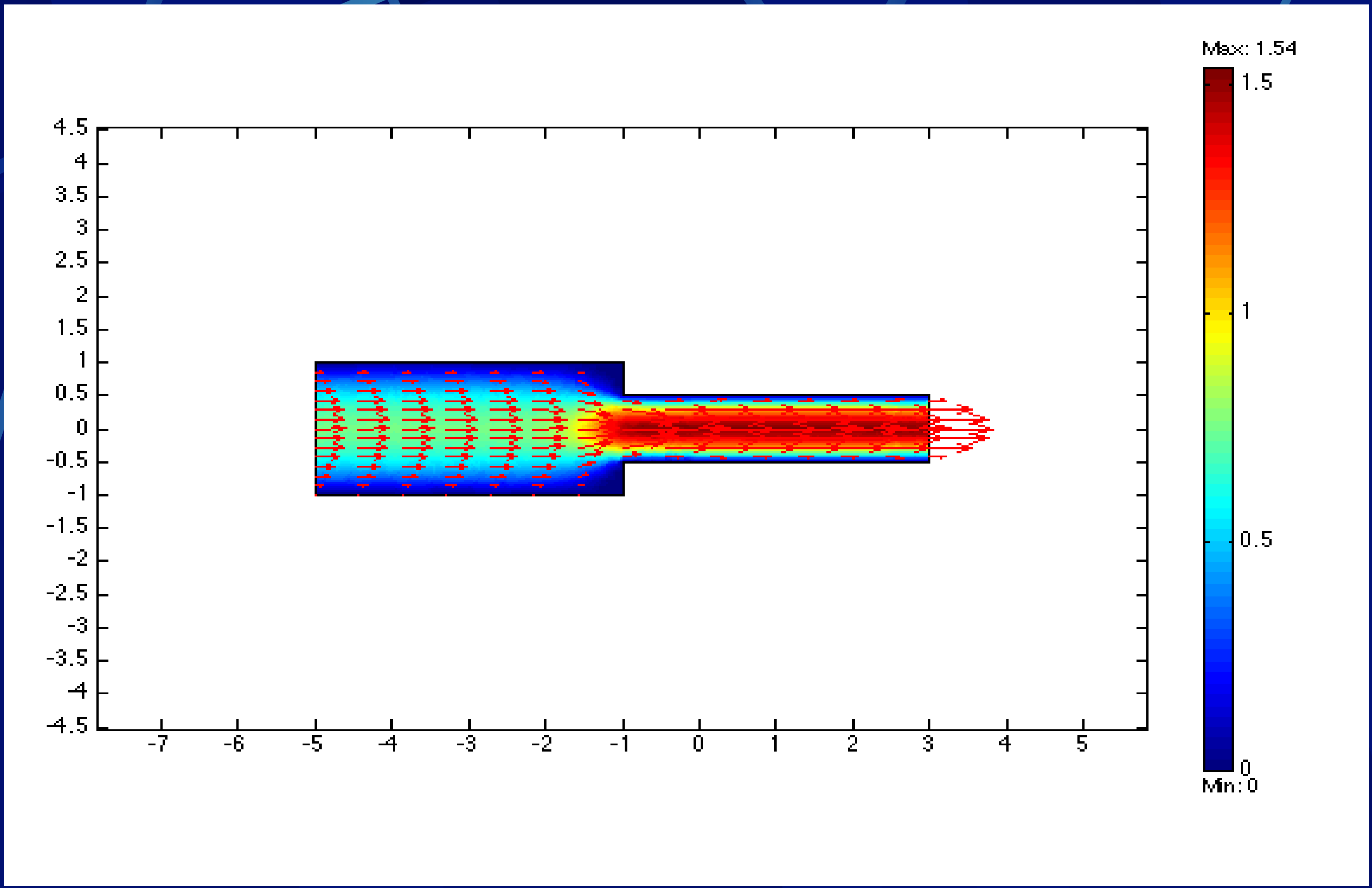
1) Set $\langle u \rangle_{outlet} = 1$

Use Bernoulli's Equation to find $\langle u \rangle_{inlet}$, then the inlet velocity profile

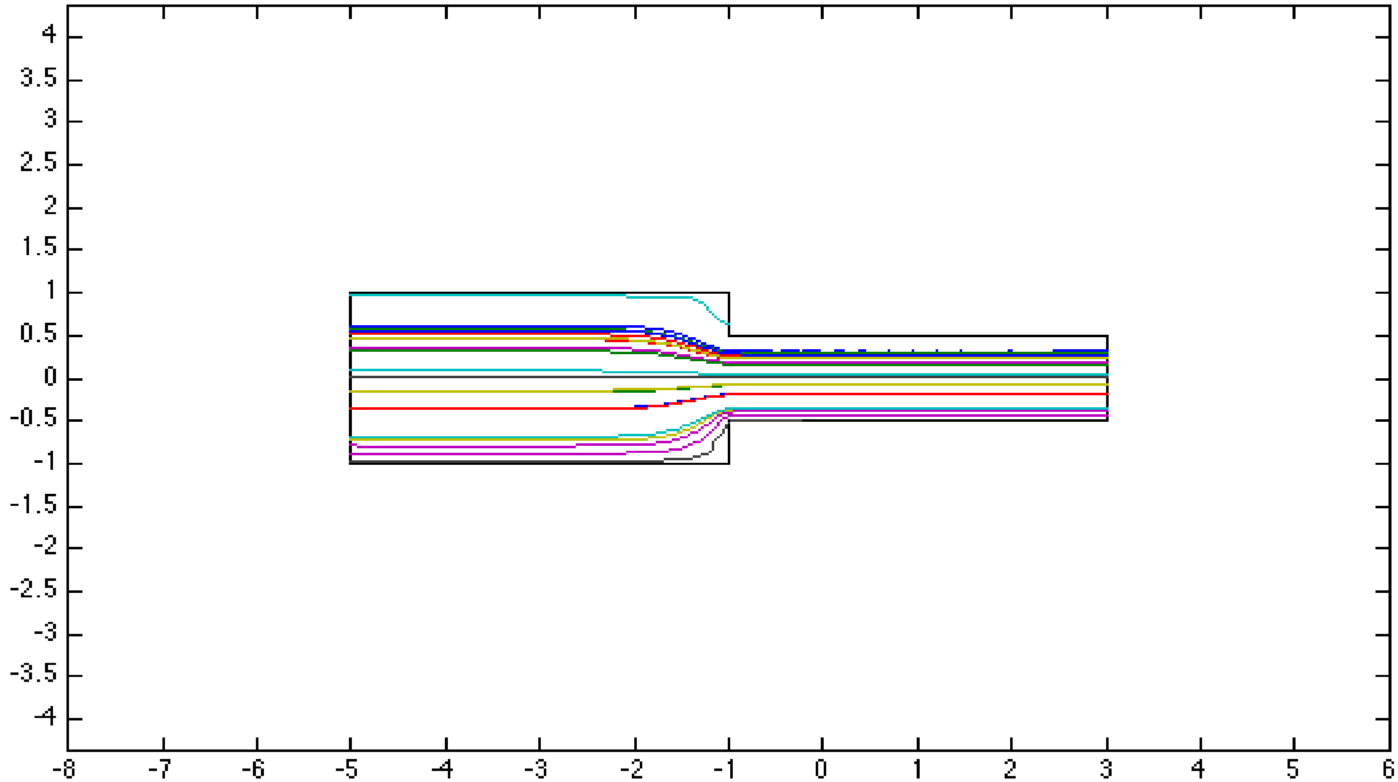
2) At the wall , $u = 0$ and $v = 0$

3) Straight out , $p = 0$

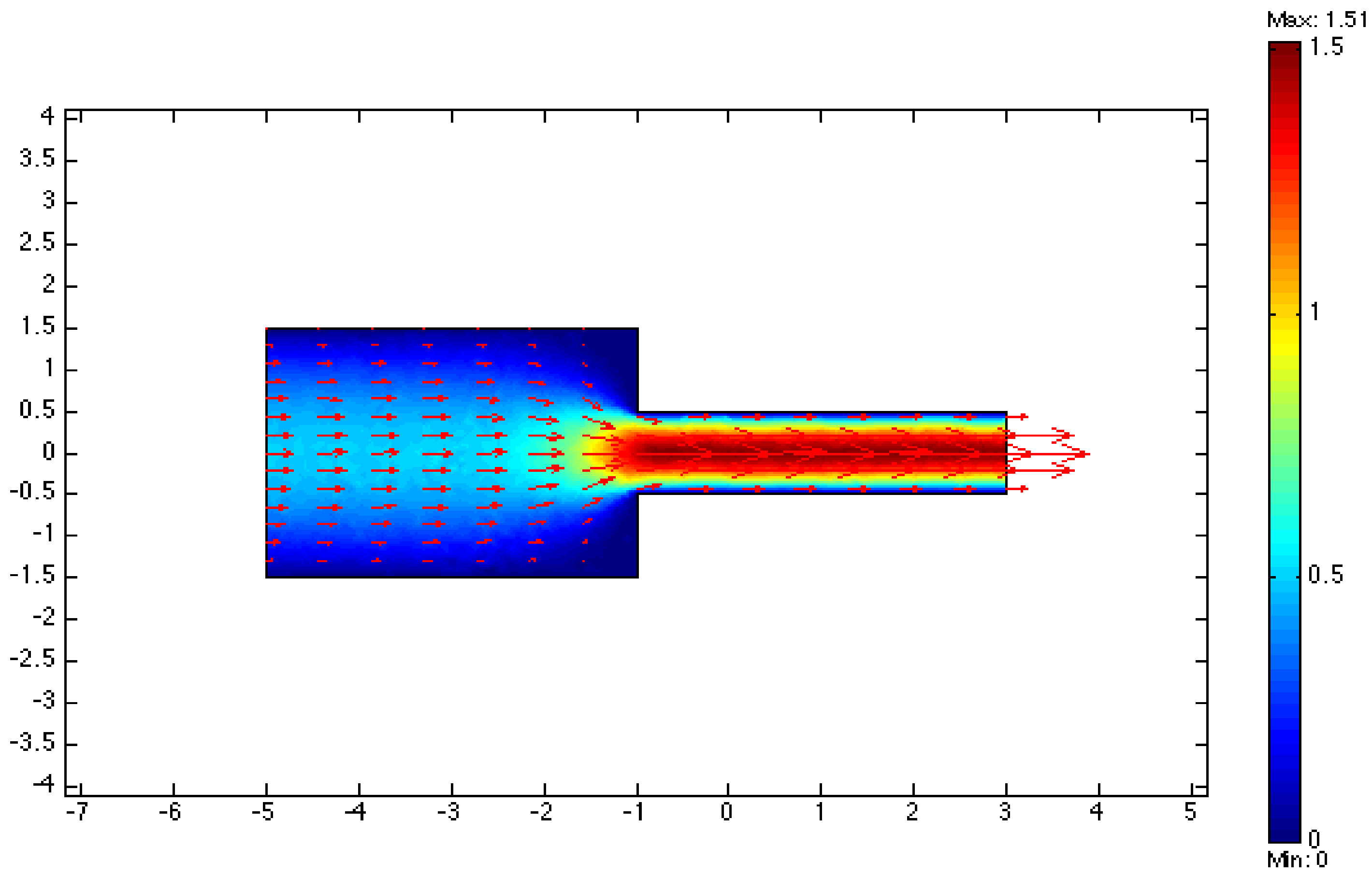
Velocity field plot for Case 1 (m/n = 2)



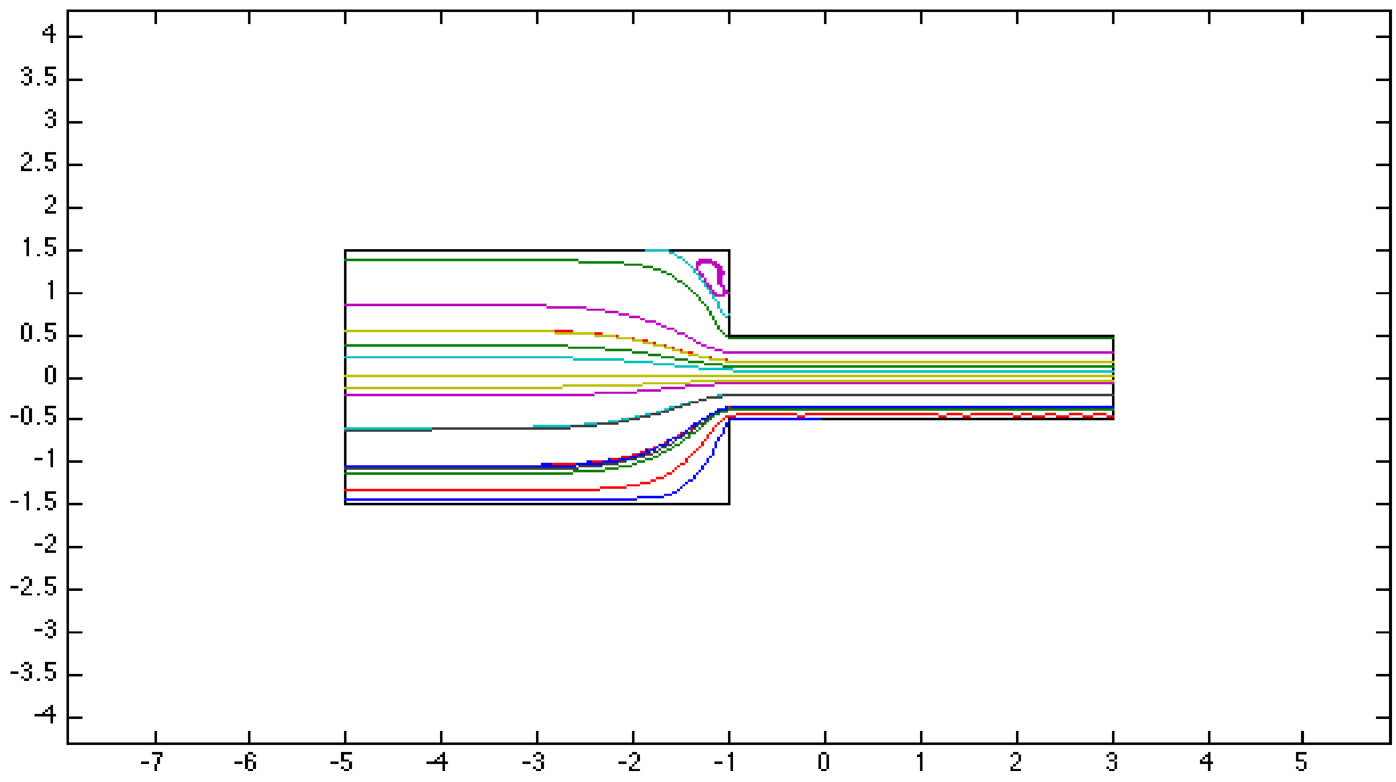
Flow lines plot for Case 1 (m/n = 2)



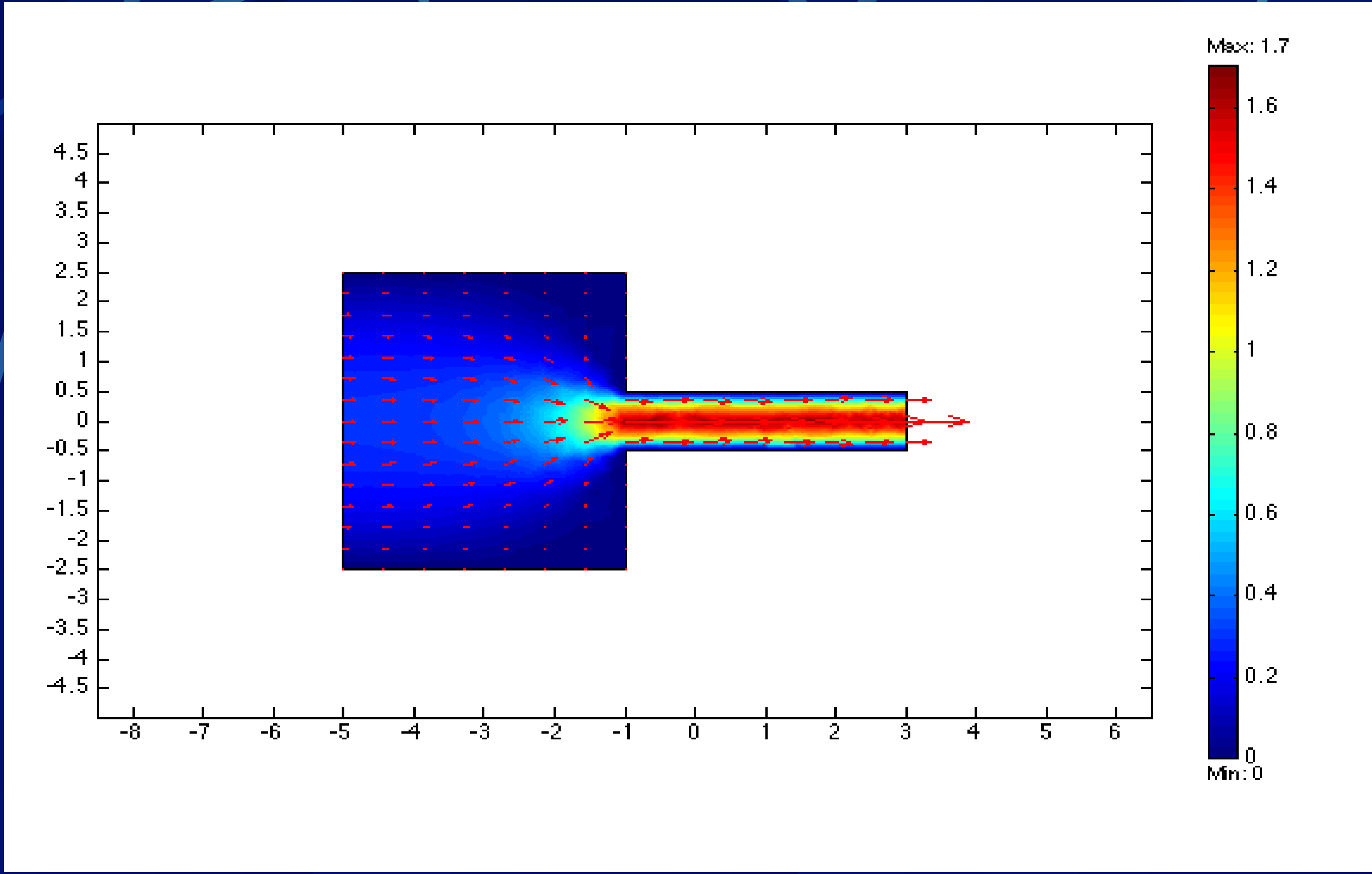
Velocity field plot for Case 2 (m/n = 3)



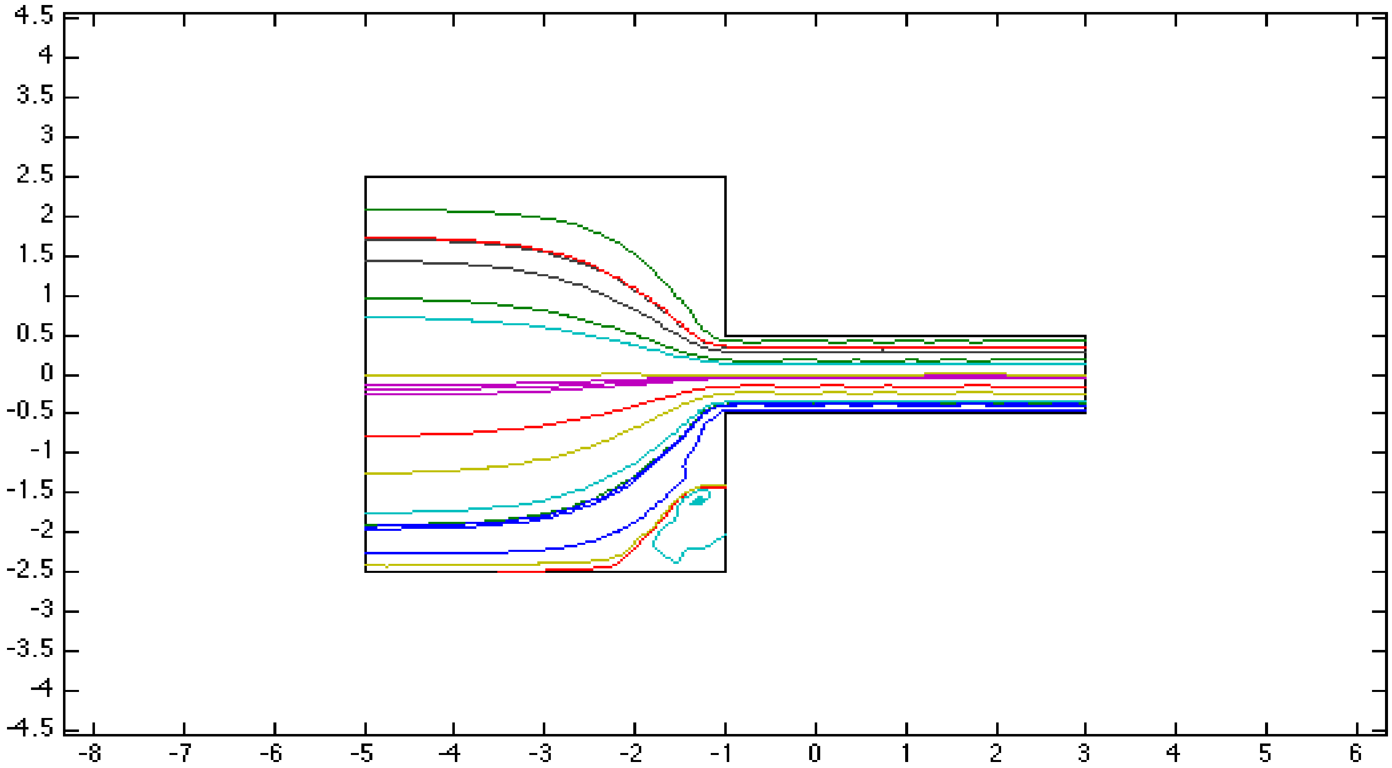
Flow lines plot for Case 2 (m/n = 3)



Velocity field plot for Case 3 (m/n = 5)



Flow lines plot for Case 3 (m/n = 5)



K values from 3 cases

Case 1 ($m/n = 2$) :	$K = 73$
Case 2 ($m/n = 3$) :	$K = 85.33$
Case 3 ($m/n = 5$) :	$K = 122.04$

Conclusions

Pressure coefficient can be calculated for laminar flow using FEMLAM

Pressure drop in laminar flow depends mostly on the pathlength

Inertia effects begin to become important at Reynolds number of 100