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Particle-Liquid Dynamics of a Heated Ferrofluid in a Rotating Magnetic Field

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Ferrofluids are colloidal suspensions of magnetite nanoparticles (10 nm), coated in a surfactant to prevent agglomeration, and suspended in a base such as oil. These fluids are of interest in the cooling of electromagnetic devices which are normally oil-cooled and insulated. In this work, a heated ferrofluid is subjected to a 60Hz rotating magnetic field, causing both natural and magnetic convection. The governing equations include a modified linear momentum equation, along with an angular momentum equation for particle spin that arises through the effect of the rotating magnetic field. The two equations are coupled through a momentum exchange between the carrier vorticity and the particle spin. This exchange varies throughout the fluid, which then governs the ferrofluid behavior. The energy equation is also coupled to the linear momentum equation through the buoyancy term. The equations are solved using the finite element CFD package FIDAP. Preliminary results from the simulation of the heat transfer will be presented.

Introduction

Magnetic fluids (ferrofluids) were first synthesized, under NASA sponsorship, in the early 1960's in an effort to create gravity-independent seals for NASA and its space shuttle applications. Commercialization of the technology was undertaken by Ferrofluidics Corporation shortly thereafter. In the 1990s, manufacturing technology for ferrofluids has advanced rapidly, resulting in a larger number of manufacturers (Xerox Corporation, USA; Liquids Research Limited, United Kingdom; Taiho Industry Company, Japan; many universities), a wider variety of stable products, and lower prices. Increasing numbers of magnetic fluids-related patents and publications in the literature point toward rapidly rising interest in magnetic fluids and their applications.

Current widespread commercial applications of ferrofluids subjected to steady magnetic fields include rotary shaft sealing applications for pumps, compressors, and crystal pulling furnaces (semiconductor manufacturing). One application that involves heat transfer is in the cooling of stereo speakers, where the fluid is injected between the permanent magnet and voice coil. Heat transfer applications for ferrofluids subjected to time-varying (rotating, alternating) magnetic fields do not appear to have been commercialized at this time. The last application represents the strongest heat transfer driving mechanism.

In the last few years, interest has been picking up in the study of ferrofluids subjected to time-varying magnetic fields. Interest lies in the cooling of electromagnetic devices which are normally oil-cooled and insulated, and in which time-varying leakage magnetic fields exist *a priori*. The chemical and mining process industries represent another promising arena of application wherein magnetic separation of slurries is currently in use. With the start of construction of the International Space Station, original sponsor NASA has shown renewed interest in ferrofluids as a heat transfer fluid in microgravity conditions.

The authors' specific interest in ferrofluids is as a heat transfer enhancement technology for earth-bound industrial processes/equipment, as well as for microgravity applications. In order to successfully implement such a technology, it is imperative to have a good understanding of the physics that give rise to the enhanced heat transfer achieved with ferrofluids and time-varying magnetic fields. The physics include the

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interaction of natural (negligible under microgravity) and magnetically driven convection, as well as conduction. A critical part of the physics arrives in the interaction between the magnetically and thermally driven ferrofluid nanoparticles and the base oil carrier. To further understand the particle-liquid dynamics in rotating fields, and to be able to numerically model it, information is drawn from a number of related interdisciplinary areas of research.

Several studies have focused primarily on the effect of time-varying magnetic fields. Rosensweig et al. (1990) experimentally and theoretically investigated the ferrofluid spin-up in a ferrofluid subjected to a rotating magnetic field. They identified the ferrofluid's free surface stress as the primary coupling mechanism between the rotating field and the ensuing ferrofluid spin-up. Zahn and Wainman (1993) experimentally and numerically investigated pumping of a ferrofluid subjected to a traveling wave magnetic field. They observed reverse pumping at low magnetic field strengths, and forward pumping at high magnetic field strengths.

Application of a time-varying magnetic field changes the fluid's viscosity, and gives rise to an effective viscosity that changes with applied field strength and frequency. As the nanoparticles align with the magnetic field flux lines, resistance to flow with the bulk carrier increases, in turn changing the effective viscosity. Shliomis et al. (1994) theoretically studied the effect of a linearly polarized (one-dimensional) alternating field on the effective viscosity of an isothermal ferrofluid. They demonstrated that at low field frequency and specific field amplitude, the viscosity attained positive values, while at high frequency and specific amplitude, the viscosity was negative. Rosensweig (1996) experimentally investigated the effect of an alternating, linearly polarized, magnetic field on the effective viscosity of an isothermal ferrofluid. Similarly to the results of Shliomis, Rosensweig shows that the ferrofluid attains both positive and negative values of viscosity, dependent upon the combination of field frequency and amplitude. Gazeau et al. (1996) and Zeuner et al. (1998) can also be referenced for additional viscosity measurements.

Studies on heat transfer enhancement are less numerous, in particular when time-varying magnetic fields are applied. Sawada et al. (1994) studied heat transfer enhancement with a magnetic fluid contained in a cubic enclosure. The magnetic fluid was subjected to a time-varying field within a range of field strengths. Their results showed regimes of both significant heat transfer enhancement and suppression. This is the only published study of heat transfer enhancement by time-varying fields known to the authors. Cader (1997) investigated a horizontal layer of ferrofluid heated from below (gravitational effects minimized) and above. A 60Hz, vertically oscillating, magnetic field was applied to the layer, and the temperature difference across the layer was measured. For both cases, significant (on the order of 25%) heat transfer enhancement was achieved, and the effect generally increased with increasing magnetic field strength. Previous work (Cader et al., 1996) suggests further heat transfer control and enhancement is achievable with changes in magnetic field frequency.

The majority of the research conducted has been by academic institutions. In recent years, industrial interest has been on the increase. Of note is the research by the authors, sponsored primarily by the Electric Power Research Institute (on behalf of electric utilities); ultimate interest is application of the technology to cooling of oil-cooled devices such as transformers. A second study has been conducted by Segal et al. (1998, 1998a) of industrial concern Asea Brown Boveri and Ferrofluidics Corporation. These researchers are studying the ferrofluid-cooling and insulation of a number of distribution transformers. They have reported significant reductions in transformer winding temperature rises and gradients, along with excellent results from the accelerated thermal aging of their ferrofluid. They also show that the ferrofluid can withstand a significantly higher impulse voltage than that of the oil. These studies are critical in that success will guarantee deployment of massive quantities of ferrofluid (compared to the "droplets" employed to-date), and will be sure to significantly advance the field.

The authors' work combines experimental, theoretical, and numerical analyses of heat transfer in ferrofluids. Presented in this paper are ongoing results from the numerical modeling of heat transfer in ferrofluids. Results for ferrofluids subjected to steady magnetic fields were reported in Tangthieng et al. (1998). Here, we report the first finite element (using commercial CFD code FIDAP from Fluent, Inc.) results for a heated ferrofluid subjected to a rotating magnetic field.

Numerical Model

The equations governing a ferrofluid in a magnetic field have been solved using commercially available FIDAP. FIDAP is a finite element code that solves the Navier-Stokes equations (for a Newtonian fluid) in two and three dimensions. It is necessary to add special body force terms and a source term to the Navier-Stokes equations, develop iteration strategies for

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$$\sqrt{Gr}\,\mathbf{u}'\bullet\nabla\mathbf{u}' = -\nabla'p' - \sqrt{Gr}\,T'\,\mathbf{e}_g + \left(1 + \frac{\zeta}{\mu}\right)\nabla'^2\,\mathbf{u}' + 2\,\frac{\zeta}{\mu}\,\nabla'\times\omega' + \sqrt{Gr}\,\frac{\mu_0 H_0^2}{\beta g T_s \rho x_s}\,M'\bullet\nabla'\;H'$$

solving the equations, then solve them. The ferrofluid selected for the simulations reported herein is a 90 Gauss fluid (1.6 % solids by volume).

(1)

Governing Equations

A magnetic fluid without a magnetic field applied behaves as a Newtonian fluid. Heat transfer within such a fluid can then be modeled by the Navier-Stokes and energy equations. When a rotating field is applied to the fluid, the particles are set into rotation. This provides a mechanism for injecting angular momentum into the fluid. Once a fluid has a structure that permits internal angular momentum, it is necessary to expand the fundamental equations to include angular momentum effects. The basic changes are that the viscous stress tensor is no longer symmetric, which changes the linear momentum equation. Then an internal angular momentum equation must be established which accounts for the interaction between the spinning particles and fluid motion. Finally, there must also be an equation for the particle magnetization, since changes will occur as the particles rotate.

The authors start with the equations for a rotating magnetic field as presented by Rosensweig (1985). The linear momentum equation is augmented by a term due to the anti-symmetric term in the shear stress; the shear stress comes from the forces applied on the surface of fluid elements, irrespective of source of the forces. The angular momentum involves couple stresses representing rotational forces applied on the surfaces of fluid elements.

The linear and angular momentum equations must be augmented by constitutive relations relating the shear stress and the couple stress to velocity, spin, and their derivatives. The symmetric portion of the shear stress is proportional to the velocity gradient, with the constant of proportionality being the coefficient of shear viscosity, μ. The anti-symmetric portion of the shear stress depends on the difference between the local rate of spin and one-half the vorticity, with the constant of proportionality being the vortex viscosity, ζ. The couple stress is proportional to the spin gradient, with the constant of proportionality being the coefficient of spin viscosity, η'.

After combining the equations with the constitutive relations, and subsequently non-dimensionalizing the result, the governing equations are given by Equations (1) and (2) (the prime [′] generally indicates a dimensionless parameter or derivative, except for spin viscosity η'):

$$\frac{I\mu}{\eta'}\sqrt{Gr}\,\mathbf{u}'\bullet\nabla'\;\omega' = \frac{\mu_0 H_0^2 \rho x_s^4}{\eta'\;\mu\sqrt{Gr}}\,M'\times H' + \nabla'^2\;\omega' + \frac{2\zeta x_s^2}{\eta'}\left(\nabla'\times\mathbf{u}' - 2\omega'\right) \qquad (2)$$

where $Gr = \frac{g\rho^2 x_s^3 \beta T_s}{\mu^2}$, g is the gravitational acceleration, $\mathbf{e}_g$ is the unit vector in the direction of gravity, μ_0 is the magnetic permeability of free space, **u** is the velocity vector, p is pressure, T is temperature, T_s is the characteristic temperature, x_s is the characteristic distance, ω is the spin velocity, β is the coefficient of thermal expansion, ρ is the density, **M** is the fluid magnetization, **H** is the magnetic field strength, H_0 is the magnetic field strength at $T=T_0$, and I is the moment of inertia of the particles.

The analysis for this study is performed per unit volume of magnetic fluid (note that there are as many as 10^{22} particles per m^3 of ferrofluid). Equation 2 represents the rate of change of internal angular momentum due to the exertion of a body couple by the externally applied magnetic field, the exertion of a surface couple due to the diffusion of internal angular momentum across the control volume surface, and finally the exchange of angular momentum between the external and the internal types.

Since heat transfer is also being considered, the energy equation (steady-state, viscous dissipation shown to be negligible) is also given as:

(3)

where $Pr = \frac{C_p\mu}{k}$.

$$Pr\sqrt{Gr}\,\mathbf{u}'\bullet\nabla T' = \nabla'^2 T'$$

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The ferrofluid magnetization equation is proposed by Rosensweig as:

$$\mathbf{u} \bullet \nabla M = \omega x M - \frac{1}{\tau}(M - M_0) \qquad (4)$$

where τ is the particle time constant and $\mathbf{M}_0$ is the fluid magnetization at $T=T_0$.

Particle-Liquid Dynamics

When a rotating magnetic field is applied to a ferrofluid, a torque is imparted to the magnetic particles. For the present study, the torque is assumed applied uniformly over a unit volume of ferrofluid (mixture of carrier and magnetic particles). The angular momentum equation (Equation 2) contains a term $\mathbf{M}'\times\mathbf{H}'$ that models this torque. The magnetic torque density, Γ, is given by:

$$\Gamma = \mu_0 \mathbf{M}'\times\mathbf{H}' \qquad (5)$$

with the time average given by (Zahn and Pioch, 1997):

$$\langle\mathbf{T}\rangle = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}\mathbf{T}(x,y,t)dt \qquad (6)$$

where Ω is the frequency of oscillation, x and y are position coordinates, and t is the time.

In the present analysis, we employ the time-average torque. At a field frequency of 60 Hz, modeling the time-dependent torque would be prohibitively expensive (i.e., computationally), and would not yield different results since equations are averaged.

To arrive at the expression for time average torque, the magnetization equation was solved along with Maxwell's equations for a non-conducting ferrofluid (following Zahn and Pioch). Equation 5 shows that the torque is a complicated function of the rotational velocity Ω, as well as the spin velocity ω. The last term in parentheses of the spin velocity equation (Equation 2) provides useful information relating to the particle-liquid dynamics. This term highlights the momentum exchange between one-half the carrier vorticity and the spin velocity, and indicates that since the vorticity varies with position, so will the spin velocity. Thus, the torque will be a complicated function of position, through the spin velocity, which in turn depends on the vorticity.

Results

Figure 1 shows preliminary results from the simulation of heat transfer. Figure 1a illustrates the geometrical configuration considered, with cooling in the left and lower walls, insulation on the top, and an axis of symmetry on the right. The heat source is shown as three internal rectangles, with cooling passages or ducts passing through the heat source. Gravity acts toward the bottom, and the rotating magnetic field always lies in the plane of the paper while rotating about on an axis perpendicular to the paper, like a clock hand.

There are two simplifications possible when a rotating magnetic field is applied to a ferrofluid. The first involves the particle time constant,

$$\tau_B = \frac{\pi\eta d^3}{2\kappa T} = 6x10^{-5}\,s \qquad (7)$$

where η is the carrier fluid viscosity, d is the particle diameter, and κ is the Boltzmann constant.

Given the small value of τ_B for the ferrofluid used, the expression for the torque on the particles can be significantly simplified. Secondly, the term involving the spin viscosity is extremely small (using Rosensweig's 1990 estimate) and is neglected here. Since this is the term that multiplies the operator with the highest derivative, of spin velocity, it may be important to spin diffusion. It is analogous to the term in the linear momentum equation that accounts for boundary layers. In essence, neglecting this term yields an inviscid solutions with regard to spin velocity. Hence, the spin boundary condition at the wall is not addressed, but the spin velocity in the bulk flow is. Using these simplifications, the linear momentum and angular momentum equations can be combined into a single modified Navier-Stokes equation,

$$\sqrt{Gr}\,\mathbf{u}'\bullet\nabla'\mathbf{u}' = -\nabla' p' - \sqrt{Gr}\,T'\mathbf{e}_g + \left(1 - \frac{Bk_1}{2+Bk_1}\frac{\zeta}{\mu}\right)\nabla'^2\mathbf{u}' \qquad (8)$$

where $B = \frac{\mu_0 H_0^2 \rho x_s^2}{2\mu^2\sqrt{Gr}}\frac{u_s \tau_B}{x_s}$, u_s is the characteristic velocity, $k_1 = \chi_0\frac{(1+2\chi_0+2\chi_0^2)}{2(1+\chi_0)^2}$, and χ_0 is the magnetic susceptibility. The simulation is then performed for a reduced viscosity:

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$$\mu - \frac{Bk_1}{2 + Bk_1}\zeta$$

For a typical ferrofluid, the value of the coefficient $Bk_1/(2 + Bk_1)$ is on the order of (10^{-1}).

Selected results from the simulation of the heat transfer in a ferrofluid subjected to a rotating magnetic field are shown in Figures 1a, 1b, 1c, 1d, and 1e. Figure 1b is a velocity vector plot, showing a strong updraft on the right, a thin downdraft on the cold left wall, and slow flow up the cooling ducts in the heat source. Figure 1c is the streamline contour plot, showing one eddy cell in the upper area. Figure 1d, a temperature contour plot, shows a stagnant pool in the bottom area with uniform temperature gradient, and nearly uniform temperature in the eddy cell in the upper area. Figure 1e, a vorticity contour plot, shows strong vorticity at the left corners of the heat source, on the right, and on the cold left wall. The reduced viscosity indicates an increased Grashof number, and this in turn indicates increased heat transfer. Thus, the final effect of the rotating magnetic field is to increase the heat transfer.

The effect of the rotating field is to thin the thermal boundary layers and increase the heat transfer rate. The particle rotation increases the mixing throughout the fluid as well.

CONCLUDING REMARKS

This paper presents the first finite-element solutions for heat transfer in the presence of a rotating magnetic field. The numerical model takes advantage of a reduced viscosity that arises under our assumptions of a negligible particle time constant for rotational relaxation and negligible particle spin viscosity.

A rotating magnetic field induces a torque on the particles within a ferrofluid. This torque in turn imparts a velocity to the fluid, increases local mixing, increases the thermal conductivity, and decreases thermal boundary-layer thickness. The overall effect is an increase in total heat transfer.

Ferrofluids offer increased cooling of electrical devices in which time-dependent magnetic fields exist *a priori*. In addition, ferrofluids in a rotating magnetic field hold the promise of convective cooling in microgravity applications.

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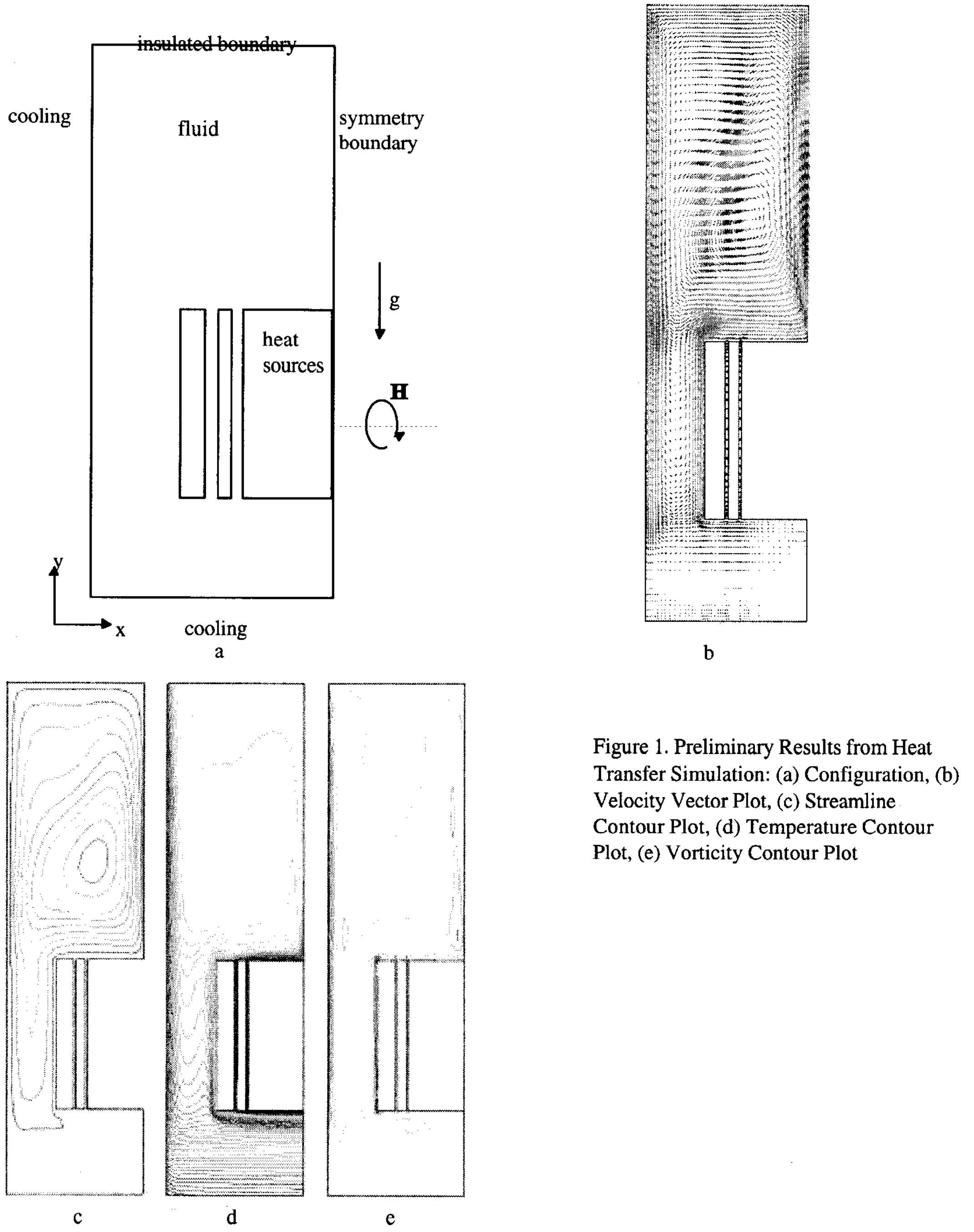

Figure 1. Preliminary Results from Heat Transfer Simulation: (a) Configuration, (b) Velocity Vector Plot, (c) Streamline Contour Plot, (d) Temperature Contour Plot, (e) Vorticity Contour Plot

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