MIXING WITH MICROPILLARS

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Spring 2008

University of Washington

I. Introduction

The objective of this research is determining the mixing efficiency and pressure drop of microfluidic devices. There are several geometries that we can use, but in this research, it will be focused in specific geometry, which is the micropillars geometry. The mixing process of this research can be used in medical fields, in sensor, etc.

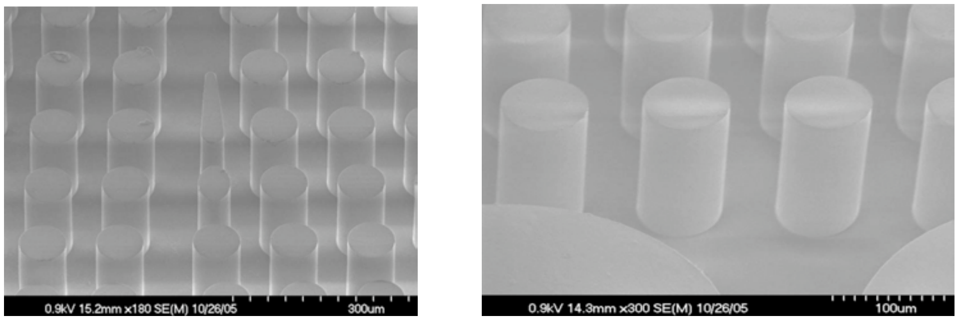

Figure1. Picture of micropillars. It shows a combination of several micropillars with certain formation.

In order to achieve the objectives, Comsol Multiohysics program will be used to visualized the geometry and to solve all the flow problems. Several advantages found in this program are it helps to give the scale concentration, velocity distribution, mixing level, etc. This program is also able to give the value for each boundary condition, and subdomain area.

At first, the micropillars model will be assumed to be 2 dimensional models. Then, this assumption will be tested at the end of the discussion section. It would be a good approximation to get the result in 3D geometry, since most of the geometry in 2D can be projected to 3D without changing the properties.

We will determine how good the mixing process by looking at the value of the mixing cup concentration, the variance, optical mixing cup concentration, optical variance and the pressure drop. *The mixing cup concentration is the concentration of the fluid that is emptied into a cup that was well stirred* (Microcomponent Flow Characterization, Finlayson, 2007). Moreover, the optical parameters are the value that is observed under florescent condition which the velocity can't be calculated or observed.

II.Problem

The geometry that I will use is the micropillars with two formations, which are the aligned formation and staggered formation. In calculating the mixing cup concentration and the variance, I will model the object in 2D and in 1 x 1 scales. In order to compare the result from one geometry to geometry, all the calculation will be dimensionless. It will make easier to compare with other geometries without worrying the dimensions.

The geometry of the micropillars has only one inlet. Therefore, two inlets are needed to mix two solutions. One approach is to use the Boolean algebra in the inlet which will set upper half to be a concentration of 1 while the lower half will be 0 concentration. The inlet flow is also to be assumed to be laminar which has a quadratic profile flow.

Convection and diffusion equation will solve the flow problem, where I set Reynold number to be constant by changing the value of Peclet Number. Therefore, on the sub domain setting I will set, Diffusivity = 1/Pe

U = u

And v = v.

The boundary conditions for convection and diffusion problems are concentration of 1 at the inlet (with the Boolean Algebra to get 2 fluids at the inlet), convective flux at the outlet and insulation/symmetry at any other boundary. In this case, the value of Peclet number will be set through solver parameter.

Next, Incompressible Navier Stoke equation will be used to solve for the condition where I set Peclet numbet to be constant, and changing the value of Reynolds Number.

On the subdomain setting, I will set,

Density = 1

Dynamic viscosity = 1

No volume force in x and y direction.

The boundary condition for Incompressible Navier Stoke equation are inlet with initital velocity of 1 m/s, outlet at 0 pressure, wall (no slip boundary) at all of the pillars, and symmetry boundary at the side. In this case, Reynolds Number will be set through the solver parameter.

On the first condition, I will set the diffusivity to be 1/Pe, and a Reynolds number of 1. The range of Pe that I will use is 100, 200, 300, 500, and 700. Furthermore, on the second condition, I will set _ to be 1/Re, set Peclet number to be 300, with Re ranging, 1, 10, 100, 1000, 2000.

There are several equations that I will use to determine how good the mixing process is. These will be calculated at the outlet boundary, which are,

The mixing cup with taking account of velocity

$$Cmix = \frac{\int C * V\, dA}{\int V\, dA}$$

The variance with taking account of velocity

$$\sigma_{var} = \frac{\int (C - Cmix)^2 * V\, dA}{\int V\, dA}$$

The optical mixing cup without taking account of velocity,

$$Cmix = \frac{\int C * V\, dA}{\int V\, dA}$$

The optical variance without taking account of velocity,

$$\sigma_{var} = \frac{\int (C - Cmix)^2 \, dA}{\int dA}$$

The pressure drop is the only equation that will be calculated at the inlet boundary, since I have set the outlet boundary to be convective flux.

$$P = P_s * P'$$

$$P_s = \frac{\int P dA}{\int dA}$$

$$P' = \rho * (U_s)^2$$

With the standard conditions that is matched with all the research group,

_ = 1000 kg / m^3

U_s = .005 m/s

III. Results

The result of this research is calculated using the boundary integration using Comsol Multiphysics.

For Re = 1 and symmetric formation, the result is the following,

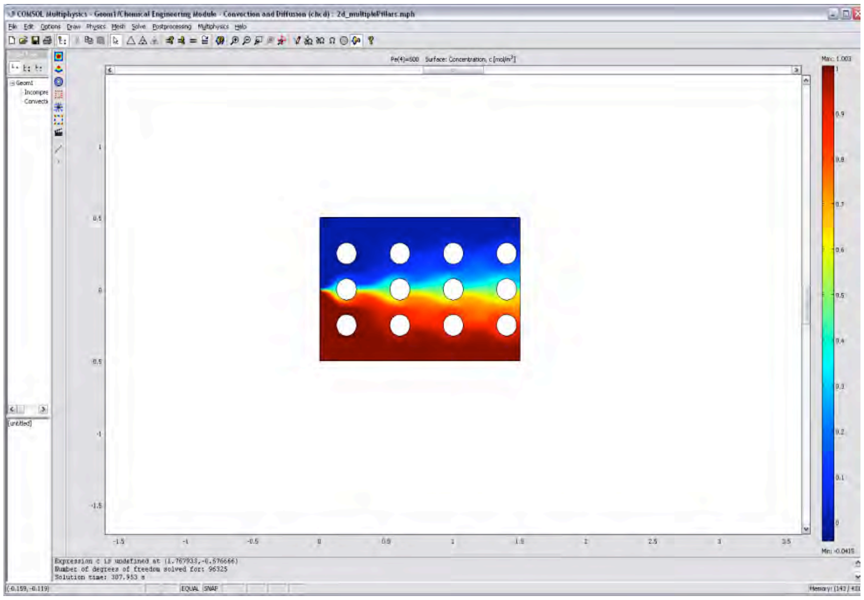

Figure 2. Picture of concentration gradient using Comsol Multiphysics program in the alligned formation when Pe = 500 and Re =1.

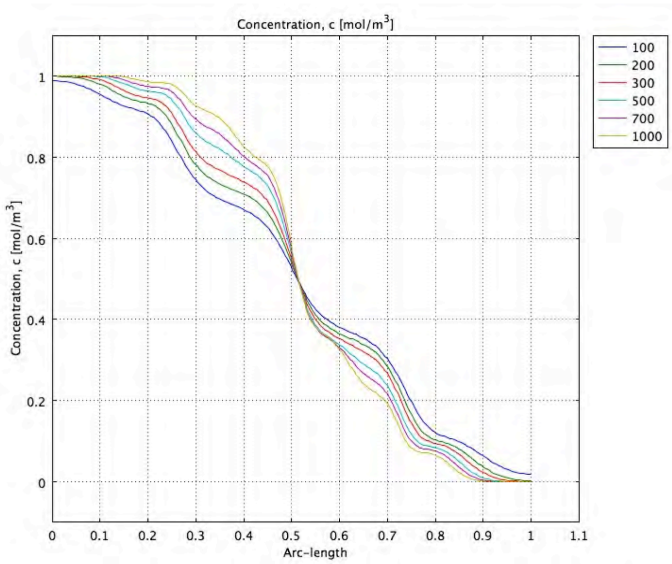

Figure3. The graph of the arc length vs concentration which shows that higher Pe means a wider spread of mixing.

Pe	**100**	**200**	**300**	**500**	**700**

∫cVdA = ∫VdA =	0.514 1.010	0.514 1.010	0.513 1.010	0.513 1.010	0.513 1.010
C_{mix} =	0.509	0.508	0.508	0.508	0.508
∫ (C-Cmix)^2.V dA = ∫VdA =	0.197 1.010	0.215 1.010	0.223 1.010	0.230 1.010	0.234 1.010
_var =	0.195	0.208	0.220	0.228	0.231
∫cdA = ∫dA =	0.517 1.000	0.523 1.000	0.526 1.000	0.532 1.000	0.535 1.000
C_{mix} =	0.517	0.523	0.526	0.532	0.535
∫ (C-Cmix)^2dA = ∫dA =	0.197 1.000	0.215 1.000	0.223 1.000	0.231 1.000	0.234 1.000
_var =	0.197	0.215	0.223	0.231	0.234
P' = Ps =	326.657 0.025	326.657 0.025	326.657 0.025	326.657 0.025	326.657 0.025
P =	8.166	8.166	8.166	8.166	8.166

Table1. Results of changing Pe by keeping Re constant in the alligned formation of micropillars. This table shows that low Pe gives a better result of mixing.

While for staggered formation,

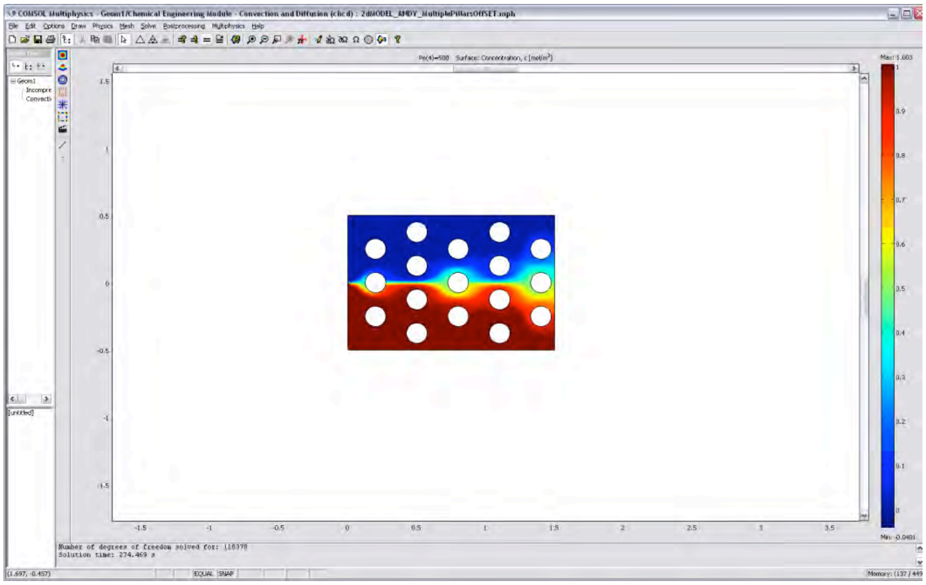

Figure4. The concentration gradient profile using Comsol Mulsiphysics program. It shows a better spread of mixing than the alligned formation. Pe = 500 and Re= 1.

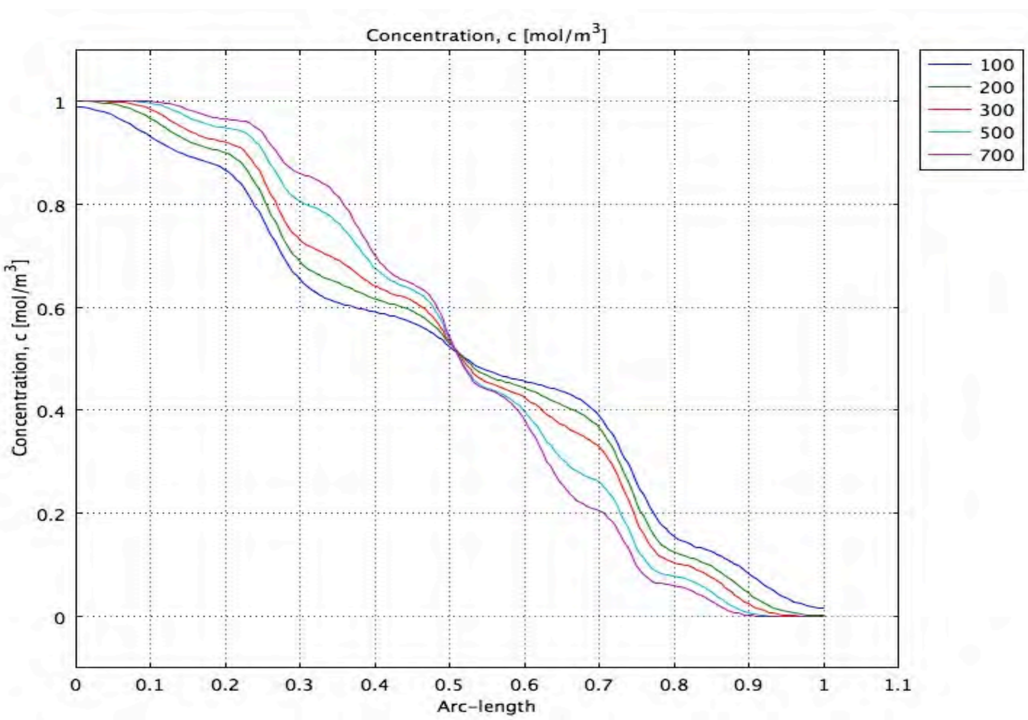

Figure5. The graph of arc-length vs concentration of the staggered formation. It shows that the staggered formation gives a better spread mixing than the alligned formation. It also shows that when Pe = 300 is the best spread mixing.

Pe	100	200	300	500	700

∫cVdA = ∫VdA =	0.516 1.013	0.516 1.013	0.516 1.013	0.516 1.013	0.516 1.013
C_{mix} =	0.510	0.509	0.509	0.509	0.509
∫ (C-Cmix)^2.V dA = ∫VdA =	0.166 1.013	0.187 1.013	0.197 1.013	0.207 1.013	0.214 1.013
__var =	0.164	0.184	0.194	0.205	0.211
∫cdA = ∫dA =	0.515 1.000	0.518 1.000	0.519 1.000	0.520 1.000	0.521 1.000
C_{mix} =	0.515	0.518	0.519	0.520	0.521
∫ (C-Cmix)^2dA = ∫dA =	0.089 1.000	0.104 1.000	0.115 1.000	0.133 1.000	0.145 1.000
__var =	0.089	0.104	0.115	0.133	0.145
P' = Ps =	1155.334 0.025	1155.334 0.025	1155.334 0.025	1155.334 0.025	1155.334 0.025
P =	28.883	28.883	28.883	28.883	28.883

Table2. Results of the staggered formation. It shows that the lower the Pe the better the mixing.

From the results above, we can conclude that they both gave almost the same results. As we compare the mixing cup concentration, it can be seen from the table that at the aligned formation, the best mixing can be found at lower Peclet Number. Moreover, the staggered formation had its best mixing at lower Peclet Number.

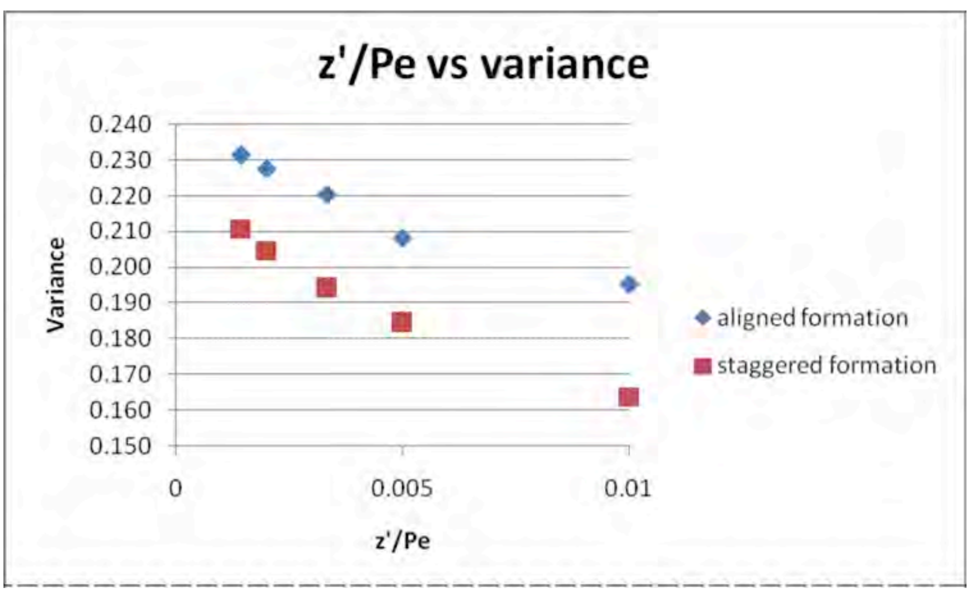

Figure6. graph of z'/Pe vs variance, while z' is the length of inlet to outlet which is 1. The variance is calculated through comsol.

Next, I will present the result from the second condition where I set the Peclet number to be constant and changing the Reynolds number.

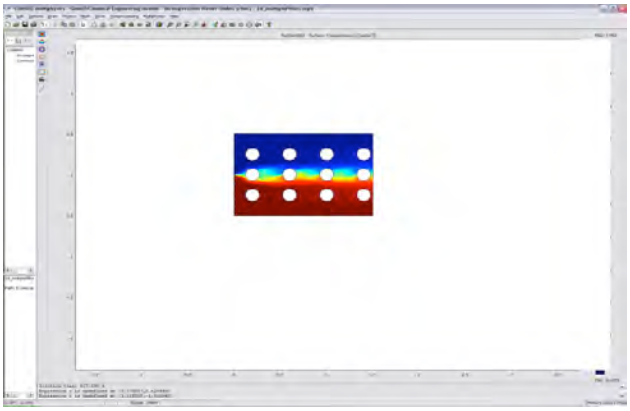

Figure7. The concentration gradient profile of the aligned formation at Re = 5000, and Pe = 300.

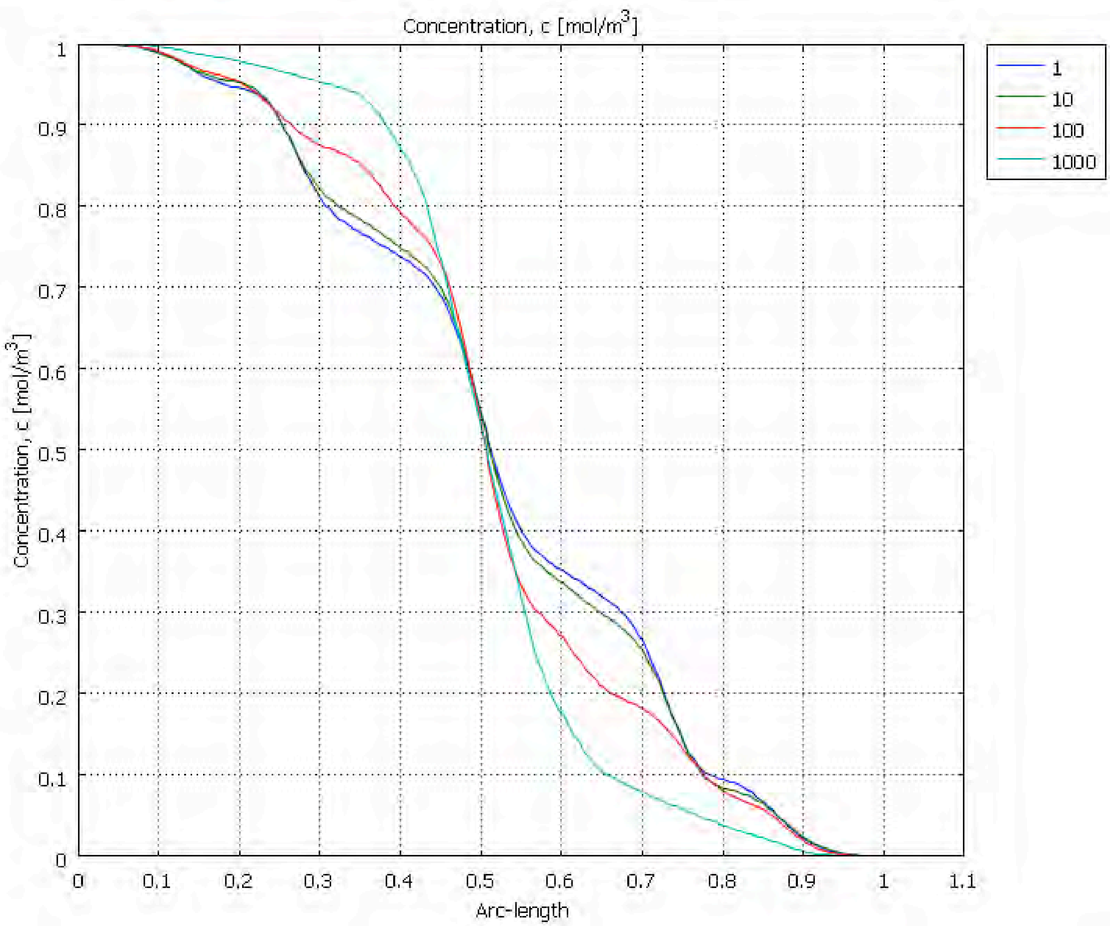

Figure8. The graph of the arc length vs concentration for aligned formation. It shows that the lower Re gave a better spread of mixing than the others.

Re	1	10	100	1000	2000	5000

∫cVdA = ∫VdA =	0.498 1.005	0.497 1.003	0.496 1.001	0.496 1.000	0.496 1.000	0.496 1.000
C_{mix} =	0.496	0.496	0.496	0.496	0.496	0.496
∫ (C-Cmix)^2.V dA = ∫VdA =	0.221 1.005	0.220 1.003	0.213 1.001	0.211 1.000	0.210 1.000	0.207 1.000
_var =	0.220	0.208	0.213	0.211	0.210	0.207
∫cdA = ∫dA =	0.487 1.000	0.488 1.000	0.490 1.000	0.494 1.000	0.494 1.000	0.496 1.000
C_{mix} =	0.487	0.488	0.490	0.494	0.494	0.496
∫ (C-Cmix)^2dA = ∫dA =	0.132 1.000	0.135 1.000	0.149 1.000	0.181 1.000	0.183 1.000	0.183 1.000
_var =	0.132	0.135	0.149	0.181	0.183	0.183
P' = Ps =	326.111 0.025	36.222 0.025	7.250 0.025	2.668 0.025	2.261 0.025	1.979 0.025
P =	13044.458	1448.870	289.987	106.704	90.442	79.170

Table4. The calculation result of the aligned formation at Pe =300 by varying the Re. It can be seen from the graph that the higher the Re the better the mixing at the outlet.

While, for the staggered formation

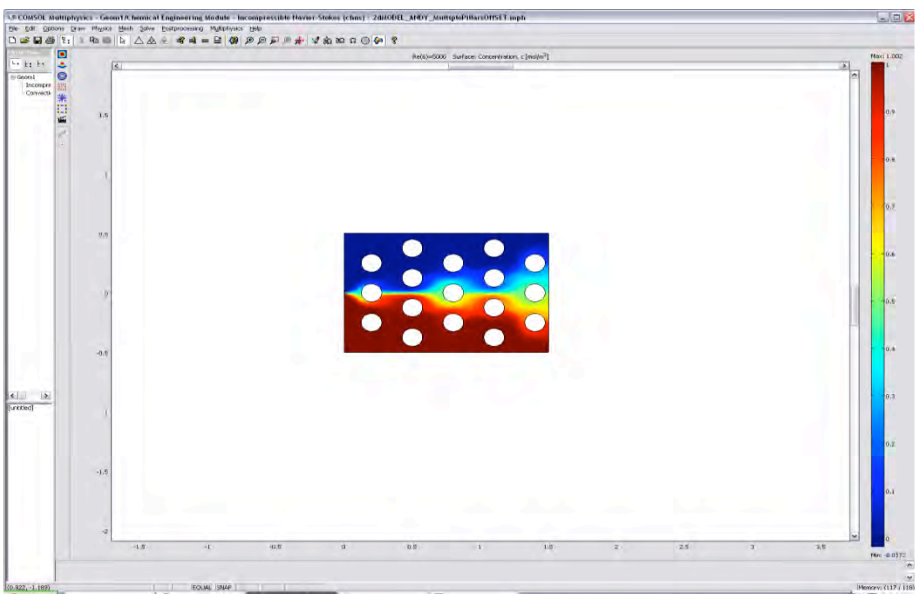

Figure9. The concentration gradient of the staggered formation which shows that the mixing is well spread at the outlet than the aligned formation.

Re	**1**	**10**	**100**	**1000**	**2000**

∫cVdA = ∫VdA =	0.500 1.006	0.499 1.004	0.496 1.001	0.496 1.001	0.496 1.001
C_{mix} =	0.497	0.496	0.496	0.496	0.496
∫ (C-Cmix)^2.V dA = ∫VdA =	0.195 1.006	0.194 1.004	0.187 1.001	0.182 1.001	0.180 1.001
_var =	0.194	0.193	0.187	0.181	0.180
∫cdA = ∫dA =	0.491 1.000	0.491 1.000	0.493 1.000	0.493 1.000	0.489 1.000
C_{mix} =	0.491	0.491	0.493	0.493	0.489
∫ (C-Cmix)^2dA = ∫dA =	0.195 1.000	0.194 1.000	0.187 1.000	0.182 1.000	0.180 1.000
_var =	0.195	0.194	0.187	0.182	0.180
P' = Ps =	1153.138 0.025	118.253 0.025	17.277 0.025	7.921 0.025	7.640 0.025
P =	46125.521	4730.118	691.099	316.844	305.619

Table5. The calculation result of the staggered formation. It shows that the best mixing is found at the higher Reynolds number.

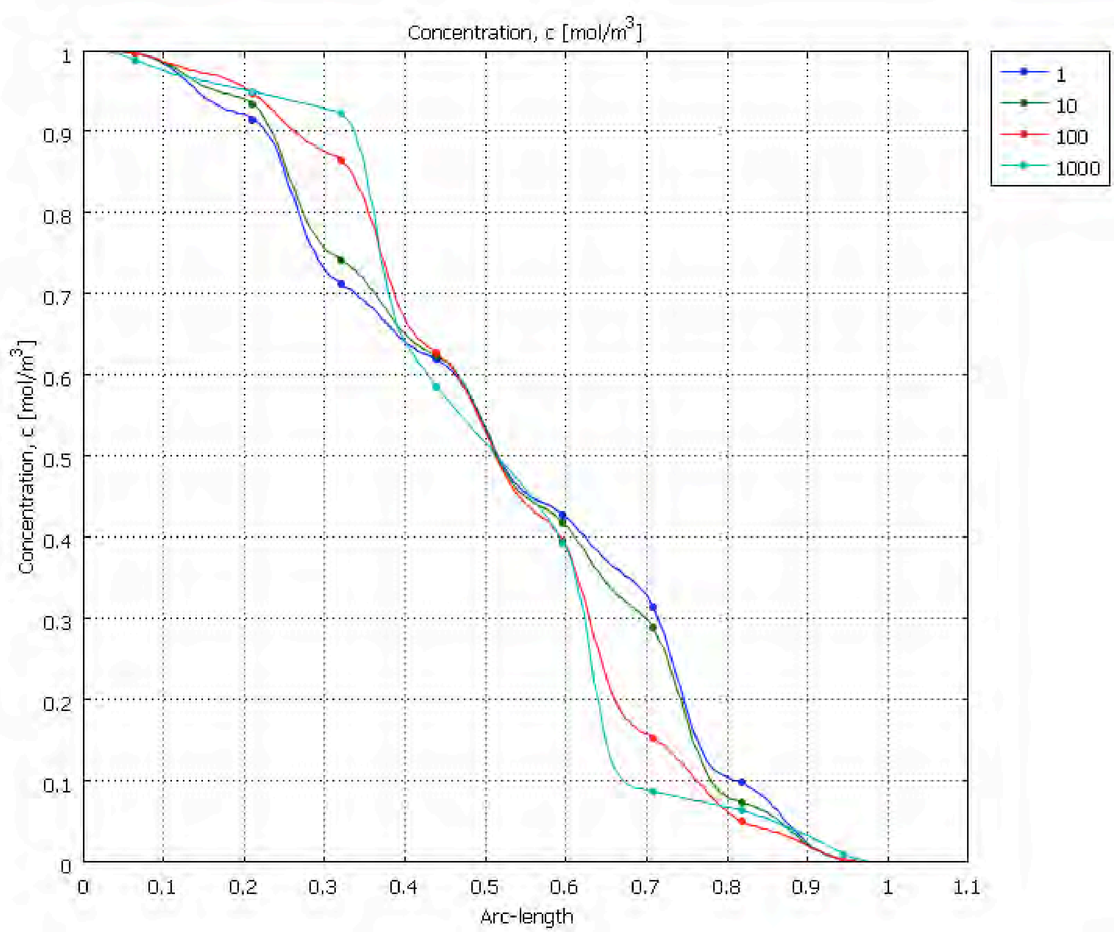

Figure10. The graph of Arc-length vs concentration for the staggered formation. It shows that the lower the Re the better spread of mixing will be found.

As seen from the table, the aligned formation gave better mixing result at higher Reynolds Number. While at the staggered formation, the lower the Reynold Number will give a better mixing.

Below, the calculation of the 3D will be performed.

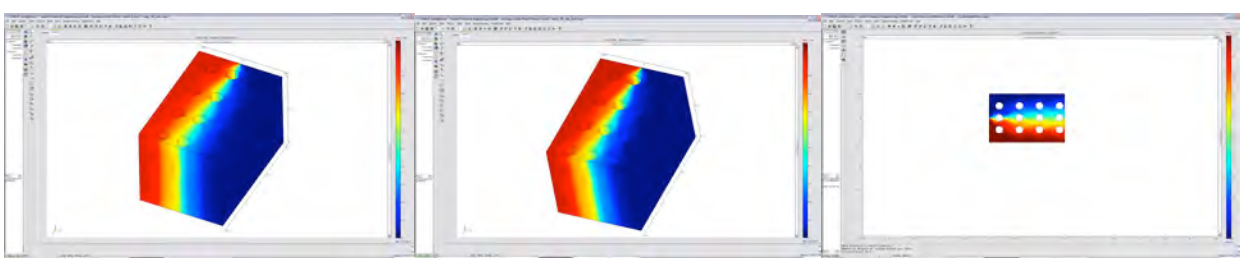

Figure11. The concentration gradient for the three conditions. The first is the 3D formation with no slip boundary condition, the second is the 3D formation with the slip condition, and the last is the 2D version. The slip and no slip boundary conditions are at the top and the bottom of the fluid flow. These pictures is taken with aligned formation at Pe=100 and Re = 1.

3D NO SLIP BC		3D SLIP BC		2D	
∫cVdA =	0.508	∫cVdA =	0.520	∫cVdA =	0.498
∫VdA =	1.016	∫VdA =	1.037	∫VdA =	1.005
C_{mix} =	0.500	**C_{mix} =**	0.502	**C_{mix} =**	0.496
∫ (C-Cmix)^2.V dA =	0.184	∫ (C-Cmix)^2.V dA =	0.189	∫ (C-Cmix)^2.V dA =	0.196
∫VdA =	1.016	∫VdA =	1.037	∫VdA =	1.005
σ_{var} =	0.181	**σ_{var} =**	0.183	**σ_{var} =**	0.195
∫cdA =	0.506	∫cdA =	0.507	∫cdA =	0.492
∫dA =	1.000	∫dA =	1.000	∫dA =	1.000
C_{mix} =	0.506	**C_{mix} =**	0.507	**C_{mix} =**	0.492
∫ (C-Cmix)^2dA =	0.136	∫ (C-Cmix)^2dA =	0.136	∫ (C-Cmix)^2dA =	0.108
∫dA =	1.000	∫dA =	1.000	∫dA =	1.000
σ_{var} =	0.136	**σ_{var} =**	0.136	**σ_{var} =**	0.108
P' =	862.774	P' =	786.464	P' =	326.111
Ps =	0.025	Ps =	0.025	Ps =	0.025
P =	34510.945	**P =**	31458.567	**P =**	13044.458

Table6. The calculation result for comparing the result for five mixing parameters for three conditions as in figure 10.

From the calculated result above, the value for 2D is so close with the result in 3D with slip boundary conditions. The only possible error is because of the coarse mesh, which it is hard to get a good mesh in 3D. While, if we see the value for 3D with slip boundary condition and no slip boundary condition, their value is off insignificantly. Therefore, the 2D result is a good approximation of the 3D results.

IV. Improvement

There are several ways to improve the mixing in the micropillars. One way is improving the mixing geometrically. Below is the best result after several experiments.

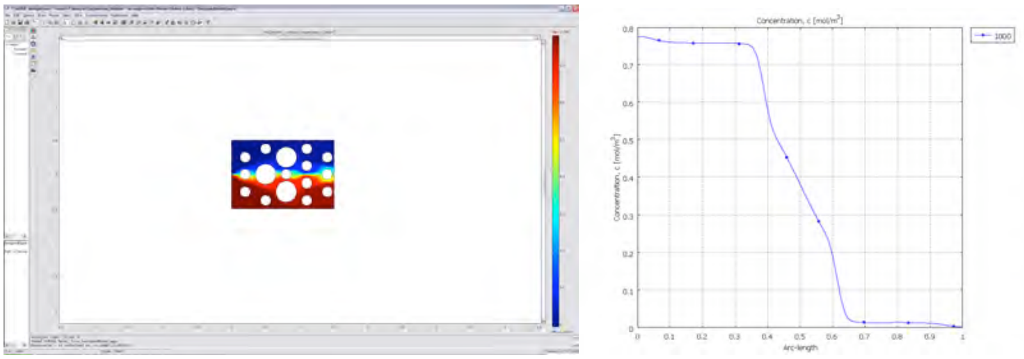

Figure12. the graph of the improved mixing geometrically. The first figure shows the concentration gradient profile which the size of the bigger pillars is twice the diameter of the regular pillars. The second graph shows the arc-length vs the concentration at the outlet for the improved geometry.

Re	**1000**
∫cVdA =	0.301
∫VdA =	0.779
C_{mix} =	0.386
∫ (C-Cmix)^2.V dA =	0.091
∫VdA =	0.779
σ_{var} =	0.117
∫cdA =	0.386
∫dA =	1.000
C_{mix} =	0.491
∫ (C-Cmix)^2dA =	0.120
∫dA =	1.000
σ_{var} =	0.120
P' =	8.068
Ps =	0.025
P =	322.731

Table7. The calculation result for determining the mixing effiiciency for the improved geometrically model. It shows a better mixing than the aligned and the staggered formation.

This improved model is one of the way to improved a better mixing. As a results, the model doesn’t need to be long in order to get a well mixed fluids than the former two models. This improved model can be used for its benefits, but it might be more expensive in the cost of productions.

V. Suggestions

The best mixing is found at the staggered formation. The variance reach 0.8. Furthermore, at the staggered formation, it is better to use high Peclet Number with a small Reynolds Number to maximize the mixing in the particular formation.

VI. Conclusions

The effects of increasing the Reynolds Number to this geometry is giving a better mixing for aligned formation. It increases the mixing in the aligned formation because it creates vortices in each area after the pillars. These also caused since the Reynolds Number is proportionally linear with the velocity of the inlet, which cause the vortex after the pillars become larger. Moreover, on the constant Peclet Number and changing Reynolds Number, the optical variance is not showing a good approximation of the regular variance, since the trend as the Re increases is not the same.

For the staggered formation, the best mixing found at the high Reynolds number and at the low Peclet number. Comparing to the aligned formation, this formation is better to get the fluid mixed.

Furthermore, the 2D results are a good representation for 3D results. Therefore, we can calculate all the models in 2D for getting all the mixing properties in 3D.

Appendix

VandenBussche, Kurt M and Melvin, Koch V. Micro Instrumentation. 2007. Wiley.

Sample calculation:

P' = _ * (Us)^2 $= 1000\ kg / m^3 * (.005\ m/s)^2$

$= 0.025\ kg/ m^2 . s^2$

$= 0.025$ Pa