Heat transfer enhancement in ferrofluids subjected to steady magnetic fields

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Received 20 May 1998; received in revised form 18 September 1998

Abstract

Finite element simulations of heat transfer to a ferrofluid in the presence of a magnetic field are presented for flow between flat plates and in a box. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Electromagnetic device; FIDAP; Heat transfer enhancement; Magnetic convection

1. Introduction

Deregulation of the United States' electric utility industry is forcing utilities to improve efficiencies in order to remain competitive. Many are seeking new technologies to improve the operation of existing oil-cooled electromagnetic equipment. One approach suggested in the literature is to replace the oil in such devices with oil-based ferrofluids, which can take advantage of the pre-existing leakage magnetic fields to enhance heat transfer processes. This paper presents results of an initial study of the enhancement of heat transfer in ferrofluids in magnetic fields which are steady but variable in space.

2. Fluids and method

Simulations are done for a hydrocarbon oil and a ferrofluid, which is the same oil containing suspension of magnetite (90 G and 1.6% by volume). Physical properties used in the simulations are shown in Table 1.

The governing equations are solved using the finite element method to solve the extended Navier-Stokes equations. Problem 1 is solved using a special program, while Problem 2 was solved by adding the appropriate body force terms to the commercial code, FIDAP.

2.1. Problem 1. Flow between vertical parallel plates

The first problem was flow between two vertical parallel plates, both at the same temperature or with the same heat flux, with flow allowed in the bottom and out the top. The heat transfer
Table 1
Physical properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Hydrocarbon oil</th>
<th>Ferrofluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (g/cm$^3$)</td>
<td>0.875</td>
<td>0.972</td>
</tr>
<tr>
<td>$\mu$ (cp at 40°C, 80°C)</td>
<td>8.5, 0.26</td>
<td>15.0, 0.39</td>
</tr>
<tr>
<td>$C_p$ (J/kgK at 40°C)</td>
<td>1928</td>
<td>1764</td>
</tr>
<tr>
<td>$k$ (W/mK at 40°C)</td>
<td>0.1255</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta$ (K$^{-1})$</td>
<td>$7.17 \times 10^{-4}$</td>
<td>$9 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K$ (A/mK)</td>
<td>-</td>
<td>30</td>
</tr>
</tbody>
</table>

The phenomenon is similar to that occurring in a vertical enclosed space in an electromagnetic device. For Problem 1, the fluid is subjected to a uniform vertical magnetic field.

The magnetization is linearized about a known magnetic field and temperature. Since the magnetic convection is driven by interactions between the temperature gradients and magnetic field gradients, it is necessary to solve for the perturbation to the magnetic field caused by the temperature gradients. Terms involving magnetic field alone (not involving a temperature gradient) are absorbed into the pressure term. The equations are taken from Finlayson [3].

$$
\text{Re} u \cdot \nabla u = - \nabla p' - \frac{\text{Gr}}{\text{Re}} (T - T_0) e_\theta + \nabla^2 u \\
- N_M (T - T_0) \nabla (e_\theta \cdot \nabla \phi),
$$

$$
\text{Pe} u \cdot \nabla T = \nabla^2 T,
$$

$$
\nabla^2 \phi = \frac{M_0}{H_0} \left( \frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi \right) - \frac{\partial T}{\partial z},
$$

where

$$
\text{Re} = \frac{\rho u_0 x_s}{\mu}, \quad \text{Gr} = \frac{g \rho^2 \beta (T - T_0)}{\mu^2}, \quad \text{Pr} = \frac{C_p \mu^2}{k},
$$

$$
N_M = \frac{K^2 \Delta T^2 \mu_0 x_s}{\mu u_s}, \quad \text{Ra} = \text{PrGr}.
$$

The parallel plates were 15 cm long and separated by 0.4 cm. The boundary conditions along the centerline are no normal velocity, no normal heat flux, and no applied vertical force. Along the solid wall all velocities are zero, and either the temperature or the heat flux is specified. As the inlet temperature is specified as 75°C, the velocity is constrained to be vertical, but its magnitude is determined by the solution. At the outlet, natural boundary conditions are used for temperature and the vertical and horizontal velocities are set to zero. The boundary conditions on magnetic potential were zero gradient on all boundaries and $\phi = 1$ at one point.

The problem was solved for $\Delta T = 1, 2, 4, 7,$ and $10$ °C, where $\Delta T = T_{\text{wall}} - T_{\text{inlet}}$. The Nusselt number is defined as $\text{Nu} = h d / k$, where $d$ is the distance between the plates and $h \Delta T = q = \Delta E d / L$, where $\Delta E$ is the energy flow per unit area open to flow at the entrance. Plots of Nusselt number versus Rayleigh number for both fluids are shown in Fig. 1. The effect of the magnetic field was minimal, even when the magnetic term was increased arbitrarily by a factor of 10. The Nusselt numbers for the oil in this geometry were slightly larger than those reported by Bar-Cohen and Rohsenow [1] for air, as is expected. It is clear from their results that the 15 cm length is in the entrance region and exit region entirely. Thus, the results should not be extrapolated to longer lengths.

2.2. Problem II. Magnetic convection in a box

The second problem was to create convection through the interaction of a magnetic field gradient.
and a temperature gradient. The flow problem is solved inside a square box, infinitely long. In this case the perturbation of the magnetic field is ignored. The dimensionless equations are:

\[
\sqrt{\text{Gr}} \mathbf{u} \cdot \nabla \mathbf{u} = - \nabla p + \nabla^2 \mathbf{u} - \sqrt{\text{Gr}} T e_y - \sqrt{\text{Gr}} A \nabla H
\]

\[
\text{Pr} \sqrt{\text{Gr}} \mathbf{u} \cdot \nabla T = \nabla^2 T,
\]

where

\[
A = \frac{\mu_0 K H_s}{x_s \rho \beta g}, \quad u_s = \frac{k}{\rho \kappa x_s} \sqrt{\text{RaPr}}.
\]

The obvious parallel between natural convection and magnetic convection is exact if the magnetic field gradient is uniform. Calculations in an annulus have been performed before [2].

The magnetic field is taken to be that of a permanent magnet placed on top of the box, with poles at \( x = 0.5, y = 1.15 \) and \( 1.85 \) [5]. The expression for the magnetic field is then [4]:

\[
H = \frac{1}{\mu} \nabla x(e_s A),
\]
\[ A = \frac{1}{2} \sum_{j=1}^{2} S_j \ln[(x - x_j)^2 + (y - y_j)^2], \]

\[ S_1 = 1, \quad S_2 = -1, \]

where \( z_j \) gives the location of the poles.

The boundary conditions are that velocity is zero on all sides of the box, the dimensionless temperature is 1.0 on the right-hand side and 0.0 on the left-hand side, with an insulated top and bottom. Results are expressed as a Nusselt number, \( \text{Nu} = hD/k = qD/k\Delta T \). Plots of the Nusselt number versus Grashof number are given in Fig. 2 for the ferrofluid with \( \text{Pr} = 176.4 \). The strength of the applied magnetic field is specified through the parameter \( A \), which varies from 0 to 2.0; \( A = 1 \) corresponds to \( H = 286 \) Gauss. For the largest magnetic field, the enhancement of Nusselt number is as high as 45\%. Shown in Fig. 3 is a velocity vector plot for a case including gravitational and magnetic effects. The magnetic field results in two major recirculation regions, instead of just one when only gravity is included. For this case the magnetic field effect dominates, since the vector plot in Fig. 3 is similar to that obtained with a magnetic field and no gravity, and is dissimilar to the figure obtained with gravity and no magnetic field.

3. Conclusions

Simulations of a ferrofluid in a magnetic field and temperature gradient show significant heat transfer enhancement due to a magnetic field gradient.

References