Modeling a Ferrofluid in a Rotating Magnetic Field

Bruce A. Finlayson
[1]University of Washington
*Corresponding author: Dept. Chemical Enrg., Box 351750, Seattle, WA 98195

Abstract: Comsol Multiphysics is used to solve the problem of a ferrofluid placed in a rotating magnetic field, known as the spin-up problem. The spin-up phenomenon occurs when the fluid is contained in a cylinder and a homogeneous magnetic field rotates uniformly in the circular cross-section. This work shows how to implement the pertinent equations in Comsol Multiphysics and shows that the simulations agree with the experimental observations made over the past 40 years. Simulations are also used to show the proper boundary condition is zero spin on solid walls, because that is the only one that gives results in agreement with experiments.

Keywords: ferrofluid, angular momentum, spin-up.

1. Introduction

A ferrofluid is a suspension of nano-sized magnetic particles, often magnetite with a diameter of 10 nm. The particles are coated with a surfactant which increases their diameter to about 25 nm; the surfactant keeps the particles from agglomerating in a magnetic field. The suspension is in an aqueous fluid or organic fluid. The spin-up phenomenon occurs when the fluid is contained in a cylinder and a homogeneous magnetic field rotates uniformly in the circular cross-section. Applications of ferrofluids include rotary seals (in computer hard drives) and multistage rotary seals (in silicon manufacture), inertia dampers and coolants (in high-end loudspeakers) (1). More recently they have been used in microfluidic devices and nanodevices (2,3) and show promise for use in biomedical fields (4).

The spin-up phenomenon was first reported in 1967 by Moskowitz and Rosensweig (5) and is illustrated in Figure 1. They found that the fluid on the top, free surface rotated faster as the strength of the magnetic field was increased, and it was larger for faster spinning of the magnetic field. Later (6) they discovered that the rate of rotation of the fluid was in the opposite direction to the direction of the rotation of the magnetic field, although the direction of torque on the wall followed the magnetic field. Brown and Horsnell (7) reported similar 'wrong-way' behavior; if the ferrofluid was contained in a freely suspended beaker, the beaker rotated in the same direction that the magnetic field was rotating, but the fluid (especially in a thin layer near the boundary) rotated in the opposite direction. They also saw violent agitation as the magnetic field was increased. Zaitsev and Shliomis (8) provided a theory for the spin-up phenomenon using the continuum theory of polar fluids developed by Dahler and Scriven (9). In the work by Dahler and Scriven, the fluid was allowed to have a structure which could rotate. The total angular momentum was conserved, and an equation was derived for the internal angular momentum; that is called here the spin equation. Zaitsev and Shliomis solved this equation under restrictive assumptions (constant torque, for example), but at that time no known value was available for the spin viscosity and the boundary condition on spin was uncertain. The results did show, however, that the fluid rotated in almost solid body rotation, except near the wall. The fluid always rotated in the same direction as the magnetic field, though.

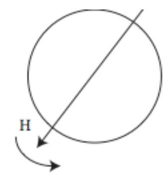


Figure 1. Diagram of the spin-up experiment

The situation was further confused by a remark by Jenkins (10) that deGennes indicated the fluid would not move in this situation. Other work showed it did, though. Glazov (11, 12) ascribed the movement to inhomogeneities in the magnetic field. Kagan (13) showed that flow reversal occurred as the magnetic field was increased, but he was using a colloidal fluid with larger particles than a typical ferrofluid. In 1989, Rosensweig and Johnson (14) measured the velocity on the free surface in an open container and found that the velocity exhibited solid body rotation except for a thin layer near the boundary (about 10% of the radius). They also found that the fluid rotation rate increased as the diameter of the vessel decreased. Rosensweig, et al. (6) presented a complete theory, but could only deduce that surface deflection would affect the direction of flow; they determined the direction of flow on the surface by experimentally monitoring small copper particles floating on the surface. Since ferrofluids are typically black, all these indications of flow were either assumed or based on observation at a top free surface. Rinaldi, *et al.* (15) put a ferrofluid into a rheometer and inserted the spindle. They did experiments in which the spindle was rotating, and others with a fixed spindle. The torque was measured when the magnetic field was rotating clockwise and counterclockwise. When the spindle was rotating, the torque was increased when the magnetic field rotated in the same direction and decreased (and reversed) when the magnetic field was rotating in the opposite direction. When the spindle was fixed, the torque was in the direction of the rotation of the magnetic field, clockwise or counterclockwise. More recently, Chaves, *et al.* (16) have used ultrasound to measure the internal velocity. They find, for their conditions, that the fluid rotates in the direction of the rotating magnetic field, except that if the top surface is exposed to the atmosphere, there is a region at the top in which flow reversal occurs.

This study was begun to utilize the power of Comsol Multiphysics to solve the continuum equations governing the situation in two- and three-dimensions without making limiting assumptions. The most common assumption is that the torque in homogeneous – identical in all regions. As we see below, that assumption alone means that certain phenomena will not be predicted that in fact occur experimentally. The Navier-Stokes equation is extended to include the spin vector and a spin equation is solved simultaneously with the flow. The magnetization for this non-conducting magnetic fluid is solved using Shliomis's magnetization equation (17). The magnetic field is represented as the gradient of a potential. Some of these equations are quasi-static, since they have no time derivative in them. This is true for the spin equation because the moment of inertial of the magnetic particles is so small. The boundary condition on spin is taken as one of four conditions: (1) zero spin on solid surfaces; (2) zero couple stress on solid surfaces; (3) Newtonian behavior at the surface; and (4) using the angular momentum equation as an algebraic equation throughout the domain so no boundary condition is needed. The work described below indicates which boundary conditions lead to results that are consistent with experimental findings and it also elucidates the mechanism that leads to the complicated flow, including flow reversal, that will not be seen in simple analyses.

2. Theoretical Development

The theoretical development generally follows Rosensweig (18) and Shliomis (17).

2.1 Governing Equations

The governing equations are (18):

Navier Stokes equation with magnetic body force and spin term added

$$\rho\frac{\partial\mathbf{v}}{\partial t} + \rho\mathbf{v}\bullet\nabla\mathbf{v} = -\nabla\mathbf{p} + 2\varsigma\nabla\mathrm{x}\,\boldsymbol{\omega} + (\eta+\varsigma)\nabla^2\mathbf{v} + \mu_{\mathbf{0}}\mathbf{M}\bullet\nabla\mathbf{H}$$

Spin equation, a representation of conservation of angular momentum:

$$0 = \mu_0\mathbf{M}\mathrm{x}\,\mathbf{H} + \zeta(2\nabla\mathrm{x}\,\mathbf{v} - 4\boldsymbol{\omega}) + \eta'\nabla^2\boldsymbol{\omega}$$

Magnetization equation, valid for small magnetic fields (17):

$$\frac{\partial\mathbf{M}}{\partial t} + \mathbf{v}\bullet\nabla\mathbf{M} = \boldsymbol{\omega}\mathrm{x}\,\mathbf{M} - \frac{1}{\tau}\left(\mathbf{M} - \mathbf{M}_{\mathbf{eq}}\right)$$

The magnetic equations are Maxwell's equations for a non-conducting fluid

$$\nabla\bullet\mathbf{B} = \mathbf{0},\ \ \nabla\mathrm{x}\,\mathbf{H} = 0,\ \ \mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}).$$

Thus, the magnetic field is given by the gradient of a potential

$$\mathbf{H} = \nabla\phi,$$

and the magnetic flux equation is rewritten as

$$\nabla^2\phi = -\nabla\bullet\mathbf{M}.$$

The applied, external magnetic field oscillates with a frequency $\mathbf{\Omega}_f$. The variables are ρ-density, **v**-velocity, t-time, p-pressure, ω-spin, η-viscosity, ζ-vortex viscosity, **M**-magnetization, **H**-magnetic field, μ_0-magnetic permeability of free space, η'-spin viscosity, τ- Brownian magnetic time constant, **B**-magnetic flux density, ϕ-magnetic potential. The spin equation is obtained by writing an equation for total angular momentum including the spin of local structure, and subtracting off the angular momentum equation derived from the linear momentum equation. Then, because the particles are so small, the time and convective derivatives are dropped since the moment of inertia of the particles is small. The result is a quasi-static equation for spin involving the vorticity, the magnetic torque, and diffusion of spin, governed by a spin viscosity. The other assumption in these equations is that the magnetic field is small enough that the magnetization-magnetic field equation is linear; this assumption will be removed in future work.

When the flow begins, it eventually approaches a steady rotational flow, mainly because the magnetic torque is constant over most of the domain. Thus, the solutions here use a quasi-static approximation for the velocity, too. Since the flow is primarily circular, in which case the convective term is zero, that term is neglected, too. Future work will include those effects as well, but they add significantly to the computational task and are not central to the conclusions.

While the equations are written in terms of spin, it is useful to replace the variable spin by the excess spin defined as the spin minus one-half the vorticity, $\boldsymbol{\omega}_{\text{excess}} = \boldsymbol{\omega} - \frac{1}{2}\nabla \text{x}\,\mathbf{v}$. The identity

$$\nabla^{\mathbf{2}}\mathbf{v} = -\nabla \times (\nabla \text{x}\,\mathbf{v}) \;\; + \nabla(\nabla \bullet \mathbf{v})$$

and continuity ($\nabla \bullet \mathbf{v} = 0$) is used to transform the momentum equation.

$$0 = -\nabla \mathbf{p} + 2\varsigma \nabla \text{x}\left[\boldsymbol{\omega} - \frac{1}{2}\nabla \text{x}\,\mathbf{v}\right] + \eta \nabla^2 \mathbf{v} + \mu_0 \mathbf{M} \bullet \nabla \mathbf{H}$$

The momentum equation and spin equation are then:

$$0 = -\nabla \mathbf{p} + 2\varsigma \nabla \text{x}\,\boldsymbol{\omega}_{\text{excess}} + \eta \nabla^2 \mathbf{v} + \mu_0 \mathbf{M} \bullet \nabla \mathbf{H}$$

$$4\,(\boldsymbol{\omega} - \tfrac{1}{2}\nabla \text{x}\,\mathbf{v}) = 4\boldsymbol{\omega}_{\text{excess}} = \frac{1}{\zeta}\Big[\mu_0 \mathbf{M} \text{x}\,\mathbf{H} + \eta' \nabla^2 \boldsymbol{\omega}\Big]$$

The equations are put into Comsol Multiphysics in non-dimensional form. In all cases, the transient form of the equation was used, and for quasi-static variables the coefficient multiplying the time derivative was set to zero. The transient Navier-Stokes equation was augmented to get the momentum equation. Two transient convective diffusion equations were augmented to get two magnetization equations (with zero diffusion). The transient diffusion equation was used for the spin equation, and a transient PDE mode was used for the magnetic potential equation. The time dependent terms were set to zero in the equations for momentum, spin, and magnetic potential. Transient simulations were done, and it took only a short dimensionless time for the magnetization to reach its cyclic situation that would repeat for many cycles; the other variables reached their quasi-static situation immediately and only changed while the magnetization was coming to its cyclic state. Thus, after a short start-up time, the results of variables other than magnetization were constant in time. All results reported here are dimensionless unless otherwise indicated. The key dimensional group was identified by Rinaldi, *et al.* (15) and varies with the square of the strength of the applied magnetic field, *K*:

$$\varepsilon = \frac{\mu_0 \chi_i K^2 \tau}{\zeta}$$

2.2 Boundary Conditions

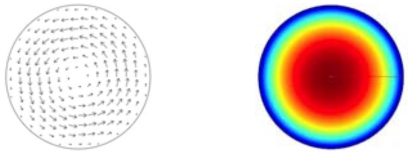
Boundary conditions must be specified for velocity, spin (unless the spin diffusion term is dropped), and magnetic potential. The velocity conditions are the usual ones: no slip on solid boundaries and slip on fluid boundaries. The spin boundary conditions are uncertain, and four are considered below. The magnetic potential boundary conditions for a rotating uniform magnetic field are

$$\phi = \phi_{applied}\left[x\cos(t) + y\sin(t)\right]$$

The pressure is specified at one point if there is no free surface.

2.3 Fluid Parameters

The estimation of parameters is an art, but we use values appropriate for a kerosene-based ferrofluid (EMG-900) manufactured by Ferrotec Corporation. A magnetic field of 50 gauss was oscillated at 85 Hz in a cylinder with a radius of 2.47 cm. The spin viscosity is taken 100 times the literature value since a small spin viscosity causes a thin boundary layer for the spin near a solid wall. While solutions (not shown) have been obtained with the literature value; the mesh must be refined near the wall and the solution takes longer to compute, but otherwise the



(a) (b)

Figure 2. (a)Arrow plot of velocity; (b) spin (from 0 (blue) to 0.202 (red).

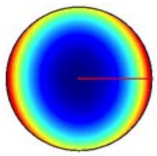


Figure 3. Torque (from 0.918 (ble) to 0.975(red).

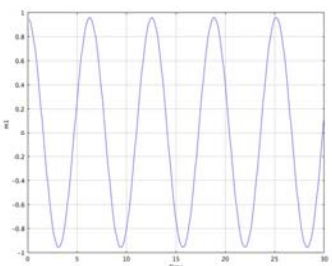


Figure 4. X-component of magnetization versus time.

phenomena are similar to the results reported here.

3. Results

3.1 Two-dimensional simulations

The first case presented is for a boundary condition of zero spin on the boundary in two dimensions. The parameter ε=0.155 in this base case. An arrow plot of the velocity is shown in Figure 2(a), and the fluid is rotating in the same direction as the magnetic field is rotating. The spin is shown in Figure 2(b): it is high in the center (0.202) but drops down to zero near the wall to fit the boundary condition. The half-vorticity is smaller; the color plot looks the same, but its peak value is only 0.0049. The excess spin gives a similar plot, but the peak value is obviously 0.197. The torque at a particular time is shown in Figure 3. The torque is constant over most of the domain, but there is a small region near the walls in which it increases; this region rotates with the magnetic field and is discussed below. The magnetic field (both *x*- and *y*-components) are varying in time,

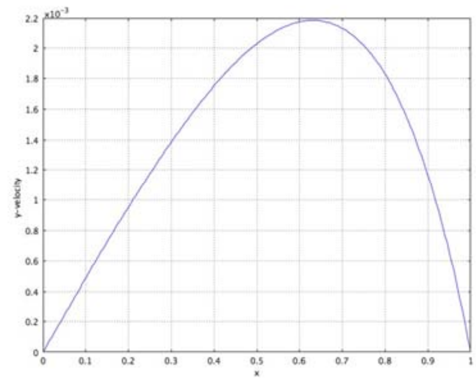


Figure 5. Vertical velocity along line y = 0.

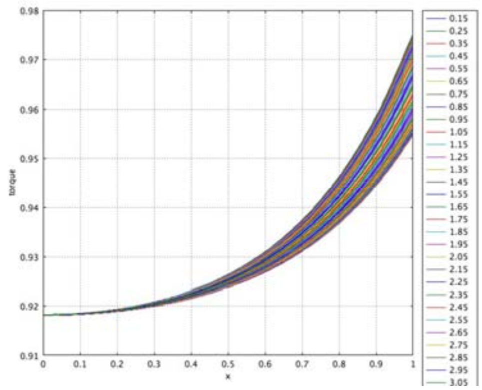


Figure 6. Torque along line y = 0.

and a typical plot of the x-component is shown in Figure 4. Even though the magnetic field is changing, the spin in most of the domain does not change in time. The spin is constant in the center and drops to zero at the boundaries. As the spin viscosity gets smaller, the boundary region becomes smaller, creating a spin boundary layer. The vertical velocity at the same point is shown in Figure 5. Note that the velocity peaks at a relative radius of about 0.63; the location at which this happens depends strongly on the spin viscosity and size of the vessel, and a measured velocity profile can be used to deduce the spin viscosity. The torque at the same point is shown in Figure 6; it varies with time near the wall, because the magnetic field is varying there.

There are two important insights given by these results. First, the torque is constant over most of the domain, but there is a thin layer near the wall that has a time-varying torque. The reason for this is illustrated in Figure 7; as the spin goes to zero near the wall, the term in the

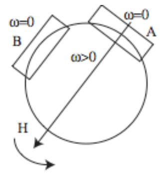


Figure 7. Diagram showing the difference in magnetization caused by the magnetic field and spin directions.

magnetization equation, $\boldsymbol{\omega} \times \mathbf{M}$, is different at the surface normal to the applied field compared with the same term at a boundary location tangent to the applied field. Furthermore, these regions rotate with the magnetic field. It is shown below that this thin layer becomes bigger as the applied magnetic field is increased and ultimately this leads to flow reversal. Thus, theories based on a constant torque everywhere have limited utility.

The second insight relates to the excess spin, $\boldsymbol{\omega}_{\text{excess}} = \boldsymbol{\omega} - \frac{1}{2}\nabla \times \mathbf{v}$. If the spin viscosity is zero, or the diffusion of spin is neglected, then this excess spin is proportional to the magnetic torque because of the spin equation.

$$4\zeta\boldsymbol{\omega}_{\text{excess}} = \mu_0 \mathbf{M} \mathrm{x} \mathbf{H}$$

The linear momentum equation is then

$$0 = -\nabla \mathbf{p} + \frac{1}{4\zeta}\nabla \mathrm{x}\left[\mu_0 \mathbf{M} \mathrm{x} \mathbf{H}\right] + \eta\nabla^2 \mathbf{v} + \mu_0 \mathbf{M} \bullet \nabla \mathbf{H}$$

Clearly, if the magnetic torque is constant in space, the curl of it is zero and the equation reduces to the standard Navier-Stokes equation. (The magnetic body force is never of importance in this flow; see below). Since the velocity is zero on the boundaries and there is no driving force, the solution will be quiescent everywhere. The results also show that the predominate effect of the magnetic field is to create the spin; the half-vorticity caused by the spin is, in this case, 40 times smaller. Thus, theories relating a rotation rate of the fluid to the applied magnetic torque are incomplete since they don't ascertain how much of the torque is transferred to the spin and how much is transferred to the half-vorticity. All the solutions show that the half-vorticity is much smaller than the non-observable spin.

Next consider solutions obtained with the other three boundary conditions. If one assumes zero couple stress on the boundary (first derivative of spin equals zero), it is not necessary for the spin to drop to zero near the wall, and the thin region in which the spin and magnetic field interact will not exist. Simulations for this case confirm that there is no flow. Thus, the boundary condition of zero couple stress is not appropriate, since experimental observations indicate there is flow. It is possible, of course, to have a boundary condition relating the spin and its derivatives (the couple stress), but that possibility is not examined here.

The third boundary condition makes the spin at the boundary equal half the vorticity, for zero excess spin. This boundary condition leads to reverse flow for all magnetic fields, no matter how small, and this is not observed experimentally. Thus, this boundary condition is not appropriate.

The fourth boundary condition uses the angular momentum equation with no spin diffusion at the boundary. Actually, to use any differential equation there one has to admit that the spin diffusion term cannot be included anywhere, since it requires a boundary condition (not a differential equation) at two points – the center and the solid wall. As soon as the spin viscosity is set to zero, and the spin is solved algebraically from the spin equation, there is no flow in the simulations. Since flow is observed experimentally, this boundary condition is rejected as well.

In conclusion, only the boundary condition of zero spin gives results that are in accord with experimental observations. There is still the possibility of the spin being proportional to the couple stress (a kind of slip for the spin vector), but that case is not considered here. What one can say is the results with zero spin probably give a larger effect than any such linear combination, since the zero couple stress condition gives no flow at all, and the results would probably be between the two special cases.

Solutions were also done in which the magnetic body force term was ignored, and the results changed only slightly, in the fourth significant figure. Those terms are left in all calculations, but they seem to be unimportant.

Next consider cases that expand the standard case to higher magnetic fields. The only parameter in which the magnetic field appears is the ε, in these non-dimensional equations. Thus, consider cases with increasing ε. The flows and torques are similar in shape, but differ in magnitude as the parameter is increased.

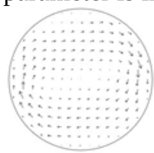


Figure 8. Arrow plot for large magnetic field, ε=155.

However, at a certain point flow reversal occurs. Figure 8 shows an arrow plot of the velocity for a larger magnetic field in which case flow reversal has completed and the flow is circular, but in the opposite direction.

Table 1 shows that as the magnetic field is increased (ε varies with K^2) the maximum spin increases, but not linearly. This occurs even though flow reversal has occurred; spin reversal does not occur.

Table 1: Maximum spin versus relative magnetic field

Relative K	ω(1/s)
1.0	16.7
3.2	129
10.0	387
13.2	431
17.8	466
31.6	504

Table 2 shows how the torque varies with magnetic field; it increases linearly with magnetic field. The angle between the magnetic field and magnetization is small but changes with magnetic field.

Table 2: Torque and angle (º) between H and M versus relative magnetic field

Relative K	Torque(Nm)*e6	Angle(º)
1.0	-1.3	0.0587
3.2	-12.0	0.0413
10.0	-74	0.0114
13.2	-104	0.0073
17.8	-145	0.0040
31.6	-252	0.0020

The rate of rotation of the fluid depends upon the magnetic field as well as other parameters. For K = 1, 3.2, 10, and 13.2 it takes values of 0.10, 0.75, 1.58, and 1.26 s^{-1}. The rate of rotation increases linearly until the magnetic field gets large enough for the flow to begin to reverse direction.

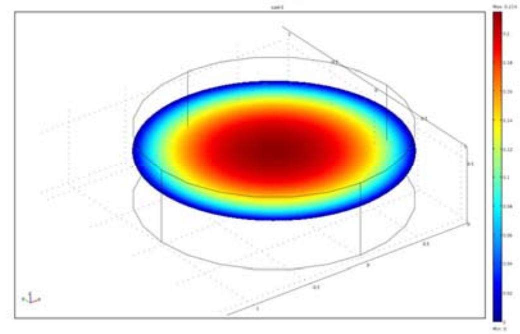


Figure 9. Spin in three-dimensional simulation at H/R = 0.4; base case.

3.2 Three-dimensional simulations

The three-dimensional problem was solved with the same equations, expanded to include the third dimension. The boundary condition on the free surface had to be adjusted to include the magnetic pressure terms. Since Comsol Multiphysics only allows a slip surface (and no pressure specification) or a pressure condition (and no restrictions on velocity), the simulations are only approximately the case desired. There is a net flow in and out of the top, free surface. The surface is also constrained to be flat. As discussed above, ignoring the convection terms in a two-dimensional circular flow is fine, but doing that in three dimensions is not valid. These limitations will be removed in future work, but the simulations reported here show interesting features.

For ε=0.155 flow is circular with little vertical flow. The spin at H=0.4R is shown in Figure 9, and this curve is similar to what is seen in two-dimensional flows. This pattern is repeated for any of the heights, and the magnitude of the spin is unchanged. Thus, the spin in this case is approximately the same from top to bottom. The vorticity shows a similar behavior (i.e. color) except that the peak vorticity increases from the bottom to the top. This is expected, since the solid boundary at the bottom constrains the flow. When the parameter ε is increased by a factor of 100, the spin reverses and is a minimum in the center. All of these results need to be confirmed by a model including the missing terms, but they are very intriguing.

4. Conclusions

Comsol Multiphysics was adapted to solve the equations governing the spin-up of a ferrofluid in a rotating magnetic field. The rate of rotation increases with magnetic field, as had been observed by Moskowitz and Rosensweig in 1967 (5). Reverse flow occurs, especially at high magnetic field, as had been observed by several authors (5, 7, 13, 16). The torque is always in the same direction, even though reverse flow occurs, as had been observed by Rosensweig's group (6, 14). Non-uniform effects are important, as surmised by Glazov (11,12) and Brown and Horswell (7). The velocity profiles are similar to those obtained experimentally by Chaves, *et al.* (16). Finally, the calculations presented here represent an improvement on the original theory due to Zaitsev and Shliomis (8) in that a constant torque is not assumed here, and the theory of Rosensweig's group (5,6,14) in that the magnetic torque mainly goes to the spin vector, not the fluid rotation as they had assumed. In addition, it is shown that the boundary condition must be either spin zero or spin proportional to the couple stress; three other options were shown to give results contrary to experiment.

This application of Comsol Multiphysics reveals its power, since previous solutions were limited in scope by the assumptions, which were necessary in order to obtain any solution at all. Much additional work remains, but the path is clear.

5. References

1. Raj, K., B. Moskowitz, and R. Casciari, Advances in Ferrofluid Technology, *J. Mag. Mag. Mat.*, **149**, 174-180 (1995).
2. Zahn, M., Magnetic fluid and nanoparticle applications to nanotechnology, *J. Nanoparticle Res.*, **3**, 73-78 (2001).
3. Sedgemore, F., On the Brink, *Nanomaterials News*, **3**, No. 7, p. 05 (2007).
4. Odenbach, S., Recent Progress in Magnetic Fluid Research, *J. Phys. Condens. Matter*, **16**, R1135-R1150 (2004).
5. Moskowitz, R. and R. E. Rosensweig, Nonmechanical Torque-driven Flow of a Ferromagnetic Fluid by an Electromagnetic Field, *Appl. Phys. Letters*, **11**, 301-3 (1967).
6. Rosensweig, R. E., J. Popplewell, and R. J. Johnston, Magnetic Fluid Motion in Rotating Field, *J. Mag. Mag. Mat.*, **85**, 171-180 (1990).
7. Brown, R., and T. S. Horsnell, The Wrong Way Round, *Elec. Rev.*, **184**, 235-6 (1969).
8. Zaitsev, V. M. and M. I.Shliomis, Entrainment of Ferromagnetic Suspension by a Rotating Field, *J. Appl. Mech. & Tech. Phys.*, **10**, 696-700 (1969).
9. Dahler, J. and L. E. Scriven, Angular Momentum of Continua, *Nature*, **192**, 36-37 (1961).
10. Jenkins, J. T., Some Simple Flows of a Para-Magnetic Fluid, *J. Physique*, **32**, 931-938 (1971).
11. Glazov, O. A., Motion of a Ferrosuspension in Rotating Magnetic Fields, *Magnetohydro-dynamics*, **11**, 140-145 (1975).
12. Glazov, O. A., Role of Higher Harmonics in Ferrosuspension Motion in a Rotating Magnetic Field, *Magnetohydrodynamics*, **11**, 434-438 (1975).
13. Kagan, I. Ya., V. G. Rykov, E. I. Yantovskii, Flow of a Dielectric Ferromagnetic Suspension in a Rotating Magnetic Field, *Magnetohydrodynamics*, **9**, 258-260 (1973).
14. Rosensweig, R. E. and R. J. Johnston, Aspects of Magnetic Fluid Flow with Nonequilibrium Magnetization, pp. 707-729 in Continuum Mechanics and Its Applications, G. A. C. graham, and S. K. Malik (ed.), Hemisphere Pub. Corp. (1989).
15. Rinaldi, C., J. H. Lee, A. D. Rosenthal, T. Franklin, M. Zahn, Ferrohdrodynamics in Time-varying Magnetic Fields, *Proc. IMECE 2002*, Nov. 17-22, 2002, New Orleans.
16. Chaves, A., C. Rinaldi, S. Elborai, He, X., Zahn, M., Bulk Flow in Ferrofluids in a Uniform Rotating Magnetic Field, *Phys. Rev. Letters*, **96** 194501-1-4 (2006).
17. Shliomis, M. I., Effective viscosity of magnetic suspensions, *Sov. Phys. JETP*, **34**, 1291-4 (1972).
18. Rosensweig, R. E., *Ferrohydrodynamics*, Dover, Mineola, NY, 1997.

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