Pressure Drop in Microchannels with Slip

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Table of Contents

1-Introduction

1.1 Purpose

The purpose of this research is to evaluate the pressure drop in various microfluidic geometries. The pressure drop was examined in two ways; how the pressure drop varies in one device as a function of the Reynolds number and how the pressure drop varies across different types of geometries with the same Reynolds number. The pressure drops were evaluated and are presented for two cases in the geometry; slip and no slip boundary conditions.

The first goal, to evaluate pressure drop as the Reynolds number changes in a device, was solved by running models in COMSOL Multiphysics. In the models, the Incompressible Navier-Stokes equation was solved, utilizing appropriate boundary settings over a range of Reynolds numbers. These sets of solutions were then plotted to determine where regions of constant pressure drop occur, making K_L values applicable.

The second goal, to compare pressure drops between different geometries, was also solved by running models in COMSOL Multiphysics. The results of the computer simulations are presented in table form for solutions with a Reynolds number of 0 in all devices. All devices have an outlet average velocity of 1 and a diameter of 1 to allow direct comparison of pressure drops.

Simulations have been compared to expected trends in literature presented in the book *Micro Instrumentation*. These results are important because they will allow future engineers to predict how pressure drops vary between different types of turns. It will also allow K_L values to be calculated easily and give limits on the regions where K_L values are applicable.

1.2 Expected Results

From literature results, pressure drop is expected to increase as the Reynolds number increases. This result is expected by comparison to Figure 8.4 in *Micro Instrumentation*

(Reprinted Below). A table showing K_L values for different geometries at negligible Reynolds numbers is presented in *Micro Instrumentation* Table 8.2 and Table 8.3. Literature and experimental results are compared in the appendix of this report.

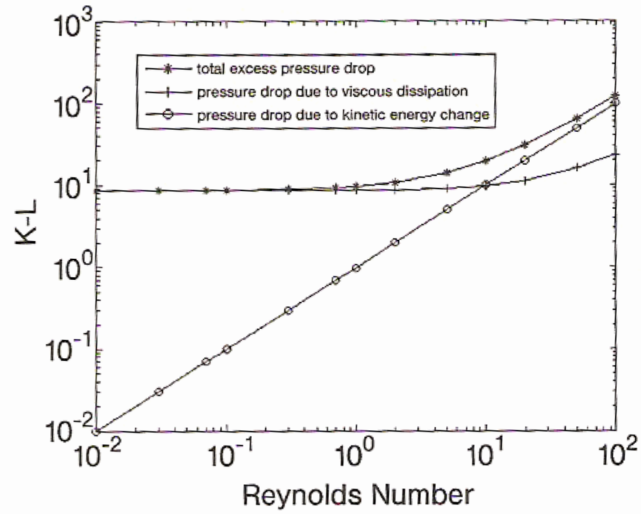


Fig. 8.4 Laminar flow excess pressure drop for a 3 : 1 contraction in a circular channel. K-L is the total pressure minus the fully developed pressure drop in the two regions, expressed as. $K_L = \Delta p_{excess}\, d_2/\eta \langle v_2 \rangle$.

2-Problem in Detail

2.1 Equations

To solve this problem, the Incompressible Navier-Stokes equation was solved in the non-dimensional forms.

For the Navier-Stokes equation, we start out with the dimensional form of the equation:

$$\rho\frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \mu \nabla^2 u$$

Equation 1

To non-dimensionalize this equation we then define the following quantities:

$$u' = \frac{u}{u_s},\ p' = \frac{p}{p_s},\ x' = \frac{x}{x_s},\ \nabla = x_s \nabla$$

Equation 2

This equation can then be arranged to give the following non-dimensional form:

$$\frac{\partial u}{\partial t} + \mathrm{Re}\, u' \cdot \nabla' u' = -\nabla' p' + \nabla'^2 u', \mathrm{Re} = \frac{\rho u_s x_s}{\mu}$$

Equation 3

In the non dimensional form of the Navier-Stokes equation, the dynamic viscosity (η) is set equal to 1 and the density (ρ) is used as a stand in for the Reynolds number which is set to 0 or varied from 0.1 to 100.

K_L is a dimensionless pressure drop coefficient. For slow flow, pressure drop is linear in the velocity and the pressure drop is given by correlations of the form:

$$\Delta p_{excess} = K_L \frac{\eta \langle v \rangle}{d}$$

Equation 4

For the geometries created, d, the diameter of the pipe, is set to 1, η the dynamic viscosity is set to 1 and $\langle v \rangle$, the outlet velocity, is set to 1. This allows the excess pressure drop of the turn to be equal to the K_L value. The excess pressure drop is defined in equation five:

$$\Delta p_{excess} = \Delta p_{total} - \Delta p_{large\_channel} - \Delta p_{small\_channel}$$

Equation 5

To find the excess pressure the pressure drop in the fully developed flow regions of the device are subtracted from the turn. In the above equation applied to the pipe this means the pressure drop in the large and small channels of the pipe.

2.2 COMSOL Model Setup

2.2.1 Slip Model Setup

To find the pressure drop in microfluidic geometries with slip, the inlet velocity is set to 1 and slip boundary conditions are applied to the walls of the device. The outlet of the device is given the boundary conditions of outlet and the option pressure. The dynamic viscosity (η) is set to 1 and the density (ρ) is used as a stand in for the Reynolds number. In parametric sweeps ρ was varied from 0.1 to 100. To determine pressure drops at very slow flows, ρ, is set to 0.

2.2.2 No Slip Model Setup

To find the pressure drop in microfluidic geometries with no slip, the inlet velocity is quadratic and no slip boundary conditions are applied to the walls of the device. The outlet of the device is given the boundary conditions of outlet and the option pressure. The dynamic viscosity (η) is set to 1 and the density (ρ) is used as a stand in for the Reynolds number. In parametric sweeps ρ was varied from 0.1 to 100. To determine pressure drops at very slow flows, ρ, is set to 0.

It is important when directly comparing pressure drops that the flow out of the device has an average velocity of 1, so that a direct comparison of pressure drops can be made. Figure 1 below shows a sample device and the settings applied.

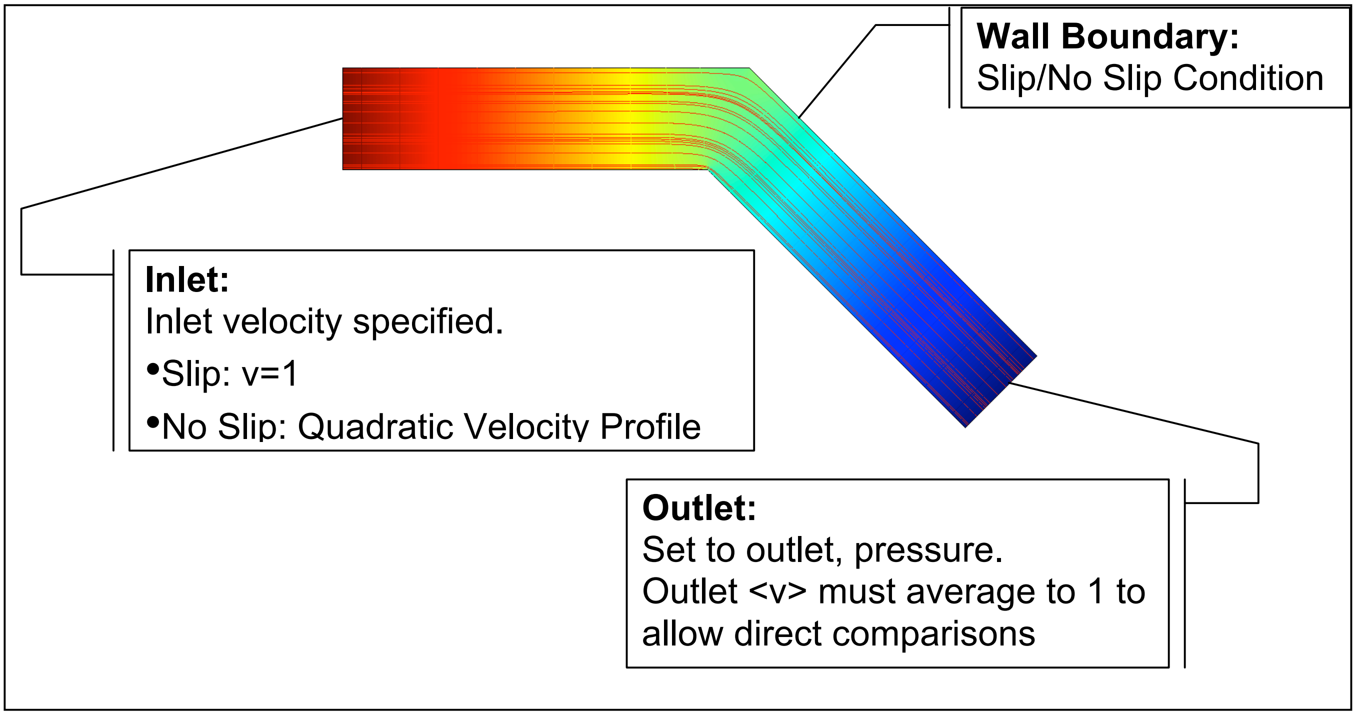

Wall Boundary:
Slip/No Slip Condition
Inlet:
Inlet velocity specified.
•Slip: v=1
•No Slip: Quadratic Velocity Profile
Outlet:
Set to outlet, pressure.
Outlet <v> must average to 1 to
allow direct comparisons

Figure 1

3-Results

3.1 Parametric Sweeps

Parametric sweeps are given which characterize the pressure drop in a device as the Reynolds number changes. The following sweeps can be used to see over what range the pressure drop remains relatively constant and calculating and using a K_L will give a good approximation to the pressure drop in the turn.

Figure 2 gives the pressure drop in the Sharp Bend, 2D geometry.

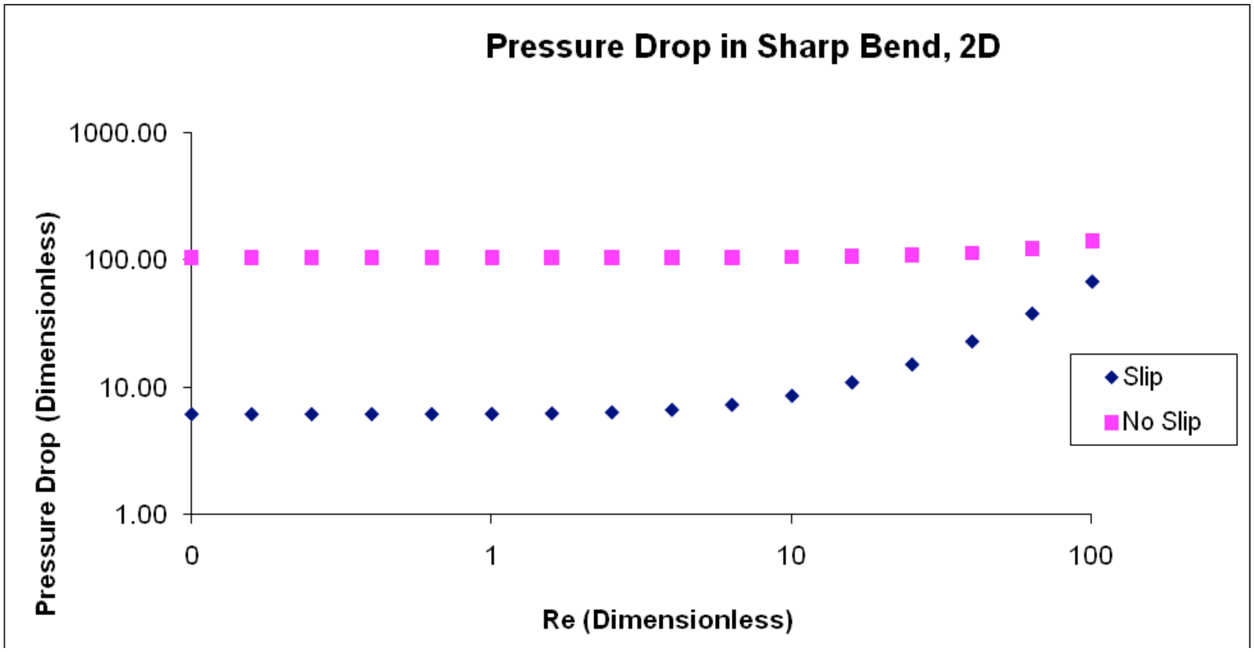


Figure 2

Notice that pressure drops between Reynolds numbers of 0 and five remain relatively constant for slip and no slip conditions. This flat region could be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 3 gives the pressure drop in the Smooth bend, 90 degrees, short radius, 2D geometry.

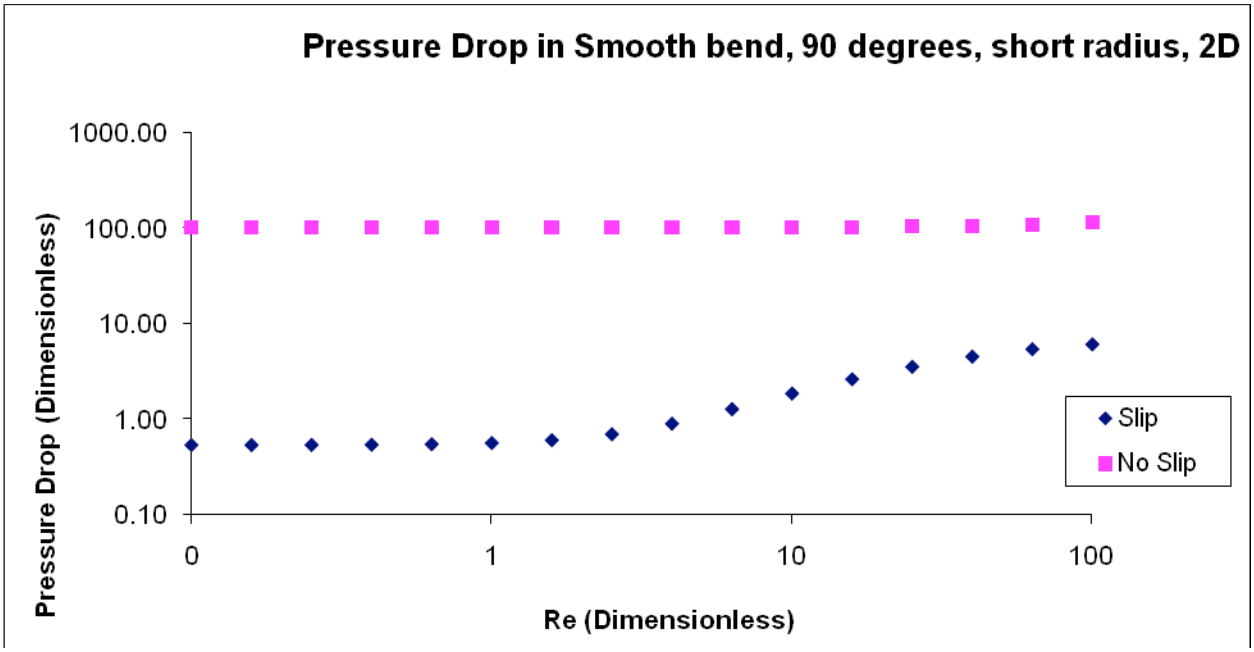


Figure 3

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant across the whole range of Reynolds numbers. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 4 gives the pressure drop in the Smooth bend, 90 degrees, long radius, 2D geometry.

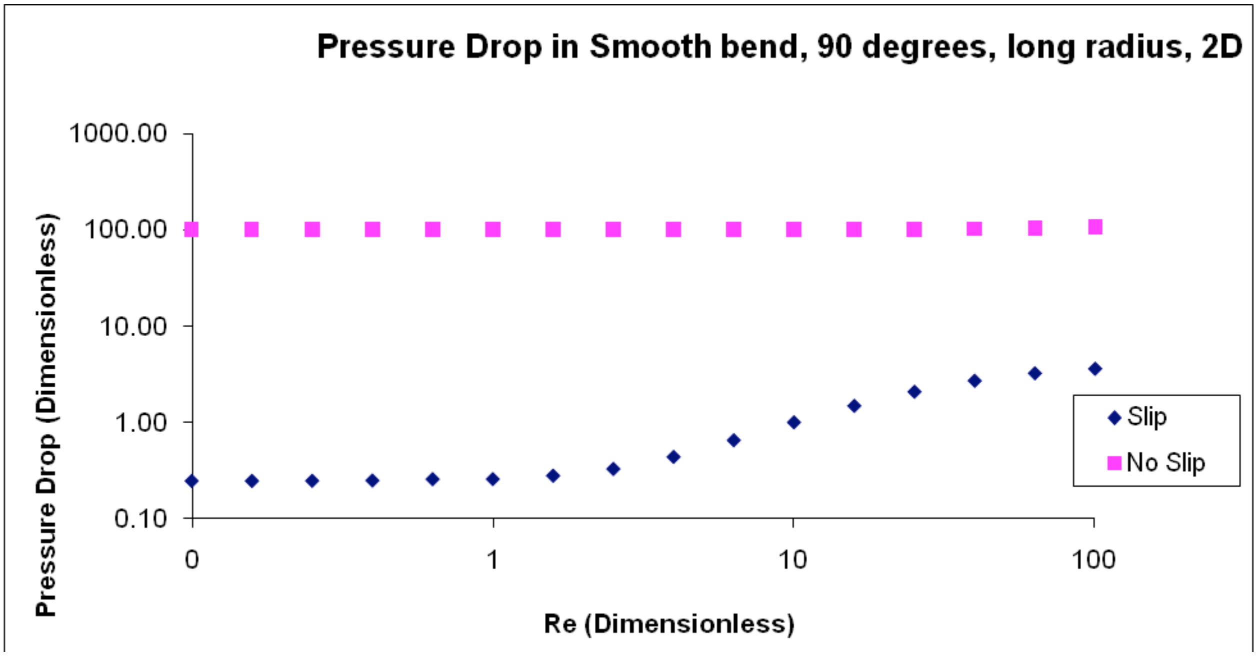


Figure 4

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant across the whole range of Reynolds numbers. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 5 gives the pressure drop in the Bend, 45 degrees, sharp change, 2D geometry.

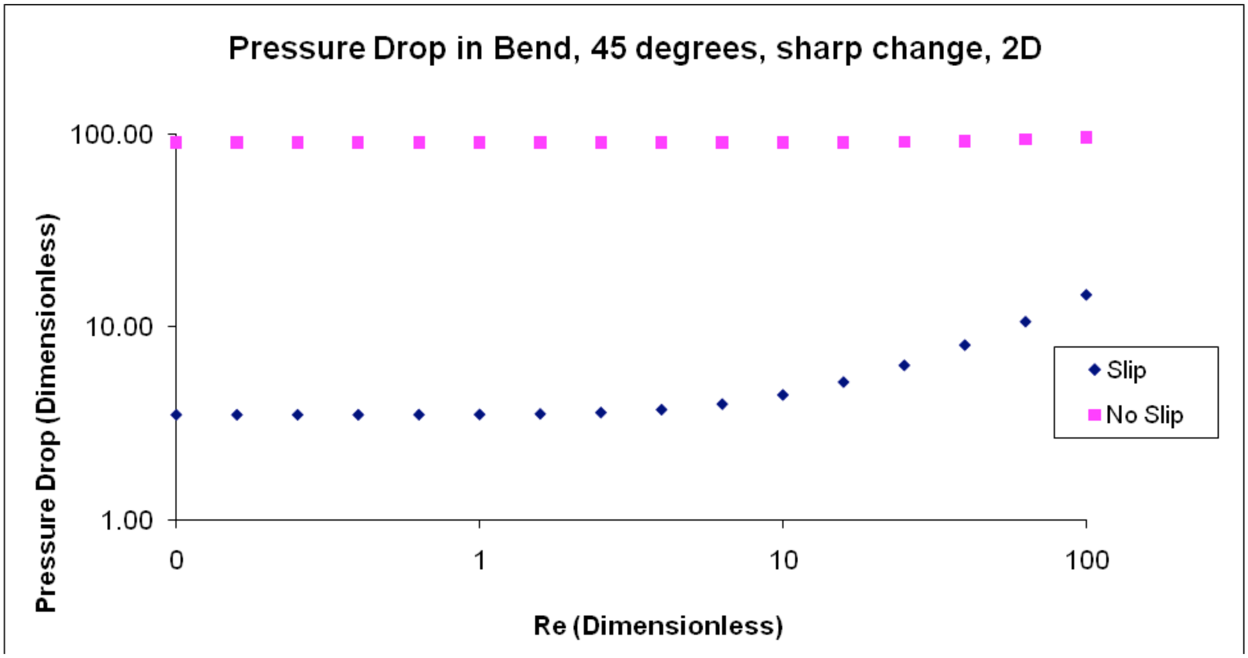


Figure 5

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant across the whole range of Reynolds numbers. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 6 gives the pressure drop in the Bend, 45 degrees, long radius, 2D geometry.

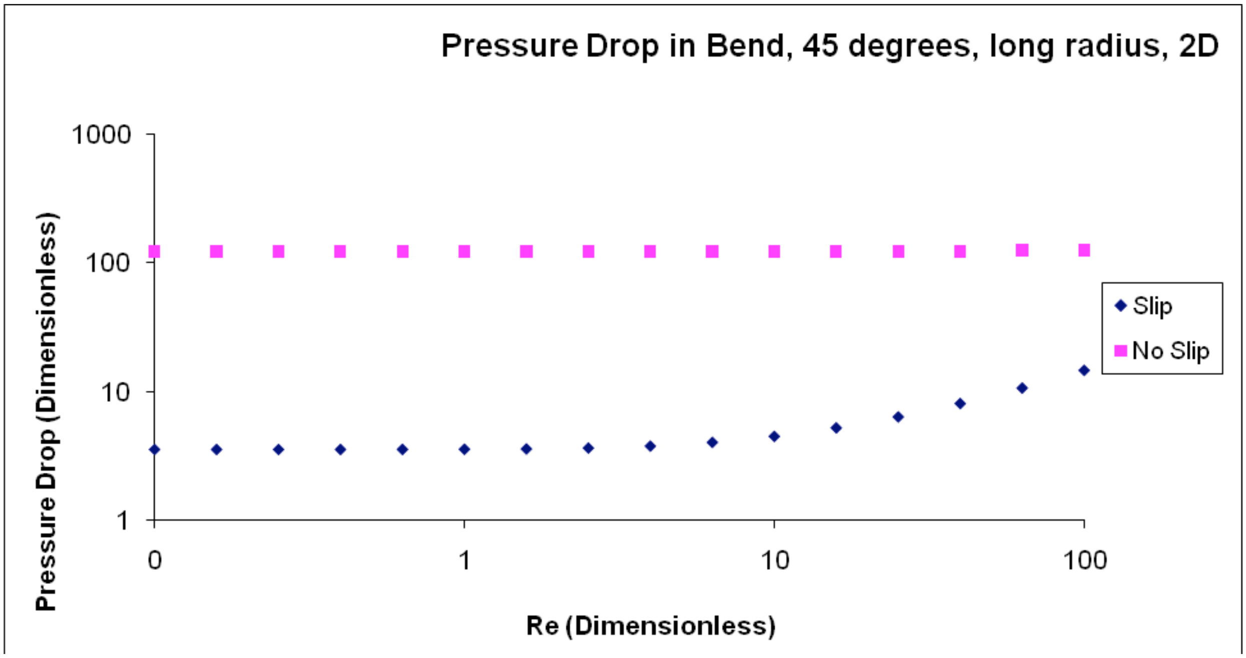


Figure 6

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant across the whole range of Reynolds numbers. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 7 gives the pressure drop in the Round pipe geometry.

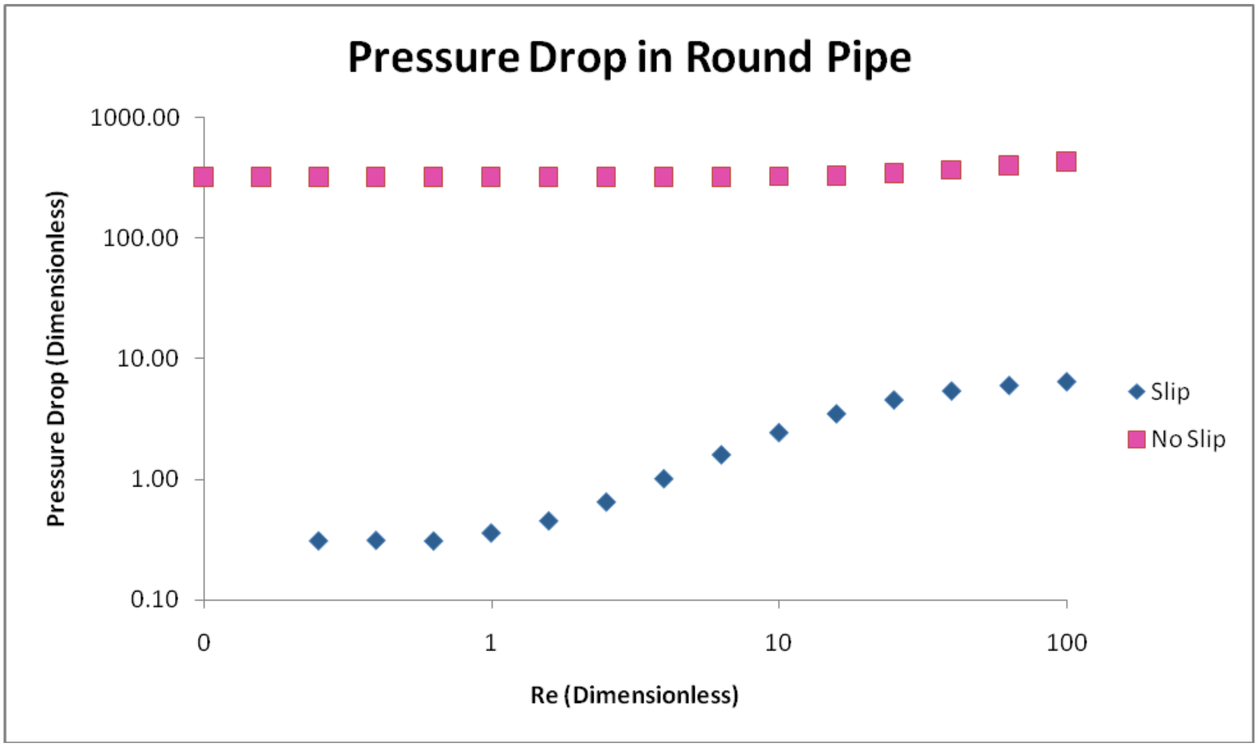


Figure 7

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains constant until a Reynolds number of 10. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 8 gives the pressure drop in the Square turn geometry.

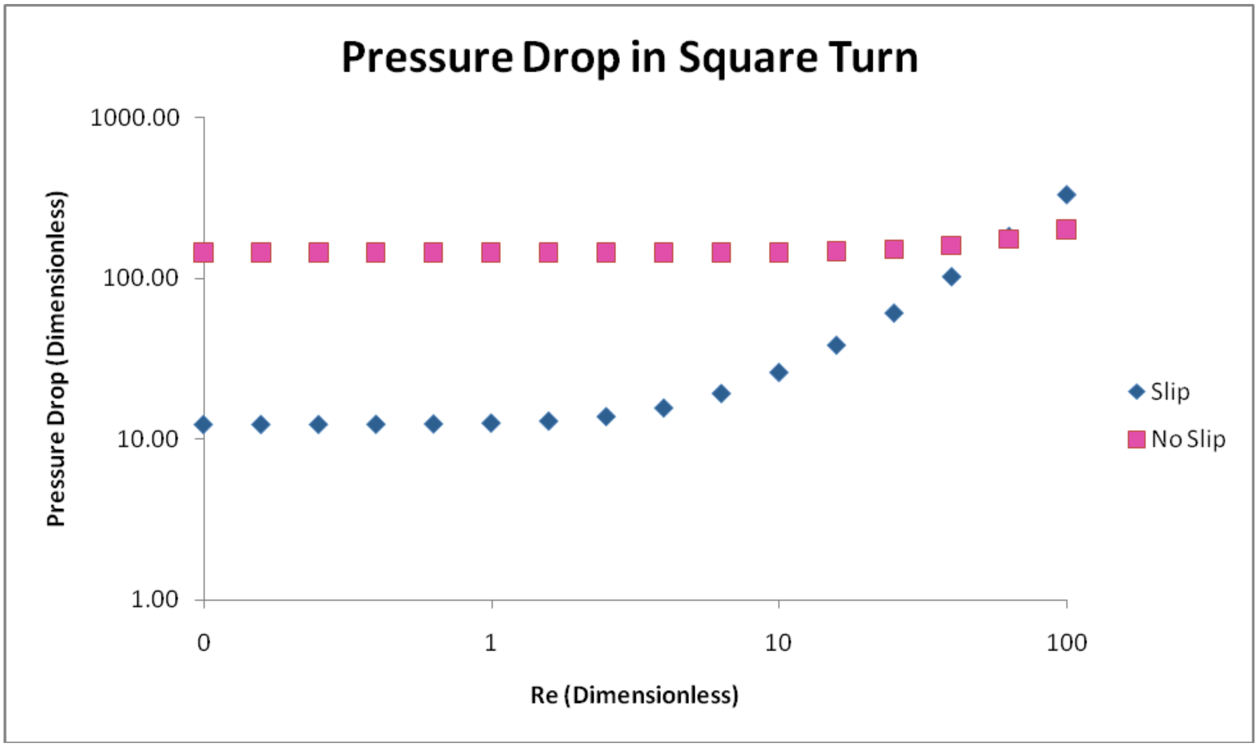


Figure 8

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant from 0 to 80. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

Figure 9 gives the pressure drop in the Square turn geometry.

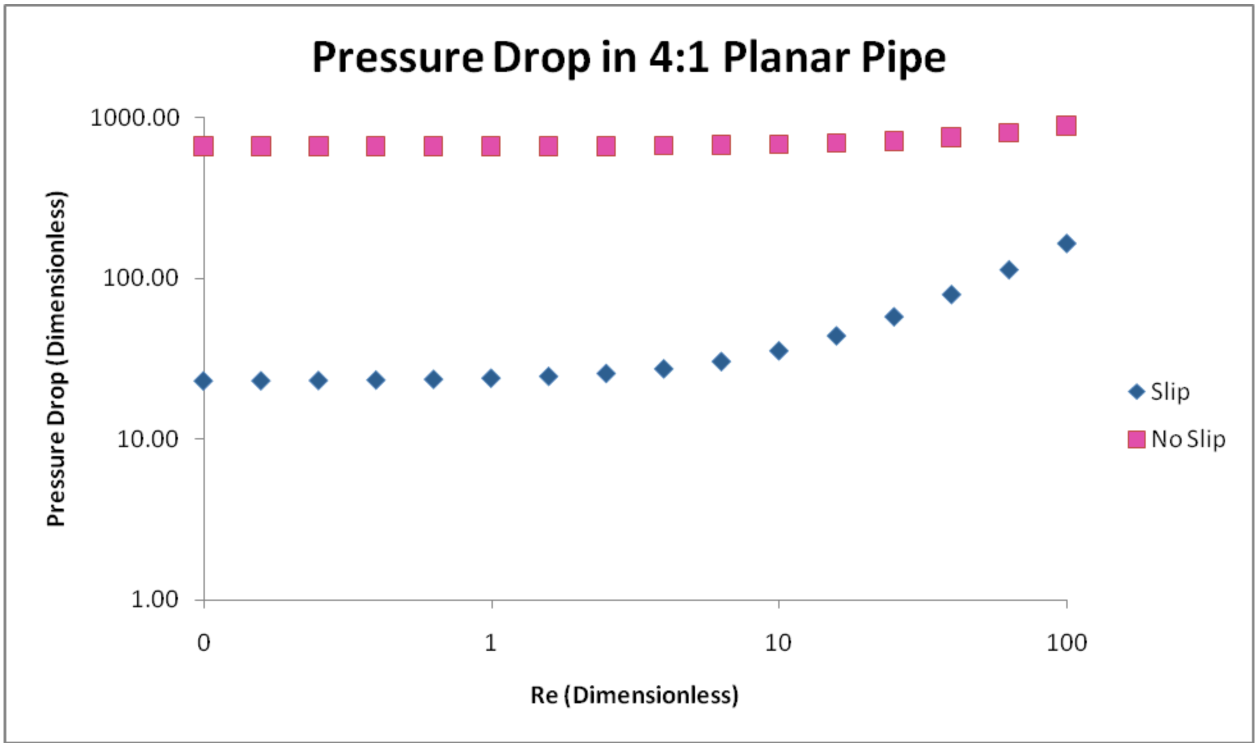


Figure 9

Notice that pressure drops between Reynolds numbers of 0 and 1 remain relatively constant for the slip solution. The no slip pressure drop remains essentially constant for Reynolds numbers between 0 and 20. These flat regions can be characterized by a constant K_L value to describe the pressure drop. At higher Reynolds numbers the viscous forces become more important and the pressure drop increases. In the slip condition solution the effect of the larger viscous dissipation forces is much greater.

3.2 Geometry Comparison

This table presents results for the slip and no slip pressure drops of different geometries. The results in the table have a Reynolds number of 0 (ρ=0, using ρ as a stand in for Reynolds number). All devices presented in the table have an average outlet velocity of 1; this allows the pressure drops in different geometries to be compared directly.

Table 1: Pressure Drops with Re=0

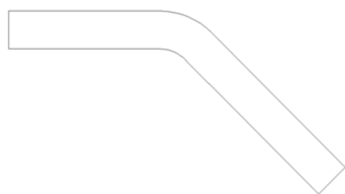
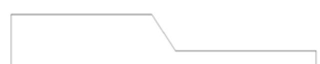
Description	Picture	Pressure Drop (No Slip)	Pressure Drop (Slip)
Bend 45, rounded		121.45	1.82
2:1 Planar Pipe		71.57	11.23
3:1 Planar Pipe		138.17	11.51
Taper 45		78.93	5.04

Table1 (continued)

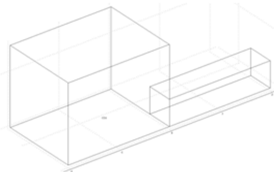
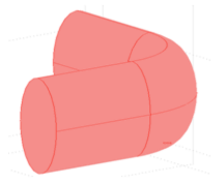
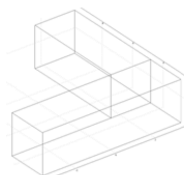
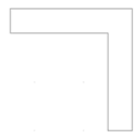
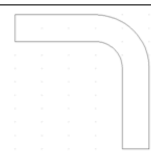
Description	Picture	Pressure Drop (No Slip)	Pressure Drop (Slip)
Square 3:1		65.63	10.93
Round Pipe		319.19	0.36
Square Turn		143.13	12.50
4:1 Planar Pipe		662.39	24.17

Table 1 (Continued)

Description	Picture	Pressure Drop (No Slip)	Pressure Drop (Slip)
Sharp bend, 2D		105.11	6.28
Smooth bend, 90 degrees, short radius, 2D (centerline radius=gap size)		103.36	0.57
Smooth bend, 90 degrees, long radius, 2D (centerline radius=1.5 x gap size)		100.63	0.26
Bend 45 degrees, sharp change, 2D		90.64	3.56
Bend, 45 degrees, long radius, 2D (centerline radius=1.5 x gap size)		122.27	1.83

4-Conclusions

From the results presented in section three we see that it is possible to characterize geometries with one K_L value over a range of Reynolds numbers. This is possible because the pressure drop remains constant over these ranges of Reynolds numbers. For most devices this range is between a Reynolds number of 0 and 1 for the slip condition and a range between 0 and 10 for no slip conditions. K_L values may be applied over a larger range of Reynolds numbers depending on the accuracy in pressure drop needed.

We also conclude that the pressure drop for slip conditions in curved devices is smaller than in turns with straight pieces. This can be seen in the very small pressure drops of the Round pipe and smooth bend geometries presented in Table 1. For no slip conditions, the pressure drop in 45 degree bend was found to be the lowest.

5-Appendices

How to calculate K_L values from pressure drops in devices.

All devices had an average outlet velocity of 1, $\langle v \rangle = 1,$ a dynamic viscosity of 1, $\eta = 1$ and a diameter of 1, $d = 1$.

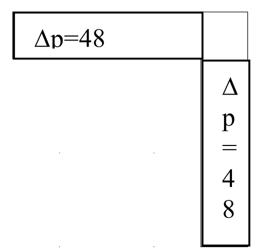
COMSOL Multiphysics gives a pressure drop of 105.1 in the Sharp bend, 2D geometry.

Pressure drop in a straight 2D pipe that is 4 units in length is 48.

To find the pressure drop due to the turn use the Δp_{excess} equation applied to the geometry:

$$\Delta p_{excess} = \Delta p_{total} - \Delta p_{channel1} - \Delta p_{channel2}$$

$$\Delta p_{excess} = 105.1 - 48 - 48 = 9.1$$

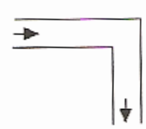


Rearranging equation 4 allows us to come up with a K_L value:

$$\Delta p_{excess} = K_L \frac{\eta \langle v \rangle}{d} \xrightarrow{Rearrange} K_L = \frac{d}{\eta \langle v \rangle} \Delta p_{excess}$$

$$K_L = \frac{1}{1\langle 1 \rangle} 9.1 = 9.1$$

This result is confirmed by the literature results from Table 8.2 in *Micro Instrumentation*

Description	Picture	K_L
Sharp bend, 2D		9.1