Flow in a Cross

Chem E 499 with Prof. Finlayson, Spring 2008
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Introduction

The objective of this experiment is to evaluate flow through two rectangular pipes laid on one another in a cross shape. The inlets consist of a fluid with a finite concentration and another fluid with zero concentration. The way that this was studied was by keeping Reynolds number (Re) constant and varying the Peclet number (Pe). For each Pe values for the mixing cup concentration, variance, optical mixing cup and optical variance are calculated.

Setup

The geometry in question is a cross shape that resembles a plus sign. The cross is a total of eight units wide and eight units tall with each "branch" of the cross being one unit wide. The resulting shape is shown in Figure 1 below.

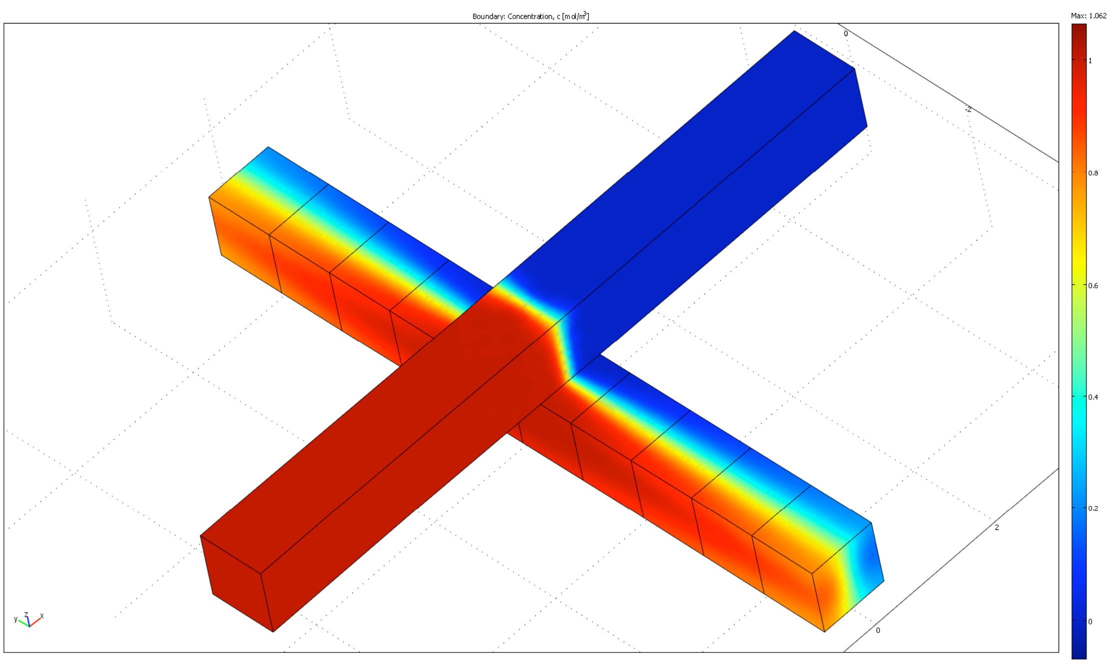

Figure 1: Three-dimensional representation of two beams on top of each other.

The entering streams are those on the bottom left and top right and the exiting streams are those on the top left and bottom right of figure 1. Inlet flows are set to be a dimensionless one unit and exit streams having boundaries of zero pressure. Concentration of the bottom left inlet stream is set to be one and the top right to be zero. Exit streams are set to boundary conditions of convective flux.

In the two dimensional case there is little control that can be exhibited over the system as there is little area for varying conditions of the device that will facilitate mixing. However, the mixing cup concentration, optical average concentration, mixing cup variance and the optical variance are calculated for the two dimensional case for Pe numbers 100, 200, 300, 500, 700, and 1000. All Pe values for the two-dimensional were evaluated at the same Re of 1.

The equations used for these calculations can be seen in equations 3-6.

$$\mathrm{Re} = \frac{\rho v_s x_s}{\eta}$$ **eq. 1**: Reynolds number

$$Pe = \frac{v_s x_s}{D}$$ **eq. 2**: Peclet number

$$c_{mixingcup} = \frac{\int c(x,y)v(x,y)dxdy}{\int v(x,y)dxdy}$$ **eq. 3**: Mixing cup concentrations

$$c_{\mathrm{var}} = \frac{\int \left[c(x,y) - c_{mixingcup}(x,y)\right]^2 v(x,y)dxdy}{\int v(x,y)dxdy}$$ **eq. 4**: Mixing cup variance

$$c_{opt.avg.conc} = \frac{\int_0^L c(x,y)dy}{\int_0^L dy}$$ **eq. 5**: Optical average concentration

$$c_{opt\,\mathrm{var}} = \frac{\int_0^L \left[c(x,y) - c_{opt.avg.conc}(x,y)\right]^2 dxdy}{\int_0^L dy}$$ **eq. 6**: Optical variance

$$\Delta P = \frac{\Delta P' \eta v_s}{x_s}$$ **eq. 7**: Pressure drop (2-D)

$$\Delta P = \frac{\Delta P' \eta v_s}{x_s^2}$$ **eq. 8**: Pressure drop (3-D)

In equations 3-6, c(x,y) is the concentration along the boundary being evaluated, v(x,y) is the velocity along the boundary, and L is the width of the opening. The three-dimensional case of this geometry was dealt with in the same way as the two-dimensional representation.

Results

Two-Dimensional Representation

By first solving the incompressible Navier-Stokes equations for laminar steady-state flow in the system the velocity in the bottom and right streams are found to be 1.014 and 0.886 respectively. For the solution of Navier-Stokes equations there were 16771 degrees of freedom and for the convective and diffusion solution there were 3584 degrees of freedom with 3584 elements. Below, in figure 2, a visual of the mesh can be seen.

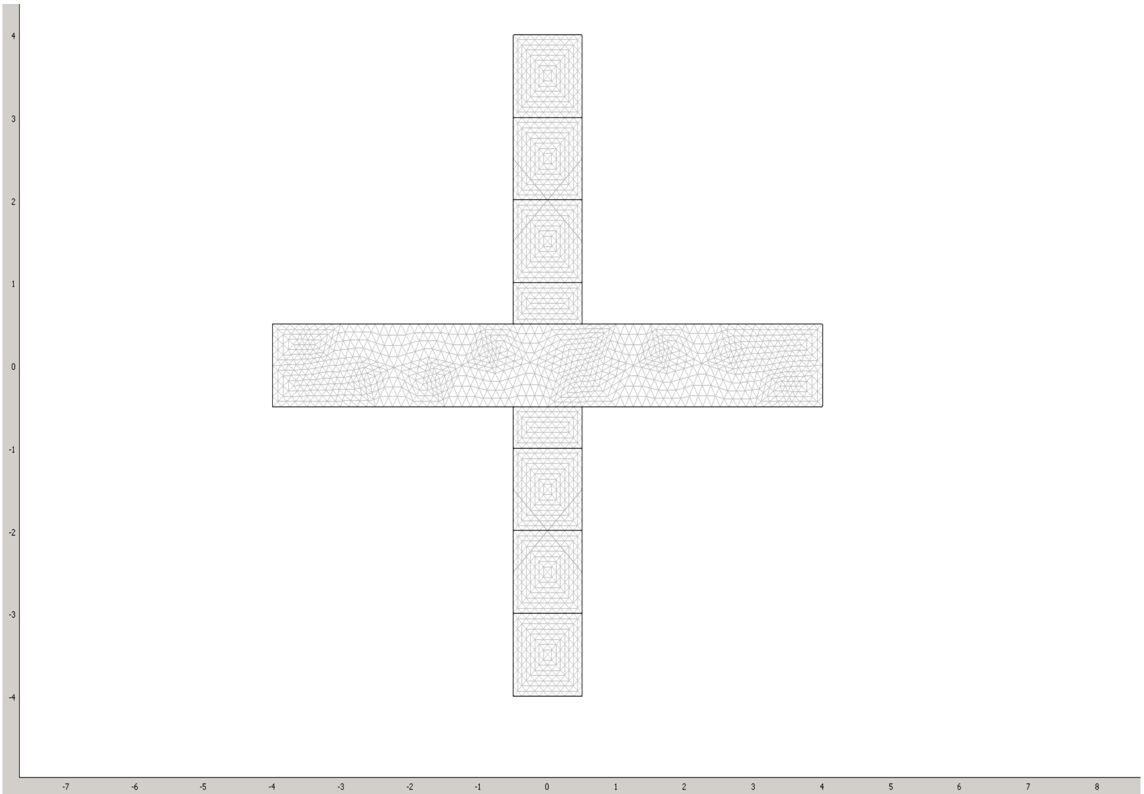

Figure 2: Typical mesh sized used for solutions in Comsol.

When plotted on a log-log axis evaluating mixing cup variance as a function of exit stream length over Pe, Figure 3 is generated. Figure 3 was constructed for the top exiting stream and approximately identical graph can be generated for the bottom stream.

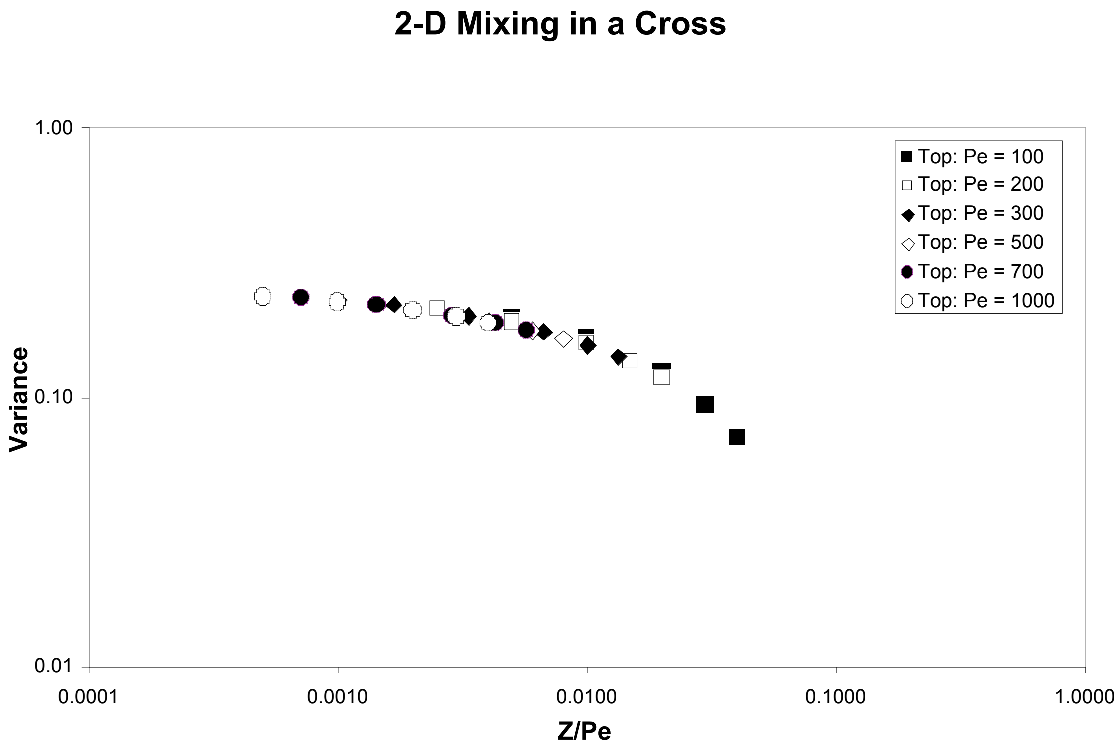

Figure 3: Mixing cup variance for flow in a two dimensional cross.

The pressure drop from the middle of the cross can be seen by examining table 1. This was evaluated for both exiting streams.

Table 1: Pressure drop at different values of Z (length/width)

	Bottom		Top	
Z	P'	dP	P'	dP
0.5	44.61	1.12	44.22	1.11
1.0	36.01	0.90	35.99	0.90
2.0	24.00	0.60	24.00	0.60
3.0	12.00	0.30	12.00	0.30
4.0	0.00	0.00	0.00	0.00

Three-Dimensional Case

For the three-dimensional case of the geometry the Navier-Stokes is solved first to characterize the flow. The Navier-Stokes equation was solved using 11,236 elements and 56,880 degrees of freedom. When the convective-diffusion problem was attempted using 18,035 degrees of freedom, Comsol was unable to converge. The meshed was refined and a solution was found with 32,475 degrees of freedom. However, there were some oscillations where the inlet streams meet and mix.

When evaluating the mixing cup variance as a function of stream length over Pe, Figure 4 is generated. It is of note that the figure generated for the three dimensional case is nearly identical to that generated for the two dimensional case.

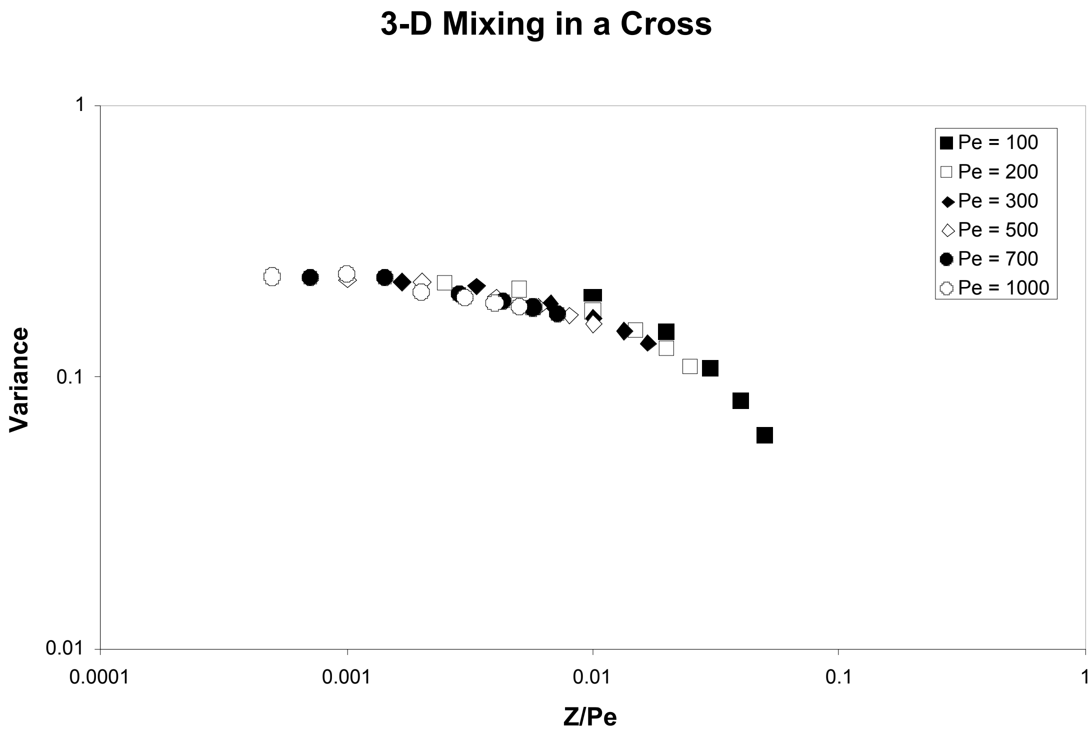

Figure 4: Three dimensional mixing cup variance as a function of stream length over Pe

The pressure drop for the three-dimensional case was also evaluated. The results can be seen in table 2 for the two exiting streams.

Table 2: Pressure drop at different values of Z (length/width). Locations refer to figure 1.

	Top Left		Bottom Right	
Z	**P'**	**dP**	**P'**	**dP**
0.5	112.94	14117.50	112.09	14011.24
1	102.31	12789.20	101.46	12682.94
2	77.97	9746.44	77.29	9661.65
3	52.00	6499.57	51.62	6452.32
4	26.09	3261.37	25.65	3206.66
5	0.00	0.00	0.00	0.00

Conclusions

It may be possible to enhance mixing by adding features in the side of the wall that will cause the width of the system at that point to be reduced. This will cause recirculation to occur in the vortices created and for the flow to increase through the smaller width. Due to the rectangular shape of the geometry an increase in Reynolds number may also increase the mixing.

In order to reach variances as low as 0.001 or even 0.01 the peclet number must be dropped to very low values. At a Pe of 10 it is possible to reduce the mixing cup variance to as low as 6.6×10^{-6} by the end of the three-dimensional case geometry.

By examining figures 3 and 4, it can be seen that for different peclet numbers, the data falls onto the same trend line when plotted on log-log axis. The results of this are similar to that of a T-sensor as would be expected to a largely similar device geometry.

From figures 3 and 4 it can also be seen that the two dimensional representation of the three dimensional geometry is an adequate model for the geometry. This can be seen in the similarities of the two figures; both of which collapse into curves of the same shape and magnitude for the mixing cup variance. Similar trends can be seen in the optical average concentration, optical variance, and mixing cup concentration.

Appendix

Table A-1: Summary of results for the top exiting stream in the two-dimensional representation.

Pe	Z	Z/Pe	v	Mixing Cup Conc	Variance	Top Optical Avg Conc	Optical Variance
100	0.5	0.0050	0.967	0.5058	0.1993	0.5040	0.2140
100	1.0	0.0100	1.000	0.5053	0.1668	0.5037	0.1926
100	2.0	0.0200	1.000	0.5053	0.1234	0.5041	0.1547
100	3.0	0.0300	1.000	0.5053	0.0935	0.5045	0.1217
100	4.0	0.0400	1.000	0.5053	0.0714	0.5048	0.0944
200	0.5	0.0025	0.967	0.5058	0.2149	0.5040	0.2251
200	1.0	0.0050	1.000	0.5053	0.1908	0.5037	0.2099
200	2.0	0.0100	1.000	0.5053	0.1595	0.5038	0.1868
200	3.0	0.0150	1.000	0.5054	0.1371	0.5040	0.1680
200	4.0	0.0200	1.000	0.5053	0.1190	0.5042	0.1503
300	0.5	0.0017	0.967	0.5056	0.2224	0.5038	0.2304
300	1.0	0.0033	1.000	0.5054	0.2016	0.5036	0.2175
300	2.0	0.0067	1.000	0.5054	0.1758	0.5037	0.1992
300	3.0	0.0100	1.000	0.5054	0.1573	0.5038	0.1851
300	4.0	0.0133	1.000	0.5054	0.1421	0.5039	0.1725
500	0.5	0.0010	0.967	0.5050	0.2301	0.5034	0.2359
500	1.0	0.0020	1.000	0.5053	0.2131	0.5036	0.2254
500	2.0	0.0040	1.000	0.5054	0.1925	0.5037	0.2111
500	3.0	0.0060	1.000	0.5054	0.1780	0.5037	0.2007
500	4.0	0.0080	1.000	0.5054	0.1659	0.5038	0.1918
700	0.5	0.0007	0.967	0.5046	0.2340	0.5031	0.2387
700	1.0	0.0014	1.000	0.5053	0.2198	0.5036	0.2300
700	2.0	0.0029	1.000	0.5054	0.2018	0.5037	0.2176
700	3.0	0.0043	1.000	0.5054	0.1893	0.5037	0.2089
700	4.0	0.0057	1.000	0.5054	0.1786	0.5037	0.2012
1000	0.5	0.0005	0.967	0.5042	0.2373	0.5028	0.2410
1000	1.0	0.0010	1.000	0.5052	0.2263	0.5035	0.2344
1000	2.0	0.0020	1.000	0.5055	0.2104	0.5037	0.2235
1000	3.0	0.0030	1.000	0.5055	0.1996	0.5038	0.2162
1000	4.0	0.0040	1.000	0.5055	0.1900	0.5038	0.2093

Table A-2: Summary of results for the bottom exiting stream in the two-dimensional representation.

					Bottom		
Pe	**Z**	**Z/Pe**	**v**	**Mixing Cup Conc**	**Variance**	**Optical Avg Conc**	**Optical Variance**
100	0.5	0.0050	-0.9657	0.5061	0.2002	0.5042	0.2146
100	1.0	0.0100	-1.0004	0.5048	0.1673	0.5034	0.1929
100	2.0	0.0200	-1.0002	0.5048	0.1237	0.5037	0.1550
100	3.0	0.0300	-1.0002	0.5048	0.0937	0.5041	0.1219
100	4.0	0.0400	-1.0002	0.5048	0.0716	0.5044	0.0946
200	0.5	0.0025	-0.9657	0.5061	0.2160	0.5042	0.2259
200	1.0	0.0050	-1.0004	0.5050	0.1912	0.5035	0.2103
200	2.0	0.0100	-1.0002	0.5050	0.1598	0.5035	0.1871
200	3.0	0.0150	-1.0002	0.5050	0.1373	0.5037	0.1682
200	4.0	0.0200	-1.0002	0.5050	0.1192	0.5039	0.1506
300	0.5	0.0017	-0.9657	0.5057	0.2235	0.5039	0.2313
300	1.0	0.0033	-1.0004	0.5050	0.2020	0.5035	0.2178
300	2.0	0.0067	-1.0002	0.5051	0.1761	0.5035	0.1994
300	3.0	0.0100	-1.0002	0.5051	0.1575	0.5036	0.1853
300	4.0	0.0133	-1.0002	0.5051	0.1423	0.5037	0.1727
500	0.5	0.0010	-0.9657	0.5048	0.2313	0.5031	0.2369
500	1.0	0.0020	-1.0004	0.5051	0.2136	0.5034	0.2258
500	2.0	0.0040	-1.0002	0.5051	0.1928	0.5035	0.2114
500	3.0	0.0060	-1.0002	0.5051	0.1782	0.5035	0.2010
500	4.0	0.0080	-1.0002	0.5051	0.1661	0.5036	0.1920
700	0.5	0.0007	-0.9657	0.5039	0.2353	0.5023	0.2398
700	1.0	0.0014	-1.0004	0.5050	0.2203	0.5032	0.2305
700	2.0	0.0029	-1.0002	0.5051	0.2020	0.5034	0.2179
700	3.0	0.0043	-1.0002	0.5051	0.1895	0.5034	0.2091
700	4.0	0.0057	-1.0002	0.5051	0.1788	0.5035	0.2014
1000	0.5	0.0005	-0.9657	0.5027	0.2387	0.5012	0.2423
1000	1.0	0.0010	-1.0004	0.5047	0.2268	0.5029	0.2349
1000	2.0	0.0020	-1.0002	0.5051	0.2106	0.5033	0.2238
1000	3.0	0.0030	-1.0002	0.5051	0.1999	0.5033	0.2164
1000	4.0	0.0040	-1.0002	0.5051	0.1902	0.5034	0.2095

Table A-3: Summary of results for the top left exiting stream in the three-dimensional case

Pe	Z	Z/Pe	v	Mixing Cup Conc	Variance	Optical Avg Conc	Optical Variance
100	0.5	0.0050	-1.5107	0.4987	0.2071	0.5000	0.2151
100	1	0.0100	-0.9701	0.5031	0.1943	0.5035	0.1941
100	2	0.0200	-0.9131	0.5038	0.1458	0.5037	0.1637
100	3	0.0300	-0.9080	0.5066	0.1081	0.5053	0.1262
100	4	0.0400	-0.9175	0.5067	0.0820	0.5056	0.0974
100	5	0.0500	-0.9082	0.5064	0.0615	0.5058	0.0738
200	0.5	0.0025	-1.5107	0.4989	0.2202	0.5003	0.2252
200	1	0.0050	-0.9701	0.5033	0.2099	0.5028	0.2113
200	2	0.0100	-0.9131	0.5035	0.1750	0.5029	0.1894
200	3	0.0150	-0.9080	0.5066	0.1476	0.5045	0.1653
200	4	0.0200	-0.9175	0.5066	0.1277	0.5046	0.1464
200	5	0.0250	-0.9082	0.5064	0.1099	0.5049	0.1283
300	0.5	0.0017	-1.5107	0.4992	0.2254	0.5006	0.2293
300	1	0.0033	-0.9701	0.5034	0.2165	0.5009	0.2165
300	2	0.0067	-0.9131	0.5032	0.1874	0.5021	0.1990
300	3	0.0100	-0.9080	0.5063	0.1652	0.5037	0.1806
300	4	0.0133	-0.9175	0.5063	0.1490	0.5037	0.1664
300	5	0.0167	-0.9082	0.5062	0.1340	0.5041	0.1525
500	0.5	0.0010	-1.5107	0.4997	0.2300	0.5012	0.2328
500	1	0.0020	-0.9701	0.5035	0.2245	0.4961	0.2208
500	2	0.0040	-0.9131	0.5031	0.1990	0.5013	0.2067
500	3	0.0060	-0.9080	0.5060	0.1823	0.5025	0.1940
500	4	0.0080	-0.9175	0.5061	0.1702	0.5028	0.1842
500	5	0.0100	-0.9082	0.5061	0.1585	0.5031	0.1744
700	0.5	0.0007	-1.5107	0.5000	0.2320	0.5017	0.2344
700	1	0.0014	-0.9701	0.5033	0.2306	0.4908	0.2245
700	2	0.0029	-0.9131	0.5033	0.2047	0.5008	0.2095
700	3	0.0043	-0.9080	0.5058	0.1910	0.5018	0.2000
700	4	0.0057	-0.9175	0.5059	0.1811	0.5022	0.1926
700	5	0.0071	-0.9082	0.5060	0.1714	0.5025	0.1846
1000	0.5	0.0005	-1.5107	0.5006	0.2334	0.5023	0.2357
1000	1	0.0010	-0.9701	0.5030	0.2379	0.4829	0.2301
1000	2	0.0020	-0.9131	0.5035	0.2093	0.5000	0.2111
1000	3	0.0030	-0.9080	0.5055	0.1983	0.5009	0.2042
1000	4	0.0040	-0.9175	0.5057	0.1904	0.5015	0.1989
1000	5	0.0050	-0.9082	0.5059	0.1825	0.5019	0.1927
10	0.5	0.0500	-1.5107	0.4992	0.0985	0.4999	0.1170
10	1	0.1000	-0.9701	0.5009	0.0688	0.5012	0.0587
10	2	0.2000	-0.9131	0.5031	0.0094	0.5034	0.0118
10	3	0.3000	-0.9080	0.5036	0.0008	0.5036	0.0010
10	4	0.4000	-0.9175	0.5036	0.0001	0.5036	0.0001
10	5	0.5000	-0.9082	0.5036	0.0000	0.5036	0.0000

Table A-3: Summary of results for the bottom right exiting stream in the three-dimensional case

Pe	Z	Z/Pe	v	Mixing Cup Conc	Variance	Optical Avg Conc	Optical Variance
100	0.5	0.0050	-1.5107	0.4987	0.2071	0.5000	0.2151
100	1	0.0100	-0.9701	0.5031	0.1943	0.5035	0.1941
100	2	0.0200	0.9030	0.4963	0.1450	0.4981	0.1629
100	3	0.0300	0.9070	0.4963	0.1079	0.4978	0.1261
100	4	0.0400	0.9031	0.4965	0.0811	0.4974	0.0965
100	5	0.0500	0.9017	0.4966	0.0609	0.4973	0.0730
200	0.5	0.0025	-1.5107	0.4989	0.2202	0.5003	0.2252
200	1	0.0050	-0.9701	0.5033	0.2099	0.5028	0.2113
200	2	0.0100	0.9030	0.4960	0.1744	0.4981	0.1888
200	3	0.0150	0.9070	0.4961	0.1479	0.4981	0.1657
200	4	0.0200	0.9031	0.4962	0.1271	0.4978	0.1459
200	5	0.0250	0.9017	0.4963	0.1095	0.4977	0.1279
300	0.5	0.0017	-1.5107	0.4992	0.2254	0.5006	0.2293
300	1	0.0033	-0.9701	0.5034	0.2165	0.5009	0.2165
300	2	0.0067	0.9030	0.4957	0.1867	0.4978	0.1982
300	3	0.0100	0.9070	0.4957	0.1656	0.4978	0.1809
300	4	0.0133	0.9031	0.4958	0.1485	0.4976	0.1659
300	5	0.0167	0.9017	0.4959	0.1337	0.4976	0.1521
500	0.5	0.0010	-1.5107	0.4997	0.2300	0.5012	0.2328
500	1	0.0020	-0.9701	0.5035	0.2245	0.4961	0.2208
500	2	0.0040	0.9030	0.4955	0.1975	0.4977	0.2042
500	3	0.0060	0.9070	0.4954	0.1821	0.4975	0.1933
500	4	0.0080	0.9031	0.4954	0.1693	0.4973	0.1827
500	5	0.0100	0.9017	0.4955	0.1578	0.4972	0.1730
700	0.5	0.0007	-1.5107	0.5000	0.2320	0.5017	0.2344
700	1	0.0014	-0.9701	0.5033	0.2306	0.4908	0.2245
700	2	0.0029	0.9030	0.4956	0.2022	0.4977	0.2050
700	3	0.0043	0.9070	0.4953	0.1900	0.4974	0.1978
700	4	0.0057	0.9031	0.4952	0.1796	0.4972	0.1896
700	5	0.0071	0.9017	0.4953	0.1700	0.4970	0.1820
1000	0.5	0.0005	-1.5107	0.5006	0.2334	0.5023	0.2357
1000	1	0.0010	-0.9701	0.5030	0.2379	0.4829	0.2301
1000	2	0.0020	0.9030	0.4958	0.2056	0.4976	0.2041
1000	3	0.0030	0.9070	0.4953	0.1961	0.4972	0.1998
1000	4	0.0040	0.9031	0.4951	0.1880	0.4969	0.1941
1000	5	0.0050	0.9017	0.4951	0.1802	0.4966	0.1884
10	0.5	0.0500	-1.5107	0.4992	0.0985	0.4999	0.1170
10	1	0.1000	-0.9701	0.5009	0.0688	0.5012	0.0587
10	2	0.2000	0.9030	0.4973	0.0092	0.4973	0.0116
10	3	0.3000	0.9070	0.4973	0.0008	0.4973	0.0010
10	4	0.4000	0.9031	0.4973	0.0001	0.4973	0.0001
10	5	0.5000	0.9017	0.4973	0.0000	0.4973	0.0000

Sample Calculations

$$\mathrm{Re} = \frac{\rho v_s x_s}{\eta} = \frac{1000\frac{kg}{m^3} \cdot 0.005\frac{m}{s} \cdot 200 \cdot 10^{-6}}{0.001 Pa \cdot s} = 1$$

$$Pe = \frac{v_s x_s}{D} = \frac{0.005\frac{m}{s} \cdot 200 \cdot 10^{-6} m}{10^{-9}\frac{m^2}{s}} = 1000$$

$$\Delta P = \frac{\Delta P' \eta v_s}{x_s} = \frac{44.61 \cdot 0.001 Pa \cdot s \cdot 0.005\frac{m}{s}}{200 \cdot 10^{-6} m} = 1.11 Pa$$