Supplemental Appendix: A new method of projection-based inference in GMM with weakly identified nuisance parameters

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This supplemental appendix contains the full set of simulation results under the settings of Section 3 of Chaudhuri and Zivot (2008). Figures 1, 2 and 3 in the original paper are numbered as figures 1, 6 and 8 in this appendix. Table-3 in the original paper is numbered as Table-1 in this appendix. Tables 1– 4 give the rate of occurrence of different types of first step confidence regions obtained by inverting the AR test (following Dufour and Taamouti (2005). These tables also contain results for sample sizes as large as 1000 and 10000 to highlight the fact that an empty confidence region is not just a finite-sample phenomenon.

References

Chaudhuri, S. and Zivot, E. (2008). A new method of projection-based inference in GMM with weakly identified nuisance parameters. University of North Carolina, Chapel Hill.

Dufour, J. M. and Taamouti, M. (2005). Projection-Based Statistical Inference in Linear Structural Models with Possibly Weak Instruments. *Econometrica*, 73: 1351–1365.

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1 Tables and Figures

n	k	Σ	WI-Case I		WI-Case II		WI-Case III		WI-Case IV	
			$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$
10^2	2	Σ_1	0	0.09	0.28	1.27	0.01	0.18	0.23	1.28
10^2	2	Σ_2	0.27	1.43	0.35	1.48	0.27	1.51	0.30	1.58
10^2	2	Σ_3	0	0.12	0.27	1.22	0.01	0.11	0.21	1.11
10^2	4	Σ_1	0.01	0.46	0.47	2.40	0.03	0.45	0.56	2.70
10^2	4	Σ_2	0.56	2.75	0.61	2.72	0.62	2.91	0.66	3.04
10^2	4	Σ_3	0.01	0.27	0.44	2.26	0.03	0.39	0.53	2.58
10^2	20	Σ_1	0.78	3.44	1.58	5.91	0.87	3.76	1.88	6.05
10^2	20	Σ_2	1.84	6.35	1.63	6.05	1.98	6.22	1.92	6.30
10^2	20	Σ_3	0.42	2.80	1.55	5.80	0.57	3.03	1.86	5.89
10^3	2	Σ_1	0.03	0.26	0.18	1.27	0.02	0.26	0.18	1.22
10^3	2	Σ_2	0.19	1.33	0.27	1.40	0.31	1.63	0.22	1.41
10^3	2	Σ_3	0.01	0.21	0.15	1.19	0.02	0.26	0.16	1.13
10^3	4	Σ_1	0.01	0.50	0.42	2.33	0.03	0.56	0.22	2.10
10^3	4	Σ_2	0.35	2.46	0.49	2.44	0.44	2.16	0.32	2.25
10^3	4	Σ_3	0.03	0.41	0.41	2.24	0.01	0.34	0.25	2.00
10^3	20	Σ_1	0.27	2.45	0.65	3.69	0.32	2.24	0.80	3.77
10^3	20	Σ_2	0.72	3.82	0.68	3.92	0.76	3.58	0.86	3.91
10^3	20	Σ_3	0.19	1.94	0.65	3.63	0.21	1.69	0.80	3.78
10^4	2	Σ_1	0	0.17	0.19	1.27	0.01	0.18	0.15	1.31
10^4	2	Σ_2	0.27	1.49	0.22	1.50	0.29	1.45	0.23	1.52
10^4	2	Σ_3	0	0.14	0.18	1.28	0	0.09	0.14	1.28
10^4	4	Σ_1	0.04	0.35	0.32	2.08	0.02	0.50	0.22	2.01
10^4	4	Σ_2	0.41	2.15	0.36	2.41	0.28	2.29	0.25	2.18
10^4	4	Σ_3	0.03	0.31	0.31	2.05	0.03	0.46	0.22	1.88
10^4	20	Σ_1	0.27	2.11	0.59	3.41	0.22	2.11	0.58	3.74
10^4	20	Σ_2	0.69	3.54	0.65	3.56	0.65	3.40	0.63	3.83
10^4	20	Σ_3	0.17	1.68	0.59	3.36	0.19	1.71	0.56	3.69

Table 1: % of times $\mathcal{C}_2(100-\zeta, \theta_{01})$ is Empty!

n	k	Σ	WI-Case I		WI-Case II		WI-Case III		WI-Case IV	
			$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$
10^2	2	Σ_1	82.81	61.50	2.31	0.52	82.15	60.38	2.52	0.52
10^2	2	Σ_2	0	0	0	0	0	0	0	0
10^2	2	Σ_3	86.03	66.04	8.05	2.16	85.88	65.95	8.57	2.09
10^2	4	Σ_1	74.35	50.20	0.06	0.01	74.71	49.51	0.02	0
10^2	4	Σ_2	0	0	0	0	0	0	0	0
10^2	4	Σ_3	80.58	58.46	0.65	0.07	80.21	56.97	0.59	0.09
10^2	20	Σ_1	20.00	7.84	0	0	19.30	7.81	0	0
10^2	20	Σ_2	0	0	0	0	0	0	0	0
10^2	20	Σ_3	34.34	15.82	0	0	33.71	15.42	0	0
10^3	2	Σ_1	80.69	57.62	0.71	0.11	80.83	57.14	0.57	0.10
10^3	2	Σ_2	0	0	0	0	0	0	0	0
10^3	2	Σ_3	85.07	63.92	3.56	0.73	85.50	64.13	3.04	0.60
10^3	4	Σ_1	73.78	48.18	0.02	0	73.38	48.28	0	0
10^3	4	Σ_2	0	0	0	0	0	0	0	0
10^3	4	Σ_3	80.46	56.55	0.26	0.01	80.04	56.52	0.20	0.01
10^3	20	Σ_1	16.64	5.25	0	0	16.64	5.49	0	0
10^3	20	Σ_2	0	0	0	0	0	0	0	0
10^3	20	Σ_3	33.16	13.53	0	0	33.51	14.24	0	0
10^4	2	Σ_1	81.25	58.77	0.86	0.10	81.44	59.17	0.92	0.10
10^4	2	Σ_2	0	0	0	0	0	0	0	0
10^4	2	Σ_3	85.58	64.86	4.09	0.83	85.51	64.95	3.96	0.99
10^4	4	Σ_1	71.17	45.33	0	0	70.35	44.92	0	0
10^4	4	Σ_2	0	0	0	0	0	0	0	0
10^4	4	Σ_3	79.05	54.42	0.06	0	78.11	53.80	0.08	0.01
10^4	20	Σ_1	17.20	5.35	0	0	16.87	5.39	0	0
10^4	20	Σ_2	0	0	0	0	0	0	0	0
10^4	20	Σ_3	34.46	14.54	0	0	34.49	14.42	0	0

Table 2: % of times $\mathcal{C}_2(100 - \zeta, \theta_{01}) = (-\infty, +\infty)$

n	k	Σ	WI-Case I		WI-Case II		WI-Case III		WI-Case IV	
			$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$
10^2	2	Σ_1	9.66	18.95	9.20	3.16	9.79	19.28	8.79	2.90
10^2	2	Σ_2	92.22	80.11	11.85	3.77	92.43	79.56	11.11	3.73
10^2	2	Σ_3	6.40	13.63	3.45	1.34	6.06	13.72	3.09	1.51
10^2	4	Σ_1	13.87	22.81	1.01	0.11	13.63	23.41	0.94	0.15
10^2	4	Σ_2	88.72	73.07	1.24	0.22	88.26	72.55	1.14	0.22
10^2	4	Σ_3	7.79	14.55	0.34	0.08	8.37	15.32	0.37	0.08
10^2	20	Σ_1	25.63	18.02	0	0	25.52	17.63	0	0
10^2	20	Σ_2	45.04	24.81	0	0	44.98	25.16	0	0
10^2	20	Σ_3	11.00	9.93	0	0	11.10	9.56	0	0
10^3	2	Σ_1	10.37	19.23	4.2	1.06	10.12	19.32	4.23	1.25
10^3	2	Σ_2	91.13	76.70	5.08	1.26	91.13	77.23	4.93	1.26
10^3	2	Σ_3	6.01	12.43	1.42	0.57	5.68	13.26	1.61	0.48
10^3	4	Σ_1	13.49	22.28	0.30	0.07	13.73	21.37	0.23	0.06
10^3	4	Σ_2	86.81	70.00	0.34	0.03	87.51	70.74	0.27	0.03
10^3	4	Σ_3	7.49	13.70	0.16	0.01	7.11	13.52	0.13	0.02
10^3	20	Σ_1	24.79	15.36	0	0	25.33	15.65	0	0
10^3	20	Σ_2	41.60	20.69	0	0	42.14	20.65	0	0
10^3	20	Σ_3	8.55	7.27	0	0	8.34	6.75	0	0
10^4	2	Σ_1	9.87	18.83	5.09	1.46	10.19	19.14	4.93	1.28
10^4	2	Σ_2	91.47	77.49	5.89	1.53	91.75	77.36	5.87	1.58
10^4	2	Σ_3	6.05	12.94	1.86	0.81	6.09	12.91	1.61	0.57
10^4	4	Σ_1	15.13	23.07	0.16	0.02	14.40	22.12	0.14	0.01
10^4	4	Σ_2	86.70	68.83	0.11	0.02	85.27	67.04	0.12	0
10^4	4	Σ_3	7.10	13.50	0.03	0	7.09	13.56	0	0
10^4	20	Σ_1	25.98	16.17	0	0	26.31	16.12	0	0
10^4	20	Σ_2	43.34	21.40	0	0	43.86	21.71	0	0
10^4	20	Σ_3	8.59	7.27	0	0	7.96	7.20	0	0

Table 3: % of times $\mathcal{C}_2(100-\zeta, \theta_{01}) = (-\infty, a] \cup [b, +\infty)$ for some $-\infty < a < b < +\infty$

n	k	Σ	WI-Case I		WI-Case II		WI-Case III		WI-Case IV	
			$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$	$\zeta = 1\%$	$\zeta = 5\%$
10^2	2	Σ_1	7.53	19.91	88.21	95.05	8.05	20.16	88.46	95.30
10^2	2	Σ_2	7.51	18.46	87.80	94.75	7.30	18.93	88.59	94.69
10^2	2	Σ_3	7.57	20.21	88.23	95.28	8.05	20.22	88.13	95.29
10^2	4	Σ_1	11.77	26.53	98.46	97.48	11.59	26.63	98.48	97.09
10^2	4	Σ_2	10.72	24.18	98.15	97.06	11.12	24.54	98.20	96.74
10^2	4	Σ_3	11.62	26.72	98.57	97.59	11.39	27.32	98.51	97.25
10^2	20	Σ_1	53.59	70.70	98.42	94.09	54.31	70.80	98.12	93.95
10^2	20	Σ_2	53.12	68.84	98.37	93.95	53.04	68.62	98.08	93.70
10^2	20	Σ_3	54.24	71.45	98.45	94.20	54.62	71.99	98.14	94.11
10^3	2	Σ_1	8.91	22.89	94.91	97.56	9.03	23.28	95.02	97.43
10^3	2	Σ_2	8.68	21.97	94.65	97.34	8.56	21.14	94.85	97.33
10^3	2	Σ_3	8.91	23.44	94.87	97.51	8.80	22.35	95.19	97.79
10^3	4	Σ_1	12.72	29.04	99.26	97.60	12.86	29.79	99.55	97.84
10^3	4	Σ_2	12.84	27.54	99.17	97.53	12.05	27.10	99.41	97.72
10^3	4	Σ_3	12.02	29.34	99.17	97.74	12.84	29.62	99.42	97.97
10^3	20	Σ_1	58.30	76.94	99.35	96.31	57.71	76.62	99.20	96.23
10^3	20	Σ_2	57.68	75.49	99.32	96.08	57.10	75.77	99.14	96.09
10^3	20	Σ_3	58.10	77.26	99.35	96.37	57.94	77.34	99.20	96.22
10^4	2	Σ_1	8.88	22.23	93.86	97.17	8.36	21.51	94.00	97.31
10^4	2	Σ_2	8.26	21.02	93.89	96.97	7.96	21.19	93.90	96.90
10^4	2	Σ_3	8.37	22.06	93.87	97.07	8.40	22.05	94.29	97.16
10^4	4	Σ_1	13.66	31.25	99.52	97.90	15.23	32.46	99.64	97.98
10^4	4	Σ_2	12.89	29.02	99.53	97.57	14.45	30.67	99.63	97.82
10^4	4	Σ_3	13.82	31.77	99.60	97.95	14.77	32.18	99.70	98.11
10^4	20	Σ_1	56.55	76.37	99.41	96.59	56.60	76.38	99.42	96.26
10^4	20	Σ_2	55.97	75.06	99.35	96.44	55.49	74.89	99.37	96.17
10^4	20	Σ_3	56.78	76.51	99.41	96.64	57.36	76.67	99.44	96.31

Table 4: % of times $\mathcal{C}_2(100-\zeta, \theta_{01}) = [a, b]$ for some $-\infty < a < b < +\infty$

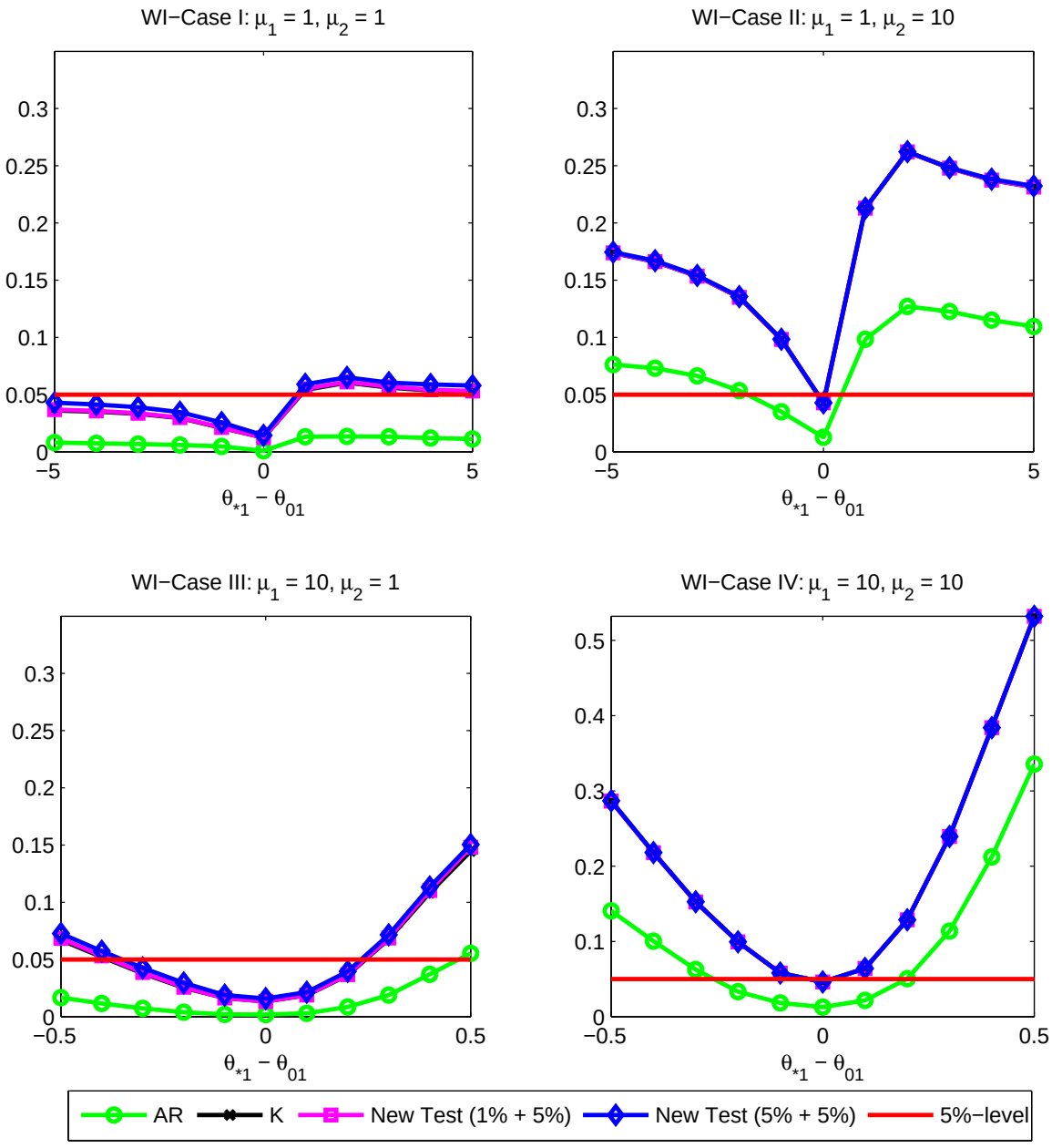

Figure 1: Sample Size = 100, Number of Instruments = 2, $\rho_{u1} = 0.5$, $\rho_{u2} = 0.5$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

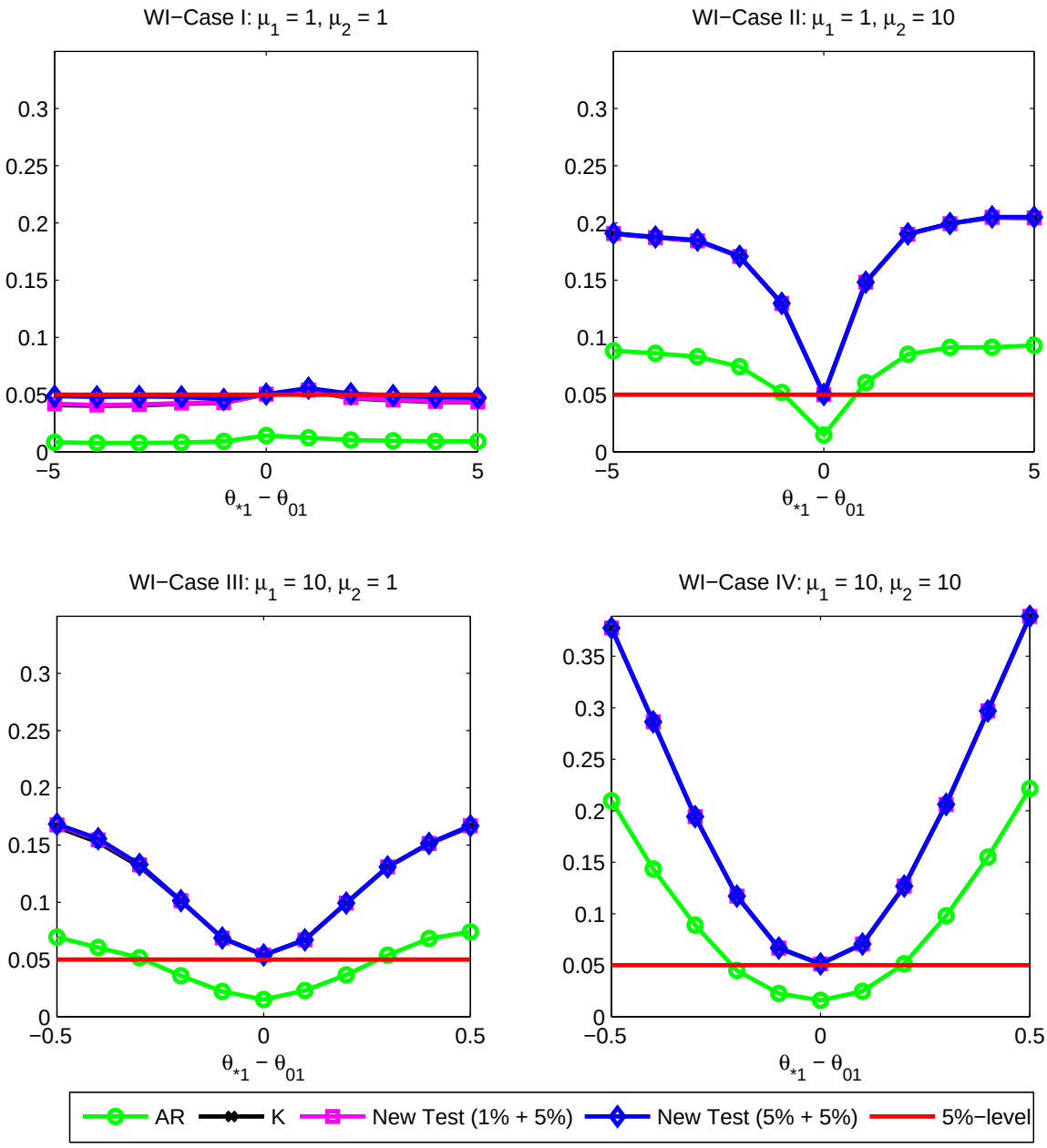

Figure 2: Sample Size = 100, Number of Instruments = 2, $\rho_{u1} = 0.1$, $\rho_{u2} = 0.99$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

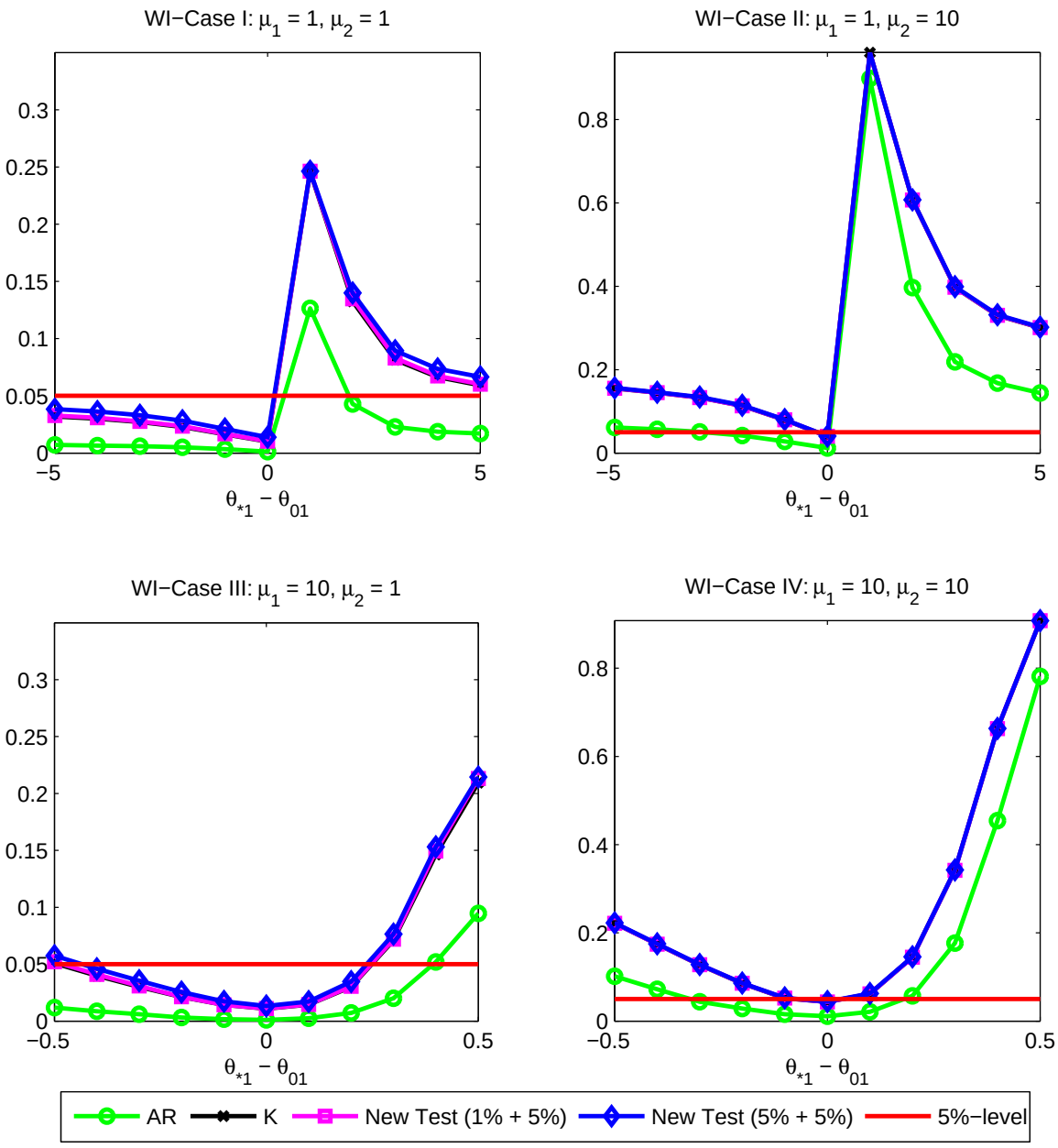

Figure 3: Sample Size = 100, Number of Instruments = 2, $\rho_{u1} = 0.99$, $\rho_{u2} = 0.1$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

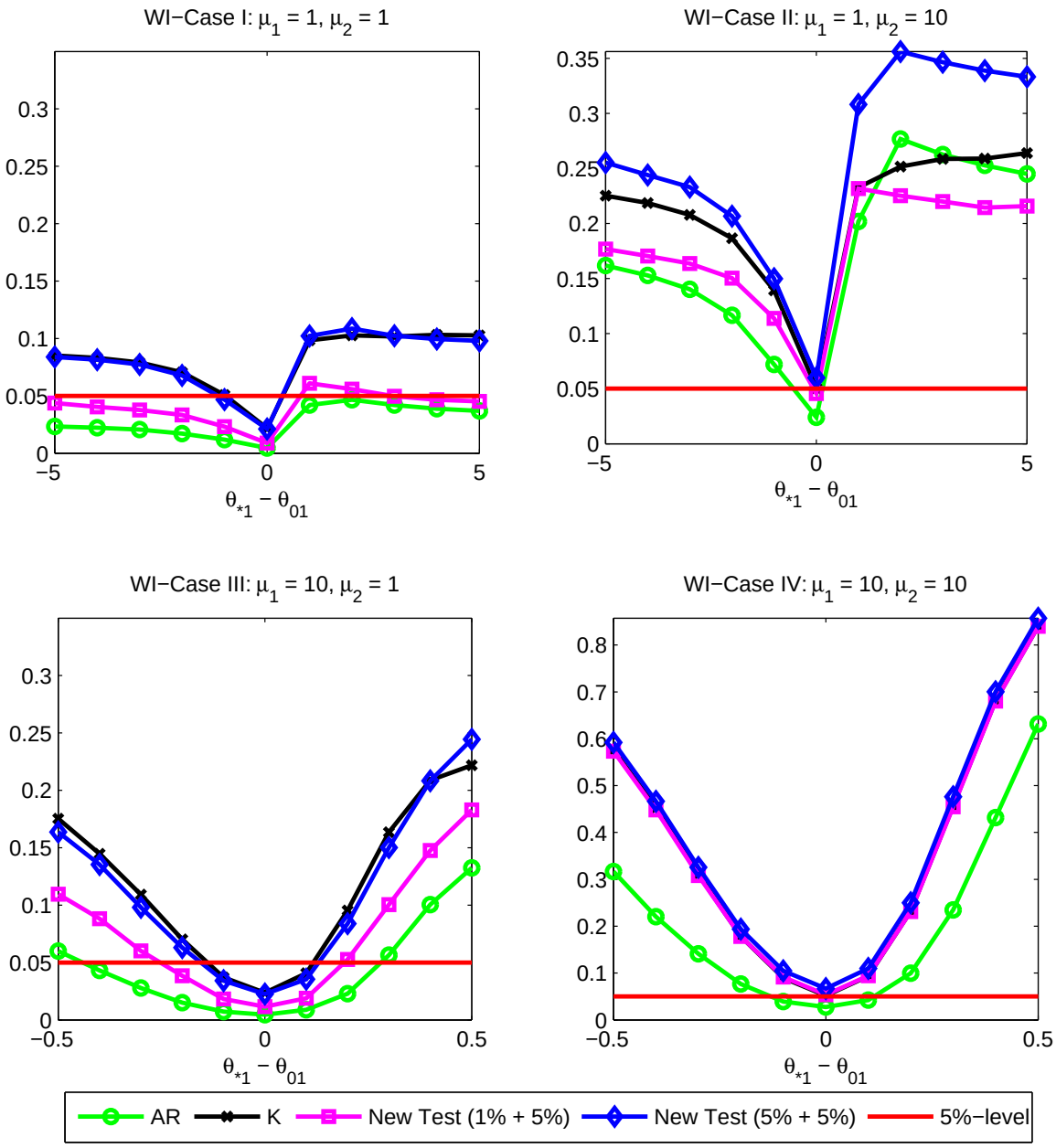

Figure 4: Sample Size = 100, Number of Instruments = 4, $\rho_{u1} = 0.5$, $\rho_{u2} = 0.5$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

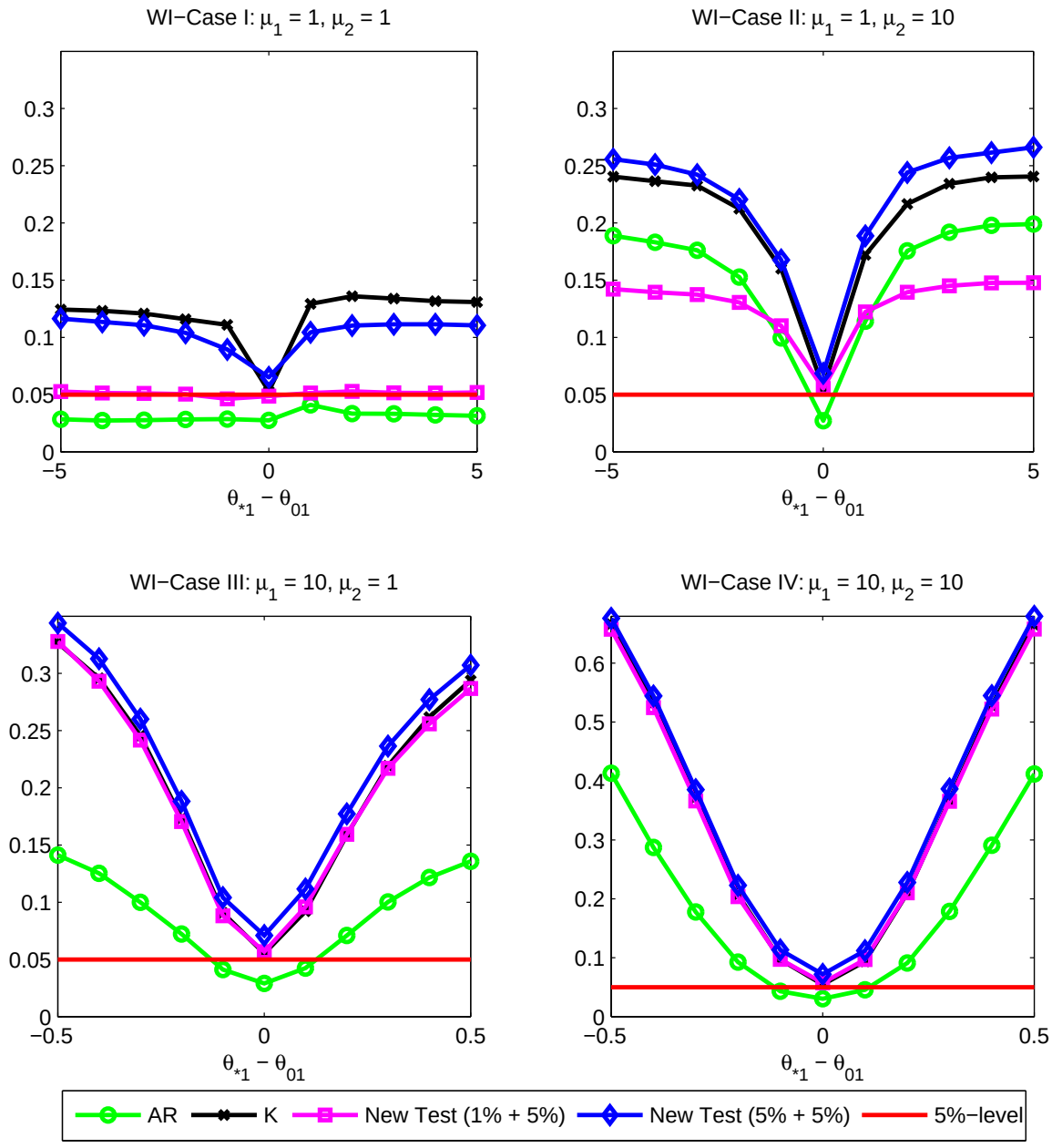

Figure 5: Sample Size = 100, Number of Instruments = 4, $\rho_{u1} = 0.1$, $\rho_{u2} = 0.99$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

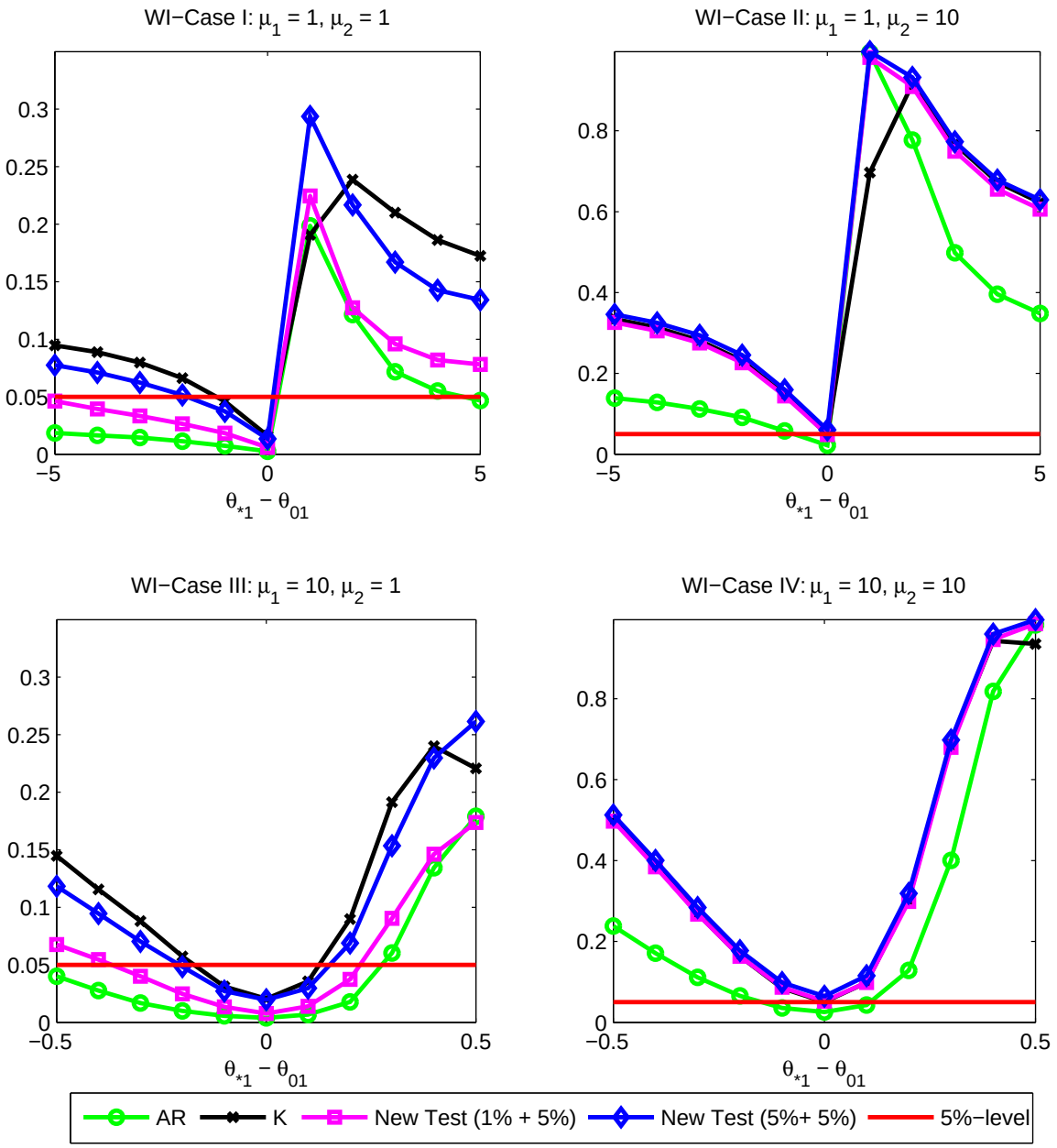

Figure 6: Sample Size = 100, Number of Instruments = 4, $\rho_{u1} = 0.99$, $\rho_{u2} = 0.1$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

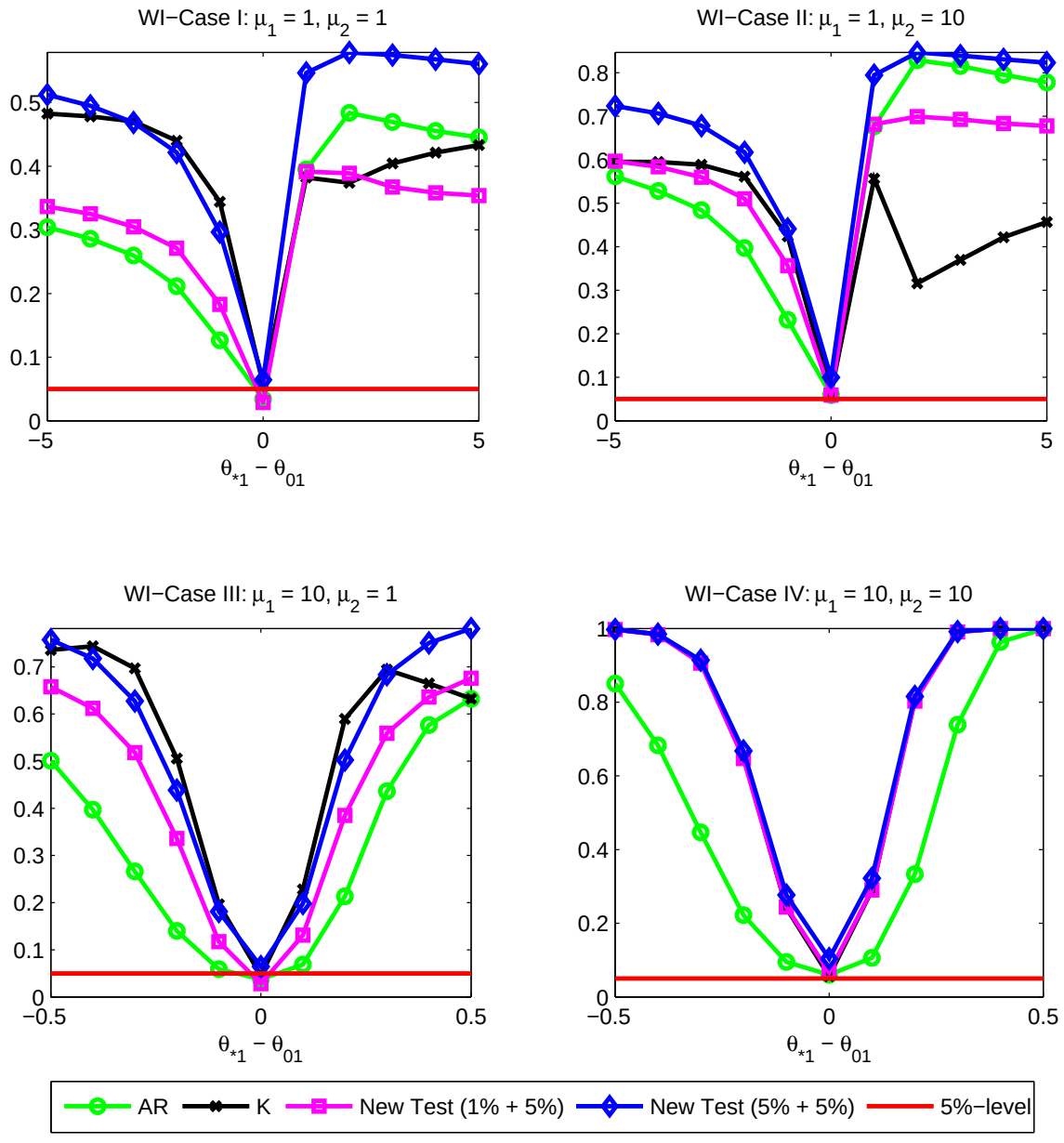

Figure 7: Sample Size = 100, Number of Instruments = 20, $\rho_{u1} = 0.5$, $\rho_{u2} = 0.5$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

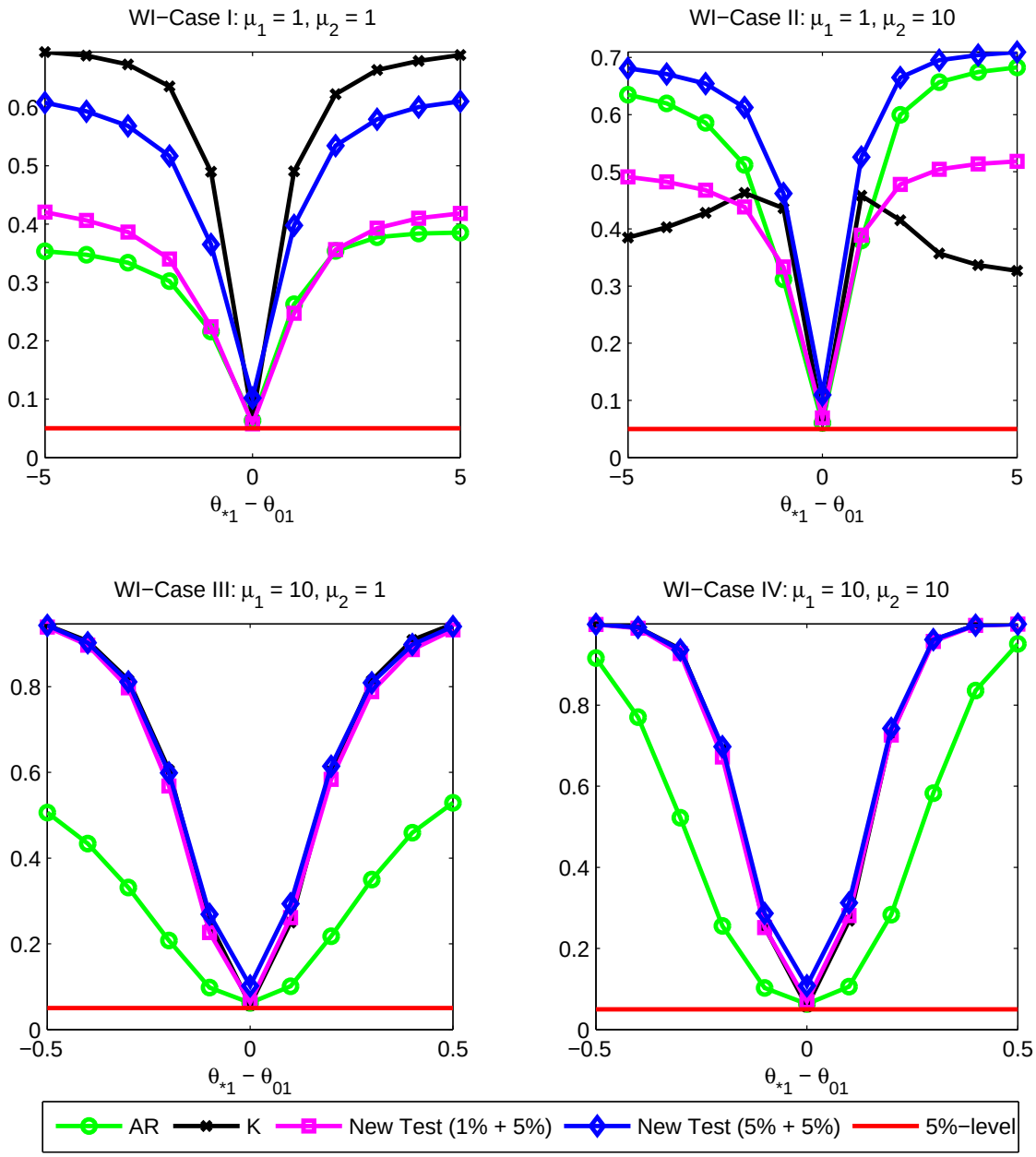

Figure 8: Sample Size = 100, Number of Instruments = 20, $\rho_{u1} = 0.1$, $\rho_{u2} = 0.99$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.

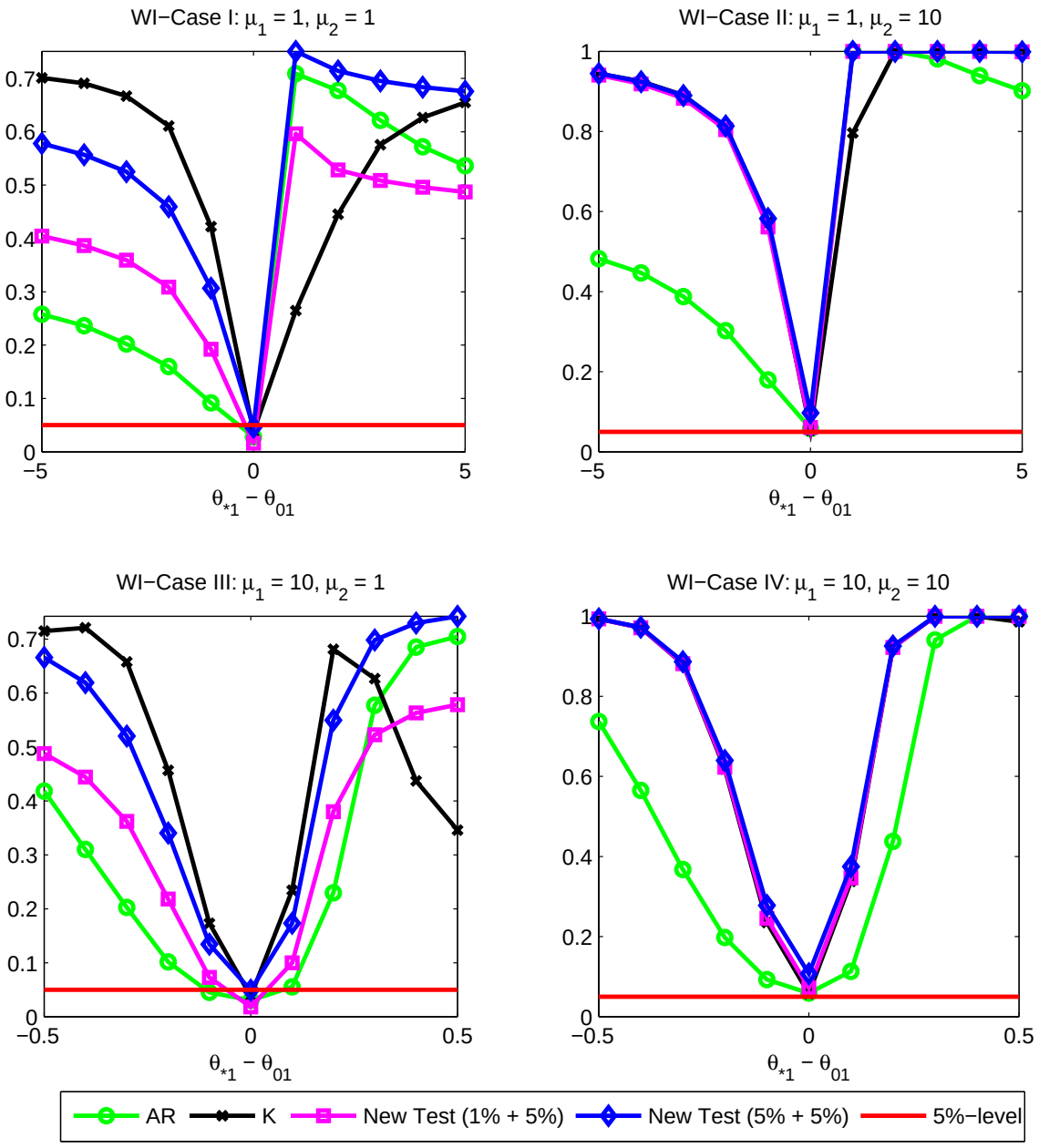

Figure 9: Sample Size = 100, Number of Instruments = 20, $\rho_{u1} = 0.99$, $\rho_{u2} = 0.1$ and $\rho_{12} = 0$. Weak instrument characterized by $\mu = 1$ and strong instrument by $\mu = 10$.