

Pressure Drop Correlations in Tapered Contractions and Expansions for Laminar Flow

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Objective

- **Objective: Find ΔP and Re correlations in the Laminar Flow Regime ($Re < 1000$)**
- **Importance: Important for analyzing flow in microchannels of various geometries (tapered contraction/expansion)**

Theory

- **Use Femlab to set up channels of various geometry and boundary conditions**
- **Find ΔP due to contraction/expansion**
- **Compare this ΔP to Hooper's and Witarsa's results**

Relevant Equations

Hooper's

$$K = [1.2 + \frac{160}{Re_1}] [(\frac{D_1}{D_2})^4 - 1] [\sqrt{\sin(\frac{\theta}{2})}] \quad \text{for } 45^\circ < \theta < 180^\circ$$

$$K = [1.2 + \frac{160}{Re_1}] [(\frac{D_1}{D_2})^4 - 1] [1.6 \sin \frac{\theta}{2}] \quad \text{for } \theta < 45^\circ$$

Contraction k-values

Witarsa'

$$K = [1.2 + \frac{15.4}{Re_1}] [(\frac{D_1}{D_2})^4 - 1] [\sqrt{\sin(\frac{\theta}{2})}]$$

$$K = [1.2 + \frac{15.4}{Re_1}] [(\frac{D_1}{D_2})^4 - 1] [1.6 \sin \frac{\theta}{2}]$$

Expansion k-values

Hooper's

$$K = 2[1 - (\frac{D_1}{D_2})^4] \quad \text{for } 45^\circ < \theta < 180^\circ$$

$$K = 2[1 - (\frac{D_1}{D_2})^4] [2.6 \sin(\frac{\theta}{2})] \quad \text{for } \theta < 45^\circ$$

Derived Equations

$$\frac{\Delta p D}{\mu v_2} = \frac{K_1 Re_1}{18}$$

$$p_2' - p_1' = Re \langle v_2' \rangle^2 (1 - \beta^2) + E_v' \quad \text{Contraction}$$

$$p_2' - p_1' = Re \langle v_1' \rangle^2 (\frac{1}{\beta^2} - 1) + E_v' \quad \text{Expansion}$$

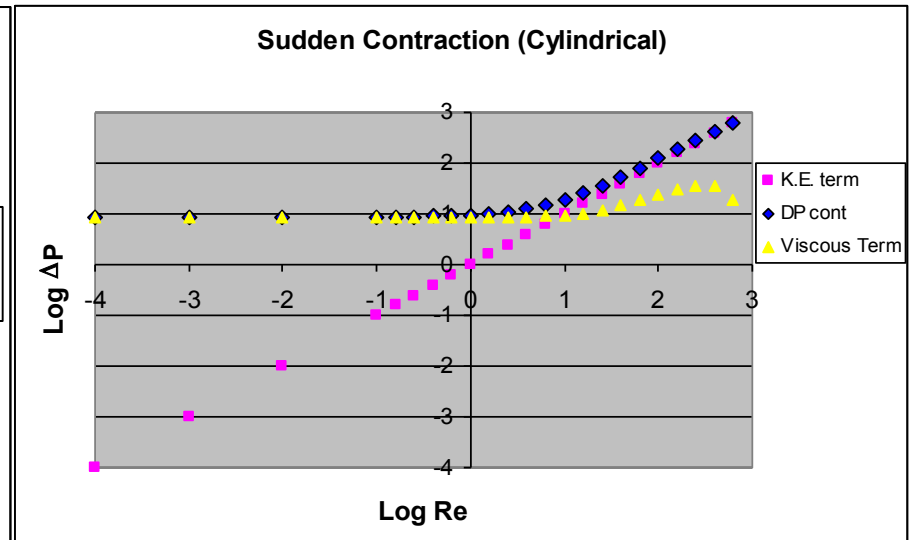
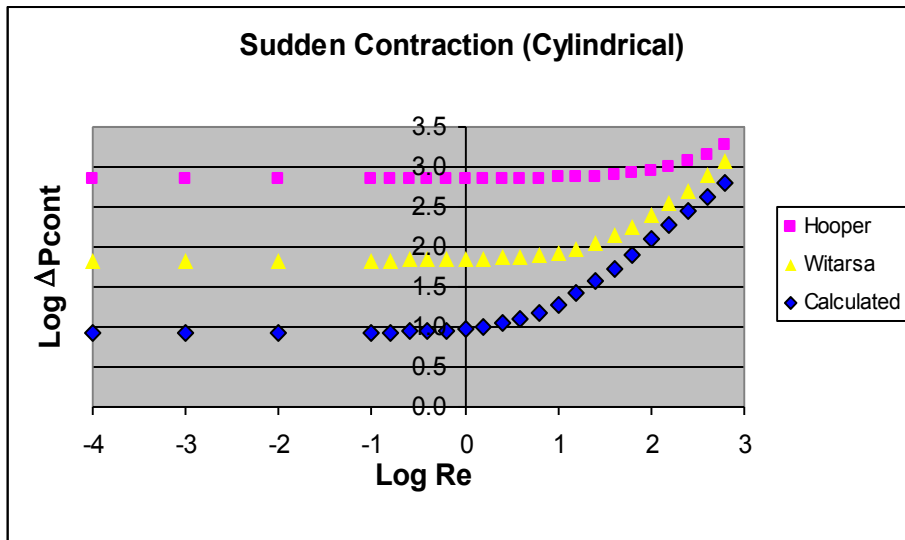
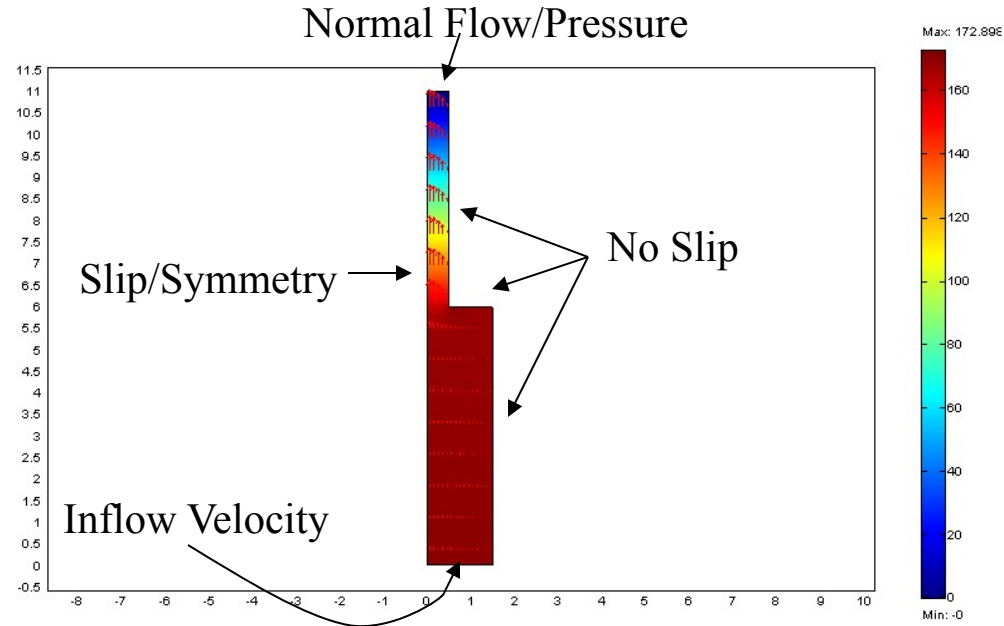
Results - Contraction

Cylindrical
Geometry

$$\theta = 180^\circ$$

2* Mesh
Refinement

1552 elements



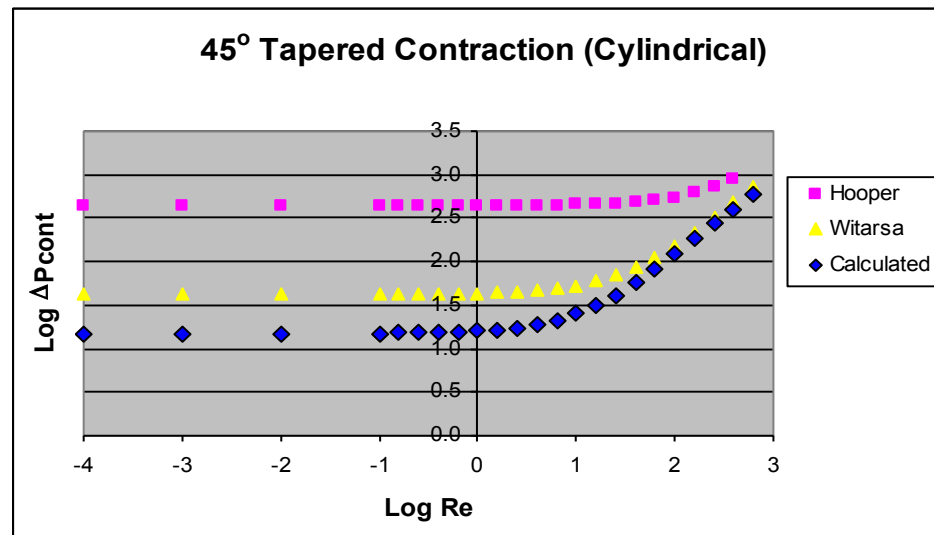
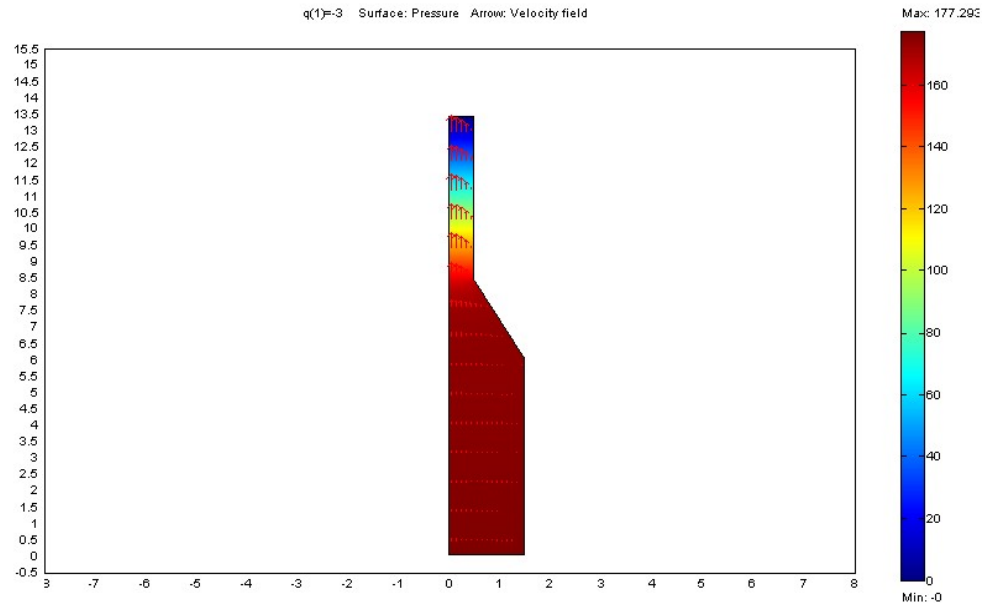
Results - Contraction

Cylindrical
Geometry

$$\theta = 45^\circ$$

2* Mesh
Refinement

1120 elements



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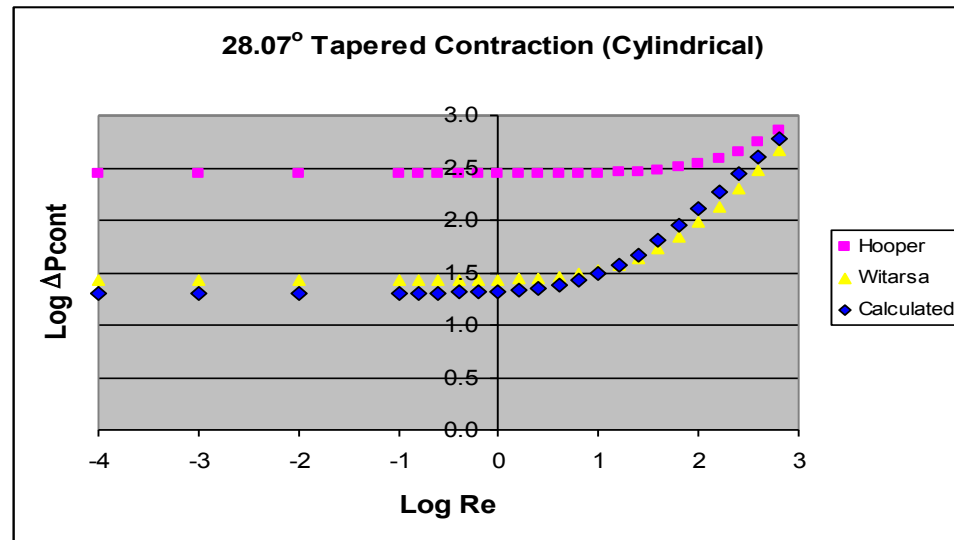
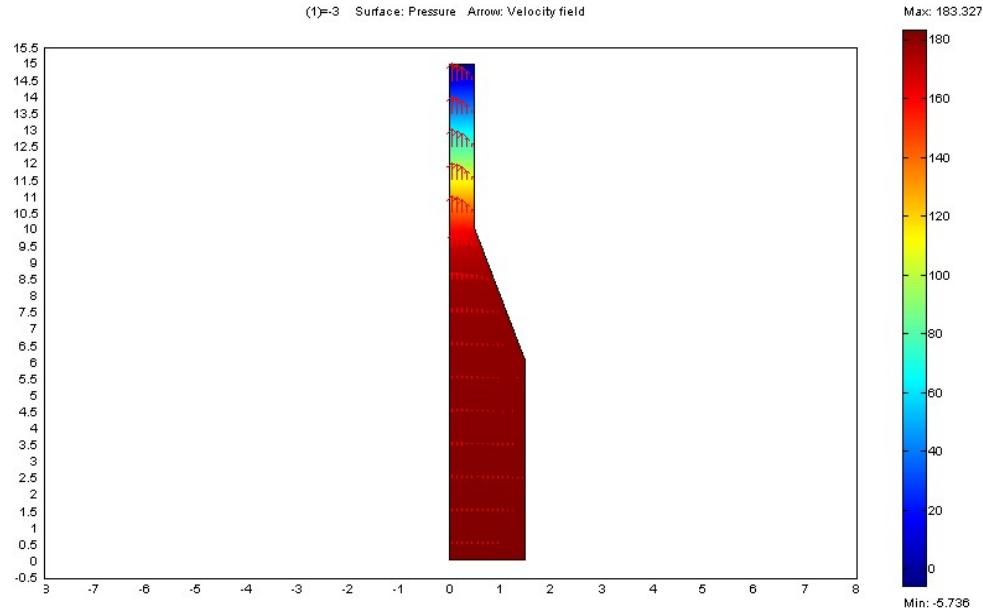
Results - Contraction

Cylindrical
Geometry

$$\theta = 28.07^\circ$$

2* Mesh
Refinement

1088 elements



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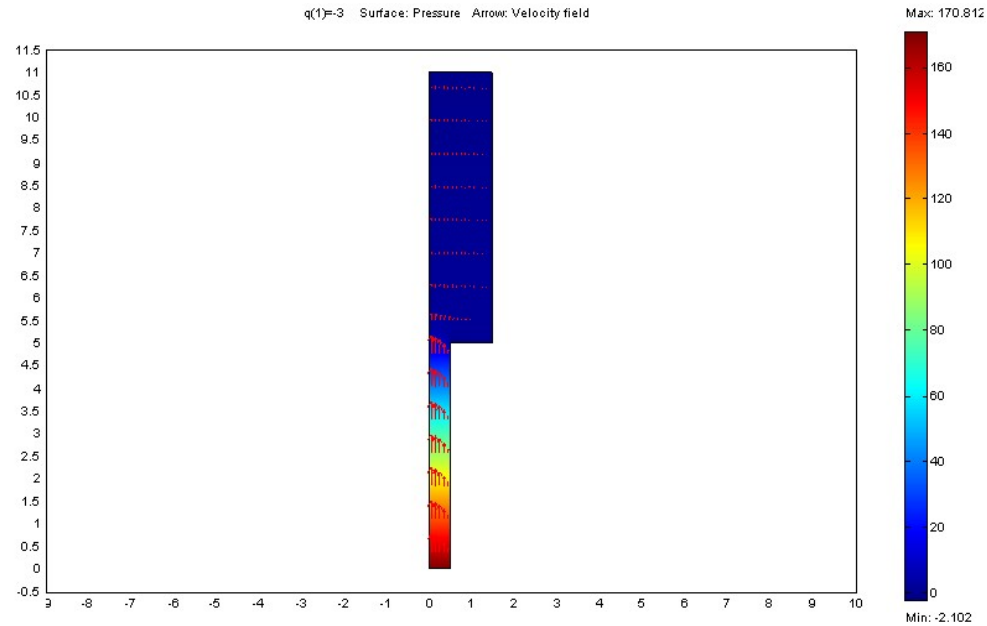
Results - Expansion

Cylindrical
Geometry

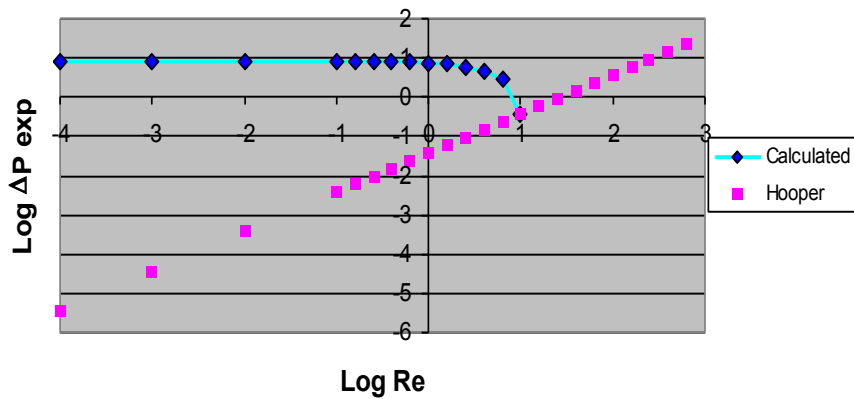
$$\theta = 180^\circ$$

2* Mesh
Refinement

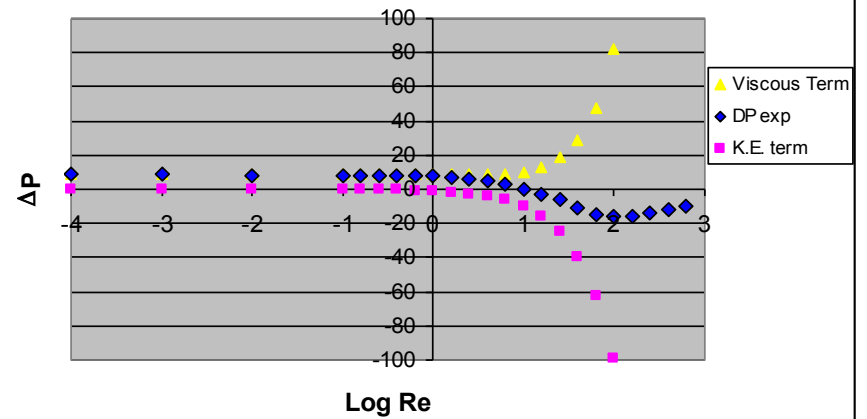
1648 elements



Sudden Expansion (Cylindrical)



Sudden Expansion (Cylindrical)



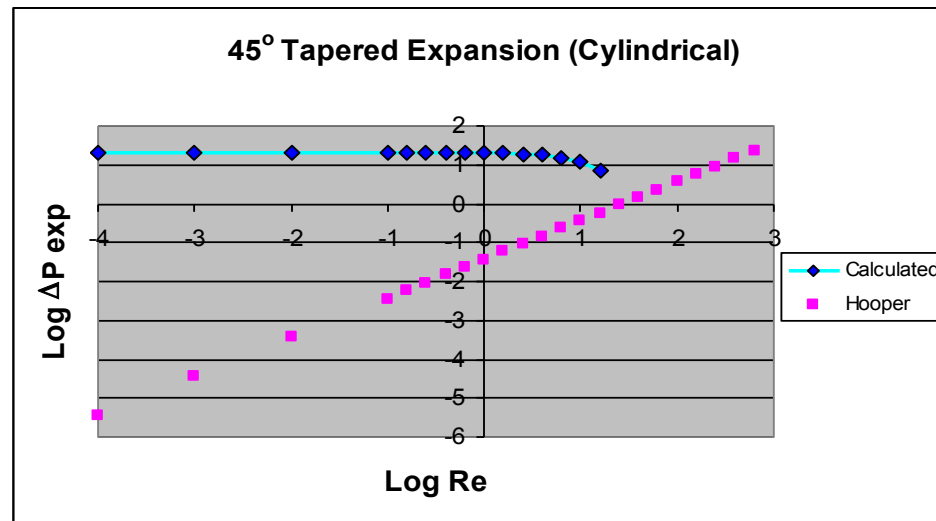
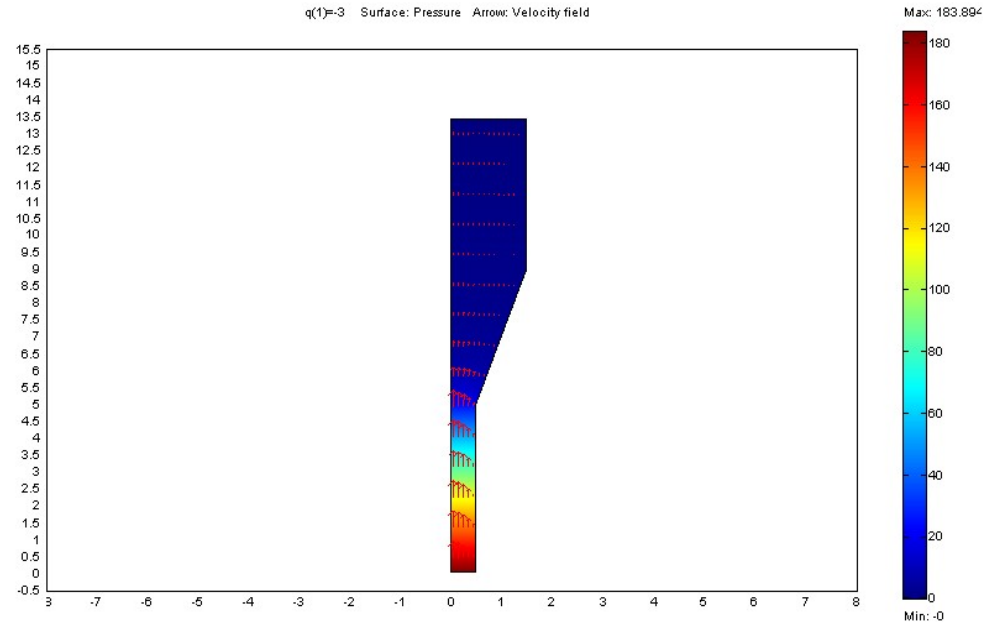
Results - Expansion

Cylindrical
Geometry

$$\theta = 45^\circ$$

2* Mesh
Refinement

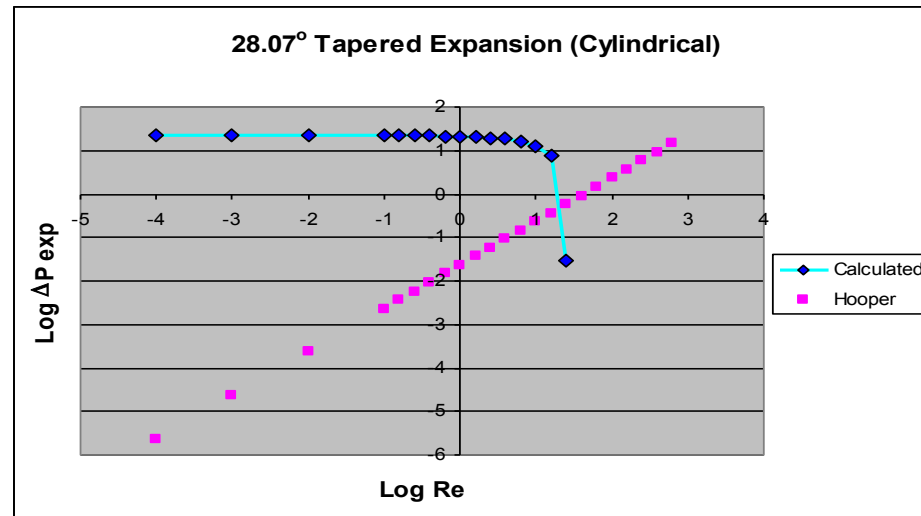
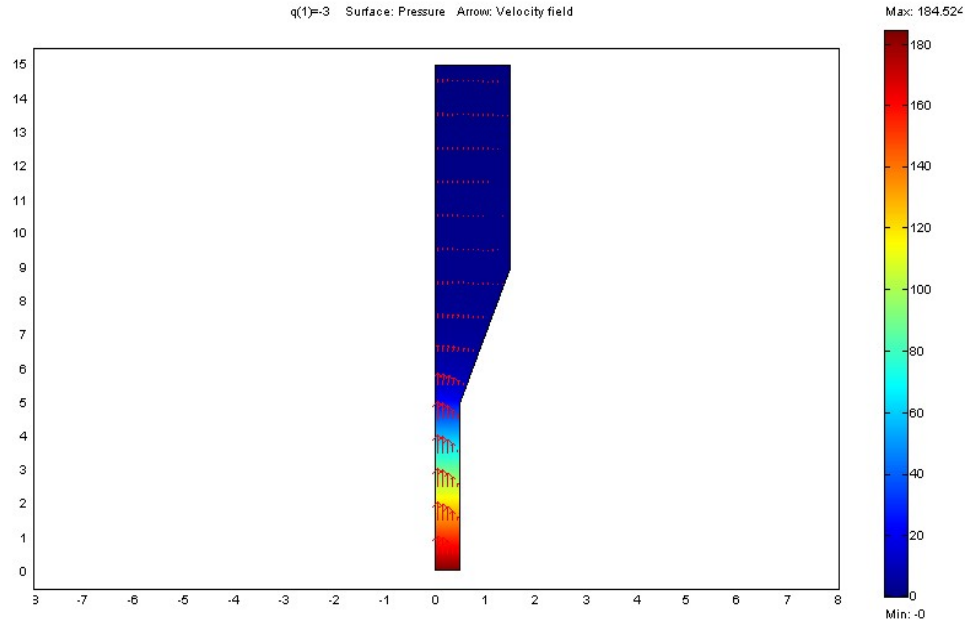
1200 elements



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Results - Expansion

Cylindrical
Geometry
 $\theta = 28.07^\circ$
2* Mesh
Refinement
1184 elements



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Results – Mesh Refinement

Contractions

180°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{con}	0.9054	0.9188	0.9293	0.9339
45°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{con}	1.1652	1.1728	1.1739	1.1739
28.07°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{con}	1.3172	1.3063	1.3041	1.3071

Expansions

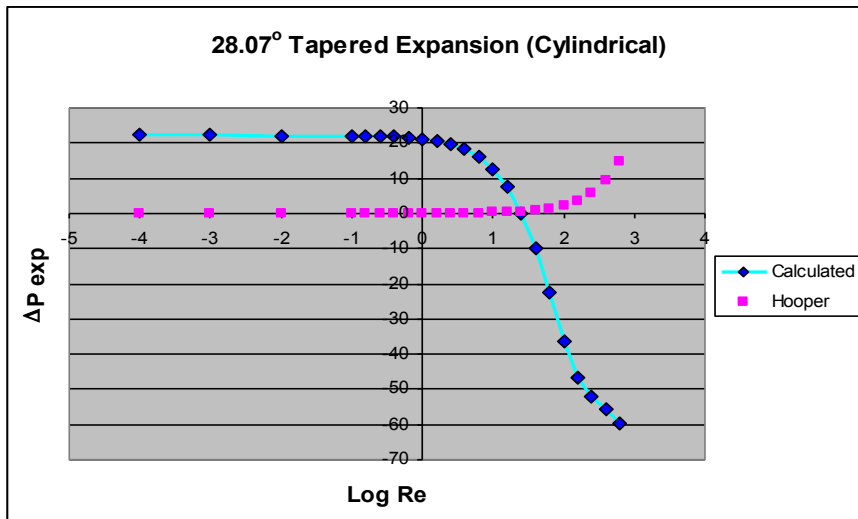
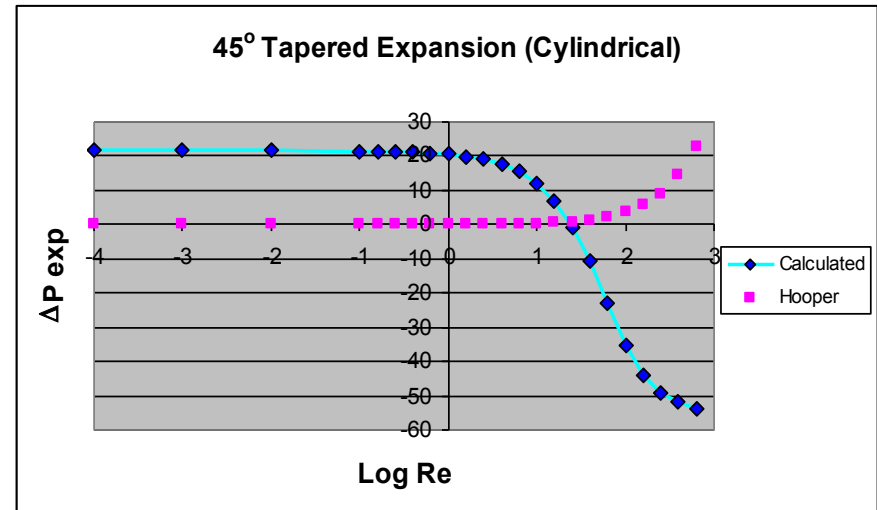
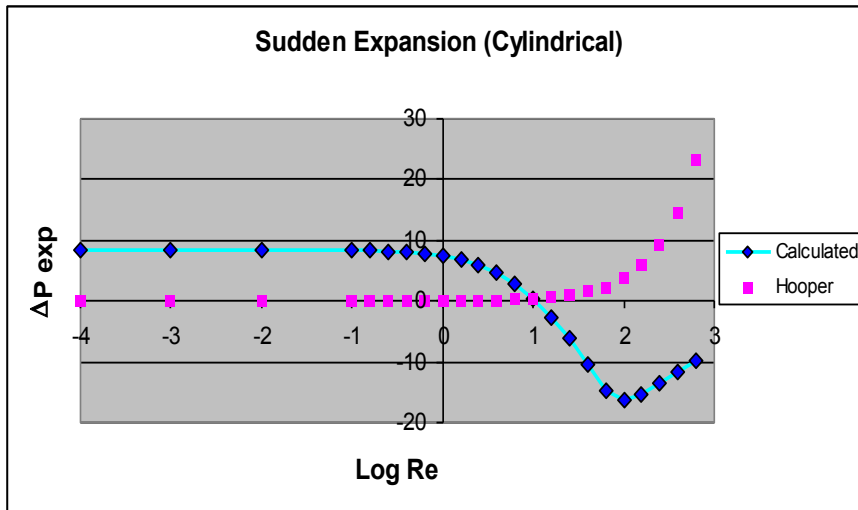
180°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{exp}	0.8980	0.9137	0.9252	0.9304
45°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{exp}	1.3283	1.3324	1.3329	1.3330
28.07°	Refinement	0*	1*	2*	3*
Re=.001	logΔP_{exp}	1.3425	1.3452	1.3454	1.3455

Discussion

- **Most ΔP values are accurate with 2 mesh refinements (3 for sudden expansions & contractions)**
- **Hooper's values are inaccurate at low Re ($Re < 900$)**
- **Δp_{cont} (28.07°) agrees well with Witarsa's results**
- **Δp_{cont} and Δp_{exp} are essentially constant and the same value for $Re < 1$ and the same geometry**

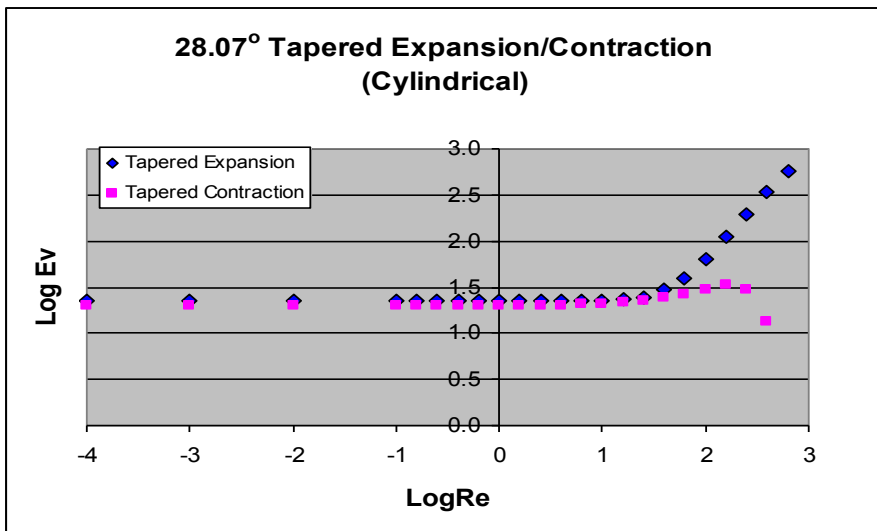
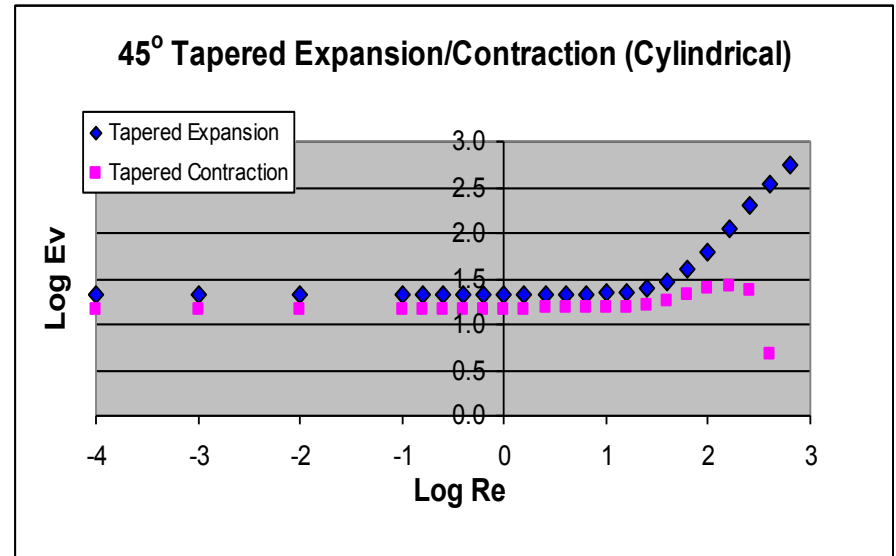
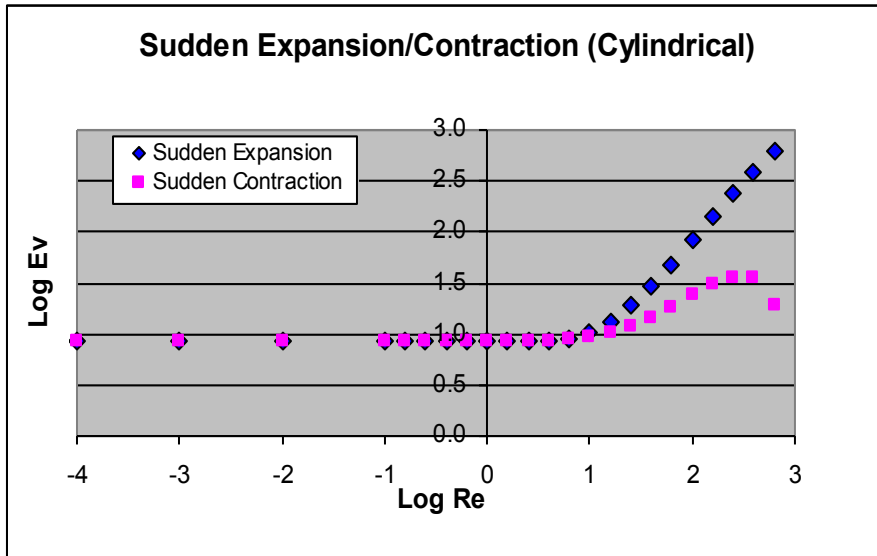
Questions?

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