

# Long Memory versus Structural Breaks in Modeling and Forecasting Realized Volatility

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## Abstract

We explore the possibility of structural breaks in the daily realized volatility of the Deutschemark/Dollar, Yen/Dollar and Yen/Deutschemark spot exchange rates with observed long-memory behavior. We find that structural breaks in the mean can partly explain the persistence of realized volatility. We propose a VAR-RV-Break model that provides superior predictive ability when the timing of future breaks is known. With unknown break dates and sizes, we find that a VAR-RV- $I(d)$  long memory model provides a robust forecasting method even when the true financial volatility series are generated by structural breaks.

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## 1. Introduction

Conditional volatility and correlation modeling has been one of the most important areas of research in empirical finance and time series econometrics over the past two decades.

Although daily financial asset returns are approximately unpredictable, return volatility is time-varying but highly predictable with persistent dynamics and the dynamics of volatility is well modeled as a long memory process<sup>4</sup>. An inherent problem for measuring, modeling and forecasting conditional volatility is that the volatility is unobservable or latent, which implies modeling must be indirect. Typically, measurements of volatility are from parametric methods, such as GARCH models or stochastic volatility models for the underlying returns. These parametric volatility models, however, depend on specific distributional assumptions and are subject to misspecification problems.

Given the availability of intraday ultra-high-frequency price data on assets, Andersen, Bollerslev, Diebold, and Labys (2003), henceforth ABDL, and Barndorff-Nielsen and Shephard (2001, 2002, 2004) introduced a consistent nonparametric estimate of the price volatility that has transpired over a given discrete interval, called *realized volatility*. They computed daily Deutschemark/Dollar, Yen/Dollar, and Deutschemark/Yen spot exchange rates realized volatilities simply by summing high-frequency finely sampled intraday squared and cross-products returns. By sampling intraday returns sufficiently frequently, the model-free realized volatility can be made arbitrarily close to underlying integrated volatility, the integral of instantaneous volatility over the interval of interest, which is a natural volatility measure.

ABDL found logarithmic realized volatility could be modeled and accurately forecast using simple parametric fractionally integrated ARFIMA models. Their low-dimensional multivariate realized volatility model (VAR-RV- $I(d)$  model, explained in Section 4) provided

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<sup>4</sup> The findings suggest that volatility persistence is highly significant in daily data but will weaken as the data frequency decreases.

superior out-of-sample forecasts for both low-frequency and high-frequency movements in the realized volatilities compared to GARCH and related approaches. Nevertheless, many studies have pointed out that observed long memory may not only be generated by linearly fractional integrated process but also by: (1) cross-sectional aggregation of short memory stationary series (Granger and Ding, 1996); (2) mixture of numerous heterogeneous short-run information arrivals (Andersen and Bollerslev, 1997); (3) non-linear models, such as structural breaks (changes) or regime switches (Granger and Hyung, 2004; Choi and Zivot, 2007; Diebold and Inoue, 2001). In particular, it has been conjectured that persistence of asset return volatility may be overstated due to the presence of structural changes.

In this paper, we focus on the possibility of structural breaks and trends in the daily realized exchange rate volatility series studied by ABDL. First, we test for long memory and estimate long memory models for the realized volatility series. We find strong evidence for long memory in exchange rate realized volatility. Second, we test for and estimate a multiple mean break model based on Bai and Perron (1998, 2003)'s method. We find several common structural breaks within these three series. Using Beran and Ocker (2001)'s semiparametric flexible trend long memory model, we do not find evidence of a smooth flexible trend in volatilities. Third, we examine the evidence for long memory in the break adjusted data. We find a partial reduction of persistence in realized volatility after the removal of breaks. The evidence suggests that part of the observed long memory may be accounted for by the presence of structural breaks in the exchange rate volatility series.

Sun and Phillips (2003) pointed out that it is very difficult to separate low-frequency dynamics and high-frequency fluctuations, in particular when short-run fluctuations have high variance. Therefore, we propose an alternative short memory model, which adapts more quickly to the current volatility, with detected break information in realized volatility. Our VAR-RV-Break model provides competitive forecasts compared with most of the forecasting models

considered by ABDL if the timing of future break dates and their sizes are known. If we have little knowledge about break dates and sizes, the VAR-RV- $I(d)$  model is the best forecasting model even when the true volatility series are generated by structural breaks. We support our findings with an extensive Monte Carlo experiment.

The rest of the paper is organized as follows. Section 2 presents results from the estimation of various long memory models for realized volatility. Section 3 presents results from the estimation of a structural breaks model and re-examines the evidence for long memory in the break-adjusted series. Section 4 reports the evaluation of out-of-sample forecasting performance of various models. Section 5 concludes.

## 2. Data and Long Memory Model

### 2.1. Data

We use the same data as ABDL, which are spot exchange rates for the U.S. dollar, the Deutschemark, and the Japanese yen from December 1, 1986 through June 30, 1999<sup>5</sup>. Their data set consists of 3,045-days of intra-day 30-minute log returns on the DM/\$ and Yen/\$ exchange rates. The intraday return is denoted  $r_t^{(h)}$ , where  $t = h, 2h, 3h, \dots, 47h, 1, 49h, \dots, 3045$ , and  $h = 1/48 = 0.0208$ . Realized daily volatilities of DM/\$ and Yen/\$ are computed as the diagonal

elements of  $\sum_{i=1}^{1/h} r_{t-1+ih}^{(h)} \cdot r_{t-1+ih}^{(h)'} .$  By absence of triangular arbitrage, the Yen/DM returns can be

calculated directly from the difference between the DM/\$ and Yen/\$ returns. Realized volatilities,

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<sup>5</sup> The raw data include all interbank DM/\$ and Yen/\$ bid/ask quotes shown on the Reuters FX screen provided by Olsen & Associates. These three currencies were the most actively traded in the foreign exchange market during the sample period. Thirty-minute prices are constructed from the linearly interpolated logarithmic average of the bid and ask quotes for the two ticks immediately before and after the thirty-minute time stamps over the global 24-hour trading day. Thirty-minute returns are obtained from the first difference of the logarithmic prices. Returns from Friday 21:00 Greenwich Mean Time (GMT) to Sunday 21:00 GMT and certain holiday periods to avoid weekend and holiday effects are excluded.

also called realized standard deviations, are calculated from the square root of the realized variances. Following ABDL, we consider models for the natural logarithm of realized volatility.

## 2.2. Long Memory Models

Before conducting further modeling and forecasting, it is crucial to determine whether the time series of log realized volatilities is stationary or not. The dichotomy between  $I(0)$  and  $I(1)$ , however, may be far too narrow. A long memory model that allows fractional orders of integration,  $I(d)$ , provides more flexibility. For an  $I(0)$  process, shocks decay at an exponential rate; for an  $I(1)$  process, shocks have permanent effect; for an  $I(d)$  process, shocks dissipate at a slow hyperbolic rate. Long memory behavior in volatility has been well established, see for example, Ding, Granger, and Engle (1993), Baillie, Bollerslev and Mikkelsen (1996), and Andersen and Bollerslev (1997).

Granger and Joyeux (1980) and Hosking (1981) showed that a long memory process for  $y_t$  can be modeled as a fractionally integrated,  $I(d)$ , process

$$(1 - L)^d (y_t - \mu) = \varepsilon_t, \quad (1)$$

where  $L$  denotes the lag operator,  $d$  is fractional difference parameter,  $\mu$  is the unconditional mean of  $y_t$ , and  $\varepsilon_t$  is stationary with zero mean and finite variance. The process  $y_t$  is stationary provided  $-\frac{1}{2} < d < \frac{1}{2}$ , and exhibits non-stationary long memory if  $\frac{1}{2} \leq d < 1$ . Two commonly used semi-parametric estimators for  $d$  are the log-periodogram estimator of Geweke and Porter-Hudak (1983) and the local Whittle estimator of Künsh (1987) and Robinson (1995b).

A flexible parametric process called the ARFIMA  $(p, d, q)$  model incorporates both long-term and short-term memory,

$$\phi(L)(1 - L)^d (y_t - \mu) = \theta(L)\varepsilon_t, \quad (2)$$

where  $\phi(L)$  and  $\theta(L)$  are autoregressive and moving average polynomials, respectively, with roots that lie outside the unit circle and  $\varepsilon_t$  is Gaussian white noise. Sowell (1992) and Beran (1995) discussed maximum likelihood estimation of (2).

To allow for a data-driven distinction of long memory, short memory, stochastic trends, and smooth deterministic trends without any prior knowledge, Beran and Ocker (2001) considered the semiparametric fractional autoregressive (SEMIFAR) model

$$\phi(L)(1-L)^\delta ((1-L)^m y_t - g(i_t)) = \varepsilon_t, \quad (3)$$

where  $\delta$  is the long memory parameter, and  $g(i_t)$  is a smooth trend function on  $[0,1]$  with  $i_t = t/T$ . In (3),  $y_t$  must be differenced to achieve stationarity by the parameter  $d = \delta + m$ . The value of  $m$  determines whether the trend should be estimated from the original data (when  $m = 0$ ) or the first difference (when  $m = 1$ ). When  $\delta > 0$ ,  $y_t$  is long memory. When  $\delta < 0$ ,  $y_t$  is antipersistent. When  $\delta = 0$ ,  $y_t$  has short memory. For more detailed discussions of long memory testing and estimation methods, we refer the reader to Baillie (1996) and Robinson (1995a).

### 2.3. Long Memory Estimation

As in ABDL, we consider modeling log realized volatility. According to the slow decay of autocorrelations in Figure 1, it is evident that log realized volatility of the exchange rate series has long memory dynamics. We estimate the long memory parameter using the local Whittle, ARFIMA( $p, d, q$ ) and SEMIFAR models mentioned in Section 2.2. The estimates of  $d$  for logarithmic realized volatility are reported in Table 1. Whether we use nonparametric, semi-parametric, or parametric methods, all of the estimates of  $d$  are in the range between 0.37 and 0.56, which confirms the existence of long memory in the logarithmic realized volatility.

[Insert Figure 1 here]

[Insert Table 1 here]

### 3. Structural Break Model

#### 3.1. Multiple Structural Break Model

It is well known that structural change and unit roots are easily confused (see Perron 1989; Zivot and Andrews 1992). Recently the confusion between long memory and structural change has been getting more and more attention. Granger and Ding (1996), Granger and Hyung (2004), and Choi and Zivot (2007) suggested that the observed long memory property in volatility can be explained by the presence of structural breaks. To investigate this conjecture for realized volatility, we use the pure multiple mean break method proposed by Bai and Perron (1998, 2003), henceforth BP. The  $m$ -break model ( $m + 1$  regimes) is defined as

$$y_t = c_j + u_t, \quad t = T_{j-1} + 1, T_{j-1} + 2, \dots, T_j, \quad (4)$$

where  $j = 1, 2, \dots, m + 1$ ,  $y_t$  is the logarithmic realized volatility, and  $c_j$  is the mean of the logarithmic realized volatility. The break points  $(T_1, T_2, \dots, T_m)$  are treated as unknown. The error term  $u_t$  may be serial correlated and heteroskedastic. Estimation is based on the least-squares principle as described in BP. We use the tests for structural change proposed in BP. Let  $\sup F_T(l)$  denote the  $F$  statistic for the null of no structural breaks versus an alternative hypothesis containing an arbitrary number of breaks, and let  $M$  denote the maximum number of breaks allowed. We set  $M = 5$  with trimming value = 0.15. Define the double maximum statistic  $UD_{\max} = \max_{1 \leq l \leq M} \sup F_T(l)$ , and the weighted double max statistic

$WD_{\max} = \max_{1 \leq l \leq M} w_l \sup F_T(l)$ , where the weights  $w_l$  are such that the marginal p-values are equal across values of  $l$ . The null hypothesis of both tests is no structural breaks against the alternative of an unknown number of breaks given some specific upper bound  $M$ . The sequential  $\sup F_T(l+1|l)$  tests the null of  $l$  breaks versus the alternative  $l+1$  breaks. To determine the number of breaks, we first use  $UD_{\max}$  and  $WD_{\max}$  to determine if at least one break occurred. If there is evidence for structural change, we select the number of structural breaks using  $\sup F_T(l+1|l)$ . To allow for a penalty factor for the increased dimension of a model, the above procedure may be complemented by selecting the number of breaks by minimizing a Bayesian Information Criterion (BIC) or a modified Schwarz Criterion (LWZ).

### 3.2. Multiple Structural Break Estimation

Table 2 displays values of all the tests used to determine the number of breaks for the logarithmic realized volatility series. The  $UD_{\max}$ ,  $WD_{\max}$  and  $\sup F_T(l)$  tests point to the presence of multiple breaks for all series. For DM/\$, the  $\sup F_T(l+1|l)$  is significant at 1% level when  $l = 4$ , which suggests 5 breaks. BIC suggests 5 breaks as well while LWZ suggests 2 breaks. Therefore, we choose 5 breaks for DM/\$. For Yen/\$,  $\sup F_T(l+1|l)$  is significant when  $l = 3$  but not significant when  $l = 4$ , which suggest 4 breaks. We follow BIC and choose 5 breaks for Yen/\$. For Yen/DM,  $\sup F_T(l+1|l)$  suggest 4 breaks as well as BIC. Hence 4 breaks are chosen for Yen/DM.

[Insert Table 2 here]

In Table 2 we also report the estimates of the break dates with their respective 90% confidence intervals. The break dates estimated for DM/\$ and Yen/\$ are very similar, which suggests common break dates for the process: May 1989, March – May 1991, March 1993, June – August 1995, and May – July 1997. The estimates of the mean parameters ( $\hat{c}_j$ ) for regimes ( $m + 1$ ) are also provided on the bottom of Table 2. Figure 2 shows the logarithmic realized volatility with the estimated  $\hat{c}$  values superimposed. We note that the mean shifts coincide with historical financial or currency crisis. For example, the highest volatility regime of DM/\$ exchange rate occurred between two breaks: March 1991 and March 1993 and the second highest volatility regime of Yen/DM exchange rate occurred between two breaks: June 1992 and May 1994. Both regimes can be attributed to the Exchange Rate Mechanism (ERM) crisis of 1992-93 in Europe. The Asian financial crisis occurred in 1997 and appears to cause the breaks in July 1997 for DM/\$, May 1997 for Yen/\$, and May 1997 for Yen/DM. Since the breaks in May 1997, Yen/\$ and Yen/DM have turned to their highest volatility regimes among the whole sample.

[Insert Figure 2 here]

### 3.3. Long Memory Estimation After Adjusting for Structural Breaks

Table 3 shows the long memory parameter estimates of the three series after adjustment for the estimated structural breaks<sup>6</sup>. The parameter  $d$  is estimated using the residual series  $y_t - \hat{c}_j$ . All estimates of  $d$  are lower after using break-adjusted series, especially in Yen/DM series. The long memory parameter of Yen/DM decreases from 0.475 to 0.283, from 0.5673 to 0.3839, and from 0.4685 to 0.3833 for Whittle, ARFIMA, and SEMIFAR methods, respectively. Figure 3

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<sup>6</sup> In contrast to our two-step approach, Hsu (2005) proposed a one-step approach which estimates the long memory parameter directly from the data with unknown mean changes. Using such a procedure, he found a smaller long memory parameter for G7 inflation rates.

displays the autocorrelation function for the adjusted volatility series. Compared to Figure 1 for the autocorrelation before adjustment for breaks, it is evident that the persistence of volatility has been reduced after removing the estimated breaks. The presence of structure breaks, however, can not totally explain the persistence of exchange rate realized volatility. Other reasons for the observed long memory might be (1) aggregation of intraday squared return series; (2) mixture of numerous heterogeneous short-run information arrivals (Andersen and Bollerslev, 1997).

[Insert Table 3 here]

[Insert Figure 3 here]

The pattern of mean shifts in Figure 2, suggests that there might be a smooth upward trend in the volatility series, especially in the Yen/DM series which has the most persistence, which is an alternative to abrupt mean breaks. To investigate this alternative, we estimate the SEMIFA model with flexible trend in (3). The results for the estimated trend are shown in Figure 4. We see that the trend is not statistically significant. Our results show that the realized volatility series is better characterized by abrupt mean shifts than by a smooth flexible trend.

[Insert Figure 4 here]

Baillie and Kapetanios (2007) proposed a test for nonlinearity of unknown form in addition to the long memory component in a time series process. Using the same ABDL data, they rejected the null of linearity for Yen/\$ and Yen/DM. Their nonlinearity forms are essentially approximations to smooth transition autoregressive (STAR) models. Such STAR models could generate series similar to structural breaks, especially with the larger speed of transition parameter in some exponential STAR (ESTAR) or logistic STAR (LSTAR) models. It is difficult

to compare the in-sample fit of the BP model and STAR model for the realized volatility in the presence of long memory. Therefore the choice between the models is likely to be settled in terms of economic plausibility and research interests. Intuitively, if the exchange rate volatility has more recurrent changes, then the STAR model is more appropriate. If changes of exchange rate volatility are more likely to be nonrecurrent, then the mean-shift model is favored.

### 3.4. Monte Carlo Simulation of Long Memory Process

We discussed previously that structural change is easily confused with long memory. Granger and Hyung (2004) pointed out that there exists another perplexity: a long memory model without breaks may cause breaks to be detected spuriously by standard estimation methods. To illustrate this phenomenon, we generated six long memory series with  $d = 0.1, 0.2, 0.3, 0.35, 0.4, 0.45$ , respectively, with mean: -0.5, standard deviation: 0.4, and sample size: 3,045. These series are similar to our sample logarithmic realized volatility. Table 4 shows results for the structural break tests of BP for the different data-generating processes (DGPs). The results suggest a positive relationship between the number of breaks and the value of  $d$  as found in Granger and Hyung (2004). This reveals a possibility that a long memory/fractionally integrated process itself contains some portion of a permanent shock, which often appears as a break in some situations<sup>7</sup>. The important implication from this Monte Carlo evidence is that the long memory DGP provides a good parsimonious alternative of in-sample fit for the true structural-break DGP when we have little knowledge for the past break dates and size<sup>8</sup>.

[Insert Table 5 here]

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<sup>7</sup> Currently there is no formal test available for multiple structural changes in the  $I(d)$  process with unknown number of breaks. The nonlinear test developed by Baillie and Kapetanios (2007) contribute the literature at this point.

<sup>8</sup> This property, which is trivial here, will become much more important when we discuss the long memory and structural breaks for out-of-sample forecasting in Section 5.

## 4. Forecast Evaluation and Simulation

### 4.1. $I(d)$ versus Breaks Model

Many models have been provided for forecasting asset return volatility and the success of a volatility model lies in its out-of-sample forecasting power. For example, ABDL proposed the following trivariate VAR-RV- $I(d)$  (fractionally integrated Gaussian vector autoregressive-realized volatility) model,

$$\Phi(L)(1-L)^d(Y_t - \mu) = \varepsilon_t, \quad (5)$$

where  $Y_t$  is  $(3 \times 1)$  vector of logarithmic realized exchange rate volatilities,  $\Phi(L)$  is a lag polynomial,  $\mu$  is the unconditional mean, and  $\varepsilon_t$  is a vector white noise process. They fixed the value of  $d$  for each series at 0.401, which is close to our long memory estimates in Table 1. They set the order of  $\Phi(L)$  to five to capture lagged effects up to a week. They compared volatility forecasts from several popular models, and they found that their VAR-RV- $I(d)$  model produced superior out-of-sample one-step-ahead and ten-step-ahead forecasts.

Granger and Joyeux (1980), however, pointed out that a long memory model would not necessarily produce clearly superior short-run forecasts, which is of interest in financial forecasting. As an alternative to (5), we consider a short memory model for break-adjusted series to produce short-run forecasts. Our alternative model, denoted VAR-RV-Break, is

$$\Phi(L)(Y_t^* - \mu) = \varepsilon_t, \quad (6)$$

where  $Y_t^*$  is the vector of break-adjusted logarithmic realized exchange rate volatilities and the order of  $\Phi(L)$  is equal to five following ABDL. Forecasts are obtained by estimating rolling models. We estimate initially over the first 2449 observations, December 2, 1986 to December 1, 1996, and using the in-sample parameter estimates one-day-ahead forecasts are made for the next

day, day 2450<sup>9</sup>. The process is then rolled forward 1 day, deleting the first observation and adding on the 2450th observation, the model is re-estimated and the second forecast is made for 2451. The rolling method is repeated until observation 3045, the end of the out-of-sample forecast period. We get 596 one-step-ahead predictions in the out-of-sample period, which is from December 2, 1996 to June 30, 1999.

## **4.2. Known versus Unknown Breaks**

We consider two situations. First, we assume that the future (out-of-sample) break dates and sizes are known. In real-time forecasts by financial practitioners, it is sometimes possible to identify breaks and make adjustments using human judgment. For example, when the Bank of Japan announced that it would abandon its zero-interest rate policy, they knew to some extent that structural breaks in Japanese bonds market's volatility would happen. In this case, when the breaks happened, they quickly adjusted the forecasts based on the given out-of-sample breaks dates and means. Second, we assume that we have little knowledge about out-of-sample breaks and sizes. For example, it was not known that a structural break occurred in 1984 for the US real output volatility until recently. As a result, many macroeconomists did not do any real-time adjustments even though they made systematic forecasts errors. They could have used the BP method to detect structural breaks by rolling over the out-of-sample period, but they would still need to wait a sufficient amount of time to estimate it<sup>10</sup>.

## **4.3. Forecast Evaluation and Comparison**

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<sup>9</sup> We choose this in-sample period to compare our result to those in ABDL.

<sup>10</sup> Andrews (1993) suggests a restricted sample interval [0.15, 0.85] instead of the full interval for trimming to avoid the much reduced power of test statistics.

Figure 5 displays the DM/\$, Yen/\$, and DM/Yen realized volatility along with the corresponding one-day-ahead VAR-RV-Break forecasts under the assumption that the future breaks are known. It appears that our forecasts capture movement of the realized volatilities well. To determine which model provides more information about the future value, we use the Mincer and Zarnowitz (1969) encompassing regression<sup>11</sup>

$$vol_{t+1,i} = \beta_0 + \beta_1 vol_{t+1|t,i}^{VAR-RV-Break} + \beta_2 vol_{t+1|t,i}^{Model} + \varepsilon_t, \quad (7)$$

where  $vol_{t+1|t}^{VAR-RV-Break}$  denotes predictions by our benchmark VAR-RV-Break model, and  $vol_{t+1|t}^{Model}$  denotes predictions from some other candidate model. The alternative models are the ones used in ABDL<sup>12</sup>.

[Insert Figure 5 here]

In addition to (7), we also evaluate out-of-sample forecasts using the relative mean squared error (MSE),

$$\frac{\sum (vol_{t+1} - vol_{t+1|t}^{Model})^2}{\sum (vol_{t+1} - vol_{t+1|t}^{Break})^2} \quad (8)$$

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<sup>11</sup> This is a regression-based method where the prediction is unbiased only if  $\beta_0=0$  and  $\beta_1=1$ . When there are more than one forecasting models, additional forecasts are added to the right-hand-side to check for incremental explanatory power. The first forecast is said to subsume information in other forecasts if these additional forecasts do not significantly increase the  $R^2$ .

<sup>12</sup> The VAR-RV- $I(d)$  model is the main model. The VAR-ABS model is a fractionally integrated vector autoregressive using daily absolute returns instead of realized volatility. The daily AR(1) GARCH (1,1) models for DM/\$, Yen/\$, and DM/Yen use AR coefficients 0.986, 0.968, and 0.99, respectively. The RiskMetrics model is an exponentially weighted moving average of the cross products of daily returns with smoothing factor  $\lambda=0.94$ . The fractionally integrated exponential GARCH (FIGARCH) (1, $d$ ,0) by Bollerslev and Mikkelsen (1996) is a variant of FIGARCH model by Baillie, Bollerslev, and Mikkelsen (1996). The last one is the high-frequency FIGARCH model using the “deseasonalized” and “filtered” 30-minutes returns.

where the denominator is the benchmark model mean squared forecast error and the numerator is the candidate method's mean squared forecast error. If the relative MSE is less than one, the candidate model forecast is determined to have performed better than the benchmark. The forecast evaluation statistics are presented in Table 5. In general, our VAR-RV-Break model out-of-sample forecasts perform as well as ABDL's VAR-RV- $I(d)$  model, and outperform most of the other models.

[Insert Table 5 here]

#### 4.4. Forecast Evaluation Results

First, the regression  $R^2$  from the VAR-RV-Break model is similar to that from VAR-RV- $I(d)$  model and is higher than most of the other models. Second, we cannot reject the hypothesis that  $\beta_0 = 0$  and  $\beta_1 = 1$  in the VAR-RV-Break model using  $t$  tests while we reject the hypothesis that  $\beta_0 = 0$  and/or  $\beta_2 = 1$  for all the other models except the VAR-RV- $I(d)$  model. Third, in the encompassing regression that includes both the break model and an alternative forecast, the estimates for  $\beta_1$  are closer to unity and the estimates for  $\beta_2$  are closer to zero. Fourth, including an alternative forecast method has little contribution to increasing  $R^2$ . Finally, most of the relative MSEs are bigger than one

The results in Table 5 show the superior forecasting ability of the VAR-RV-Break model when the future break dates and sizes are known in the out-of-sample period. This result is consistent with Hyung, Poon and Granger (2006), who investigated out-of-sample forecasting of S&P 500 return volatility. Pesaran, Pettenuzzo, and Timmermann (2006) provided a Bayesian estimation and prediction procedure allowing breaks to occur over the forecast horizon. They

found that their method, which has similar inference with known break information (prior), leads to better out-of-sample forecasts than alternative models.

As shown in Table 6, without knowledge of out-of-sample breaks, unsurprisingly, the prediction ability of the VAR-RV-Break deteriorates. The degree of deterioration depends on the numbers and sizes of the out-of-sample breaks. For the DM/\$ series, the VAR-RV-Break model still outperforms all models except the VAR-RV- $I(d)$  model because the out-of-sample break is not large as shown in Figure 2. But for the Yen/\$ and Yen/DM, which show larger breaks, the VAR-RV-Break model's prediction ability becomes inferior to the other models (except VAR-ABS). In this case, the VAR-RV- $I(d)$  is the best forecasting model.

[Insert Table 6 here]

#### **4.5. Forecast Simulation for Break and Long Memory Models**

To check the robustness of our comparison of the VAR-RV-Break and the VAR-RV- $(d)$  out-of-sample forecasts, we perform the following simulation experiment. To mimic the daily logarithmic exchange rate realized volatility data, we generate 3000 observations from an AR(1) process with autoregressive coefficient equal to 0.41 and unconditional variance equal to 0.16. We partition the series into six periods by four ad hoc breaks as shown in Figure 6.A. Each period's range and mean are as follows: Period 1[1:700; 0.5], Period 2[701:1500; -1.3], Period 3[1501:2000; -0.5], Period 4[2001:2300; -0.5], Period 5[2301:2700; -1.2], and Period 6[2701:3000, 0.7]. The 2000 observations in Period 1 to Period 3 are treated as the in-sample period and the 1000 observations in Period 4 to Period 6 are treated as the out-of-sample period.

[Insert Figure 6 here]

For the AR-Break model, we perform one-step-ahead forecasts based on the true DGP. Again, we consider two situations. First, when the out-of-sample breaks are known, we adjust the mean for the forecast evaluation. If the break information is not known, we don't do any adjustment. For the AR- $I(d)$  model, we use in-sample data (Figure 6.A P1 to P3) to initially estimate the long memory parameter and the AR(1) coefficient based on the SEMIFAR model (3)<sup>13</sup> We get  $d = 0.2697$  and  $AR(1) = 0.2137$ . We then compute rolling one-step-ahead forecasts using the updated SEMIFAR model. Figure 6.B shows the results for Period 4 in which an out-of-sample break has not occurred. Whether breaks are known or not, the break model performs slightly better than the  $I(d)$  model. Surprisingly, in Period 5 and 6 after breaks occurred, the  $I(d)$  model still accurately predicts while the break model deteriorates substantially when the breaks are unknown.

Table 7 contains the root mean squared error (MSE),

$$\left[ \sum (y_{t+1} - y_{t+1|t}^{Model})^2 / N \right]^{1/2}, \quad (9)$$

and relative mean squared error,

$$\frac{\sum (y_{t+1} - y_{t+1|t}^{I(d)})^2}{\sum (y_{t+1} - y_{t+1|t}^{Break})^2}, \quad (10)$$

to evaluate  $N$  out-of-sample forecasts from the known break model, unknown break model, and  $I(d)$  model. In Period 4, the relative MSEs for the known break and unknown break models are equal to 1.02 implying that break models perform slightly better than the  $I(d)$  model. In period 5, 6, and the whole sample, the relative MSEs for the known break model are 1.117, 1.589, and 1.222, respectively. Unsurprisingly, the known break model forecasts better for the true break process than the  $I(d)$  model. Compared with the results in Table 6 for the exchange rate realized

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<sup>13</sup> The number of lagged values to use for prediction is 50.  $k=100000$  is chosen to determine the Fourier frequencies to use in evaluating the theoretical spectrum of an ARFIMA model.

volatility data, the relative MSE values are 0.98, 1.02, and 1.04, respectively, which means the known break model's forecasting is not as good as that for the simulated data. This is reasonable because the true series in Table 5 is not a pure AR-Break process. For the unknown break model, the relative MSEs in Period 5, 6 and the whole ample are 0.301, 0.155, and 0.257, respectively. The  $I(d)$  model performs far better than the unknown break model.

[Insert Table 7 here]

The lower panel of Table 7 shows the forecasts evaluation statistics for ten-step-ahead forecasts. We see a similar pattern in these forecast evaluation statistics. The known break model is better than the  $I(d)$  model, and the  $I(d)$  model is better than the unknown break model. It is worth noting that when we compare the root MSE results in Table 7, we see that the  $I(d)$  model's 10-step-ahead forecasts worsen more than break model's forecasts. For the  $I(d)$  model, its whole out-of-sample root MSE increases from 0.453 to 0.634 when it forecasts from one-step-ahead to ten-step-ahead. In contrast to the known break model, its whole out-of-sample root MSE only increases from 0.410 to 0.441 when it forecasts from one-step-ahead to ten-step-ahead. Although the long memory is expected to have better multi-step-ahead forecasts compared to other models, the findings here do not support this expectation.

In summary, when the DGP is a pure mean break series without any long memory, we can get very good out-of-sample forecast performance using a simple AR- $I(d)$  model. This result shows that a long memory/fractional integrated model may still be the best forecasting model when the true financial volatility series are generated by structural breaks and we have little knowledge about the timing and sizes of the breaks.

## 5. Conclusions

Modeling realized volatility constructed from intraday high-frequency data provides improved out-of-sample forecasts compared with traditional volatility models. ABDL's VAR-RV- $I(d)$  model beats popular GARCH-type models for forecasting daily realized volatilities for the DM/\$, Yen/\$ and Yen/ DM spot exchange rates. The main reason is that the former model, which exploits the intraday volatility information, provides an accurate and fast-adapting estimate of current volatility while the latter model, depending on slowly decaying past squared returns, adapts only gradually to current volatility shocks. In light of this property, we show that superior volatility forecasts could also be produced using a short memory model for realized volatility which incorporates mean shift information on realized volatility instead of long memory. Our short-memory-Break model generally performs the best provided the timing of future break dates and their sizes are known. With little knowledge about out-of-sample break dates and their sizes, we show that a parsimonious long memory model is a robust forecasting model even when the true financial volatility series are generated by structural breaks.

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## References

- Andrews, D. W. K. (1993). "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica*, 61:4, pp. 821-856.
- Andersen, T. G., and T. Bollerslev. (1997). "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns," *Journal of Finance*, 52:3, pp. 975-1005.
- Andersen, T. G., T. Bollerslev, F.X. Diebold, and P. Labys. (2003). "Modeling and Forecasting Realized Volatility," *Econometrica*, 71, pp. 529-626.
- Baillie, R. T. (1996). "Long Memory Processes and Fractional Integration in Econometrics," *Journal of Econometrics*, 73, pp. 5-59.
- Baillie, R.T., T. Bollerslev, and H.O. Mikkelsen. (1996). "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 74, pp. 3-30.
- Baillie, R.T., and G. Kapetanios. (2007). "Testing for Neglected Nonlinearity in Long-Memory Models," *Journal of Business and Economic Statistics*, 25, pp. 447-61.
- Bai, J., and P. Perron. (1998). "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica*, 66, pp. 47-78.
- Bai, J., and P. Perron. (2003). "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics*, 18, pp. 1-22.
- Barndorff-Nielsen, O. E., and N. Shephard. (2001). "Non-Gaussian Ornstein-Uhlenbeck-Based Models and Some of Their Uses in Financial Economics," *Journal of the Royal Statistical Society, Series B*, 63, pp. 167-241.
- Barndorff-Nielsen, O. E., and N. Shephard. (2002). "Econometric Analysis of Realised Volatility and its Use in Estimating Stochastic Volatility Models," *Journal of the Royal Statistical Society, Series B*, 64, part 2, pp. 253-280.

- Barndorff-Nielsen, O. E., and N. Shephard. (2004). "Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics," *Econometrica*, 72:3, pp. 885-925.
- Beran, J. (1995). "Maximum Likelihood Estimation of the Differencing Parameter for Invertible Short and Long Memory Autoregressive Integrated Moving Average Models," *Journal of Royal Statistical Society Series B*, 57:4, pp.659-72.
- Beran, J., and D. Ocker. (2001). "Volatility of Stock-Market Indexes—An Analysis Base on SEMIFAR Models," *Journal of Business and Economic Statistics*, 19:1, pp 103-116.
- Bollerslev, T., and H. O. Mikkelsen. (1996). "Modeling and Pricing Long Memory in Stock Market Volatility," *Journal of Econometrics*, 73, pp. 151-84.
- Choi, K., and E. Zivot. (2007). "Long Memory and Structural Changes in the Forward Discount: An Empirical Investigation," *Journal of International Money and Finance*, 26, pp. 342-363.
- Diebold, F. X., and A. Inoue. (2001). "Long Memory and Regime Switching," *Journal of Econometrics*, 105:1, pp. 131-59.
- Ding, Z., C. W. J. Granger, and R. F. Engle. (1993). "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance*, 1, pp. 83-106.
- Geweke, J., and S. Porter-Hudak (1983). "The Estimation and Application of Long Memory Models," *Journal of Time Series Analysis*, 4, pp. 221-238.
- Granger, C.W.J., and R. Joyeux. (1980). "An Introduction to Long Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, 1, pp 15-39.
- Granger, C.W.J., and Z. Ding. (1996). "Varieties of Long Memory Models," *Journal of Econometrics*, 73, pp. 61-77.

- Granger, C.W.J., and N. Hyung. (2004). "Occasional Structural Breaks and Long Memory with an Application to the S&P 500 Absolute Stock Returns," *Journal of Empirical Finance*, 11, pp. 399-421.
- Henry, M. (2001). "Robust Automatic Bandwidth for Long Memory," *Journal of Time Series Analysis*, 22:3, pp 293-316.
- Hosking, J. R. M. (1981). "Fractional Differencing," *Biometrika*, 68, pp. 165-76.
- Hyung, N., S. H. Poon, and C. W. J. Granger. (2006). "A Source of Long Memory in Volatility," Working paper.
- Hsu, C.C. (2005). "Long Memory or Structural Changes: An Empirical Examination on Inflation Rates," *Economics Letters*, 88, pp. 289-94.
- Künsch, H. (1987). "Statistical Aspects of Self-Similar Processes," *Proceedings of the First World Congress of the Bernoulli Society, Vol, I*, VNU Science Press, Utrecht, pp. 67-74.
- Mincer, J., and V. Zarnowitz. (1969). "The Evaluation of Economic Forecasts," in *Economic Forecasts and Expectations*, edited by W. F. Sharpe, and C. M. Cootner. Englewood Cliffs, New Jersey: Prentice-Hall.
- Perron, P. (1989). "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," *Econometrica*, 57, pp. 1361-401.
- Pesaran, M. H., D. Pettenuzzo, and A. Timmermann. (2006). "Forecasting Time Series Subject to Multiple Structural Breaks," *Review of Economic Studies*, 73:4, pp. 1057-84.
- Robinson, P. M. (1995a). "Gaussian Semiparametric Estimation of Long Range Dependence," *Annals of Statistics*, 23, pp. 1630-1661.
- Robinson, P. M. (1995b). "Log-Periodogram Regression of Time Series with Long-Range Dependence," *Annals of Statistics*, 23, pp. 1048-72.
- Sowell, F. (1992). "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models," *Journal of Econometrics*, 53, pp. 165-188.

Sun, Y., and P. Phillips. (2003). "Nonlinear Log-periodogram Regression for Perturbed Fractional Processes," *Journal of Econometrics*, 115, pp. 355-89.

Zivot, E., and D. W. K. Andrews. (1992). "Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Journal of Business and Economic Statistics*, 10:3, pp. 251-70.

**Table 1. Estimation for Long and Short Memory of Log Realized Volatility**

		d	AR(1)	MA(1)
Local	DM/\$	0.395 (0.0269)	N/A	N/A
	Yen/\$	0.446 (0.0269)	N/A	N/A
Whittle	Yen/\$	0.475 (0.0269)	N/A	N/A
	Yen/DM	0.475 (0.0269)	N/A	N/A
ARFIMA	DM/\$	0.3817 (0.0142)	0	0
	Yen/\$	0.4107 (0.0142)	0	0
	Yen/DM	0.5673 (0.0316)	0.28 (0.05)	0.49 (0.02)
SEMIFAR	DM/\$	0.3778 (0.0142)	0	N/A
	Yen/\$	0.4096 (0.0142)	0	N/A
	Yen/DM	0.4685 (0.0212)	-0.1018 (0.03)	N/A

1. The numbers in the parentheses indicate standard errors.
2. Local Whittle's method is based on Gaussian maximum likelihood estimation with the choice of bandwidth equal to 0.5, suggested by Henry (2001).
3. ARFIMA model is based on Beran (1995).  $\phi(L)(1-L)^\delta [(1-L)^m y_t - \mu] = \theta(L)\varepsilon_t$  where  $-0.5 < d < 0.5$ . The integer  $m$  is the number of times that  $y$  must be differenced to achieve stationarity, and the long memory parameter is given by  $d = \delta + m$ . The method uses BIC to choose the short memory parameters  $p$  and  $q$ . When  $m = 0$ ,  $\mu$  is the expectation of  $y_t$ ; when  $m = 1$ ,  $\mu$  is the slope of linear trend component in  $y_t$ .
4. SEMIFAR (Semiparametric Fractional Autoregressive) model is based on Beran and Ocker (2001).  $\phi(L)(1-L)^\delta [(1-L)^m y_t - g(i_t)] = \varepsilon_t$ . By using a nonparametric kernel estimate of  $g(i_t)$  instead of constant term  $\mu$ . The method uses BIC to choose the short memory parameter  $p$ .

**Table 2. Multiple Structural Changes Test Results**

\ Series			
Statistics	DM/\$	Yen/\$	Yen/DM
Tests			
sup $F_T(1)$	52.58	133.36	305.13
sup $F_T(2)$	55.64	76.86	246.36
sup $F_T(3)$	42.37	54.06	188.79
sup $F_T(4)$	36.92	50.74	149.35
sup $F_T(5)$	33.94	42.29	110.46
UDmax	55.64**	133.36**	305.13**
WDmax	84.95**	133.36**	323.48**
sup $F_T(2 1)$	36.23	17.88**	131.56
sup $F_T(3 2)$	12.83	7.19	71.95
sup $F_T(4 3)$	16.29	26.37	21.31
sup $F_T(5 4)$	16.29	8.57	0
Numbers of Changes Selected			
BIC	<b>5</b>	<b>5</b>	<b>4</b>
LWZ	2	2	2
Sequential	4	2	4
Multiple Structural Changes Dates Estimation			
$\hat{T}_1$	1989.5.11 [89.2.14-89.11.1]	1989.5.11 [88.12.21-89.9.6]	1989.11.21 [89.11.7-89.12.15]
$\hat{T}_2$	1991.3.18 [90.9.24-91.8.20]	1991.5.16 [91.3.20-91.10.2]	1992.6.10 [92.3.4-92.10.20]
$\hat{T}_3$	1993.3.5 [92.12.7-93.5.14]	1993.3.30 [92.11.25-93.5.21]	1994.5.4 [94.3.24-94.6.1]
$\hat{T}_4$	1995.8.24 [95.6.15-95.11.29]	1995.6.15 [94.8.24-96.6.19]	1997.5.8 [97.4.17-97.5.28]
$\hat{T}_5$	1997.7.10 [97.4.9-89.10.27]	1997.5.7 [97.4.3-97.6.17]	
Estimations of Mean for Each Regime			
$\hat{c}_1$	-0.662 (0.015)	-0.686 (0.017)	-0.924 (0.012)
$\hat{c}_2$	-0.472 (0.017)	-0.457 (0.018)	-0.5 (0.013)
$\hat{c}_3$	-0.334 (0.016)	-0.687 (0.017)	-0.342 (0.015)
$\hat{c}_4$	-0.553 (0.015)	-0.446 (0.017)	-0.659 (0.012)
$\hat{c}_5$	-0.770 (0.017)	-0.580 (0.018)	-0.205 (0.014)
$\hat{c}_6$	-0.585 (0.017)	-0.205 (0.018)	

1. \* indicates 5% significance level
2. \*\* indicates 1% significance level
3. In bracket are the 90% confidence intervals
4. In parentheses are standard errors
5. Number of Changes Selected From Sequential Method is based on 1% level

**Table 3. Estimation for Long and Short Memory of Log Realized Volatility  
After Adjustment for Breaks**

		d	AR(1)	MA(1)
Local	DM/\$	0.306 (0.0675)	N/A	N/A
	Yen/\$	0.304 (0.0675)	N/A	N/A
Whittle	Yen/DM	0.283 (0.0675)	N/A	N/A
ARFIMA	DM/\$	0.3671 (0.0142)	0	0
	Yen/\$	0.3945 (0.0142)	0	0
	Yen/DM	0.3839 (0.0142)	0	0
SEMIFAR	DM/\$	0.367 (0.0142)	0	N/A
	Yen/\$	0.3976 (0.0142)	0	N/A
	Yen/DM	0.3833 (0.0142)	0	N/A

1. The numbers in the parentheses indicate standard errors.
2. The Whittle, AFIMA, and SEMIFAR models are explained in the details below Table 1.

**Table 4. Estimated Spurious Breaks for Long Memory Simulation**

d	Breaks exist or not		Number of Breaks Selected			
	$Udmax$	$Wdmax$	$\sup F_T(l+1 l)$	BIC	LWZ	Sequential
0.1	No	No	0	0	0	0
0.2	Yes	Yes	0	2	0	1
0.3	Yes	Yes	3	4	2	3
0.35	Yes	Yes	3	3	3	3
0.4	Yes	Yes	2	4	2	2
0.45	Yes	Yes	3	4	3	3

1. Six different long memory parameters DGP based on Monte Carlo Simulation for 3045 observations.
2. Structural breaks tests are based on Bai and Perron (1998, 2003).
3. The tests are based on 1% significance level.

**Table 5. Out-of-Sample Forecast Evaluation When Future Breaks Are Known**

	$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	Rel MSE
<b>DM/\$</b>					
VAR-RV-Break	0.036 (0.048)	0.978 (0.091)	--	0.246	--
VAR-RV- $I(d)$	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV- $I(d)$	0.021 (0.049)	0.366 (0.332)	0.628 (0.327)	0.250	0.98
VAR-RV-Break + VAR-ABS	0.037 (0.046)	0.980 (0.102)	-0.009 (0.096)	0.246	3.86
VAR-RV-Break + Daily GARCH	-0.041 (0.060)	0.907 (0.120)	0.189 (0.137)	0.249	1.23
VAR-RV-Break + Daily RiskMetrics	-0.004 (0.047)	0.906 (0.119)	0.139 (0.098)	0.250	1.22
VAR-RV-Break + Daily FIEGARCH	0.046 (0.052)	0.987 (0.109)	-0.024 (0.100)	0.246	1.38
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.066 (0.059)	0.369 (0.207)	0.689 (0.217)	0.274	1.08
<b>Yen/\$</b>					
VAR-RV-Break	-0.030 (0.106)	1.090 (0.144)	--	0.330	--
VAR-RV- $I(d)$	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0.189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV- $I(d)$	-0.024 (0.101)	0.603 (0.564)	0.490 (0.662)	0.331	1.02
VAR-RV-Break + VAR-ABS	-0.058 (0.109)	1.044 (0.148)	0.166 (0.136)	0.331	3.01
VAR-RV-Break + Daily GARCH	-0.103 (0.141)	0.734 (0.131)	0.432 (0.263)	0.348	1.03
VAR-RV-Break + Daily RiskMetrics	-0.048 (0.112)	0.842 (0.102)	0.245 (0.134)	0.340	1.13
VAR-RV-Break + Daily FIEGARCH	-0.279 (0.209)	0.384 (0.260)	0.962 (0.484)	0.385	0.95
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.395 (0.252)	-0.007 (0.375)	1.656 (0.734)	0.380	1.06
<b>DM/Yen</b>					
VAR-RV-Break	-0.047 (0.096)	1.097 (0.132)	--	0.353	--
VAR-RV- $I(d)$	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV- $I(d)$	-0.054 (0.099)	0.483 (0.452)	0.650 (0.548)	0.357	1.04
VAR-RV-Break + VAR-ABS	-0.044 (0.094)	1.107 (0.148)	-0.028 (0.140)	0.353	3.60
VAR-RV-Break + Daily GARCH	-0.021 (0.082)	0.816 (0.135)	0.235 (0.167)	0.365	1.16
VAR-RV-Break + Daily RiskMetrics	-0.029 (0.089)	0.860 (0.117)	0.199 (0.121)	0.362	1.21
VAR-RV-Break + Daily FIEGARCH	-0.063 (0.106)	0.978 (0.118)	0.141 (0.143)	0.355	1.01
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.228 (0.156)	0.232 (0.294)	1.197 (0.530)	0.407	1.08

1. In parentheses are standard errors.

**Table 6. Out-of-Sample Forecast Evaluation When Future Breaks Are Unknown**

	$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	Rel MSE
<b>DM/\$</b>					
VAR-RV-Break	0.057 (0.050)	0.879 (0.088)	--	0.214	--
VAR-RV- $I(d)$	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV- $I(d)$	0.018 (0.050)	0.057 (0.179)	0.933 (0.196)	0.249	0.95
VAR-RV-Break + VAR-ABS	0.065 (0.047)	0.895 (0.103)	-0.056 (0.104)	0.214	3.73
VAR-RV-Break + Daily GARCH	-0.002 (0.060)	0.814 (0.131)	0.160 (0.161)	0.216	1.19
VAR-RV-Break + Daily RiskMetrics	0.029 (0.046)	0.814 (0.131)	0.115 (0.116)	0.216	1.18
VAR-RV-Break + Daily FIEGARCH	0.069 (0.051)	0.890 (0.111)	-0.029 (0.108)	0.214	1.33
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.072 (0.059)	0.140 (0.189)	0.888 (0.210)	0.267	1.05
<b>Yen/\$</b>					
VAR-RV-Break	-0.040 (0.127)	1.419 (0.218)	--	0.262	--
VAR-RV- $I(d)$	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0.189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV- $I(d)$	-0.000 (0.120)	-0.042 (0.151)	1.111 (0.146)	0.329	0.67
VAR-RV-Break + VAR-ABS	-0.156 (0.137)	1.242 (0.212)	0.577 (0.155)	0.282	1.98
VAR-RV-Break + Daily GARCH	-0.153 (0.147)	0.688 (0.129)	0.686 (0.203)	0.327	0.68
VAR-RV-Break + Daily RiskMetrics	-0.096 (0.132)	0.841 (0.137)	0.471 (0.108)	0.319	0.74
VAR-RV-Break + Daily FIEGARCH	-0.360 (0.185)	0.458 (0.146)	1.084 (0.295)	0.387	0.62
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.399 (0.190)	-0.371 (0.260)	1.961 (0.450)	0.384	0.70
<b>DM/Yen</b>					
VAR-RV-Break	-0.052 (0.138)	1.486 (0.248)	--	0.227	--
VAR-RV- $I(d)$	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV- $I(d)$	-0.030 (0.131)	-0.087 (0.175)	1.190 (0.096)	0.355	0.56
VAR-RV-Break + VAR-ABS	-0.106 (0.142)	1.259 (0.269)	0.484 (0.143)	0.247	1.93
VAR-RV-Break + Daily GARCH	0.004 (0.123)	0.655 (0.162)	0.519 (0.110)	0.325	0.62
VAR-RV-Break + Daily RiskMetrics	-0.041 (0.132)	0.762 (0.177)	0.485 (0.085)	0.324	0.65
VAR-RV-Break + Daily FIEGARCH	-0.140 (0.148)	0.795 (0.194)	0.631 (0.107)	0.303	0.54
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.188 (0.133)	-0.261 (0.192)	1.608 (0.295)	0.407	0.58

1. In parentheses are standard errors.

**Table 7. Out-of-Sample Forecast Evaluation for Simulations**

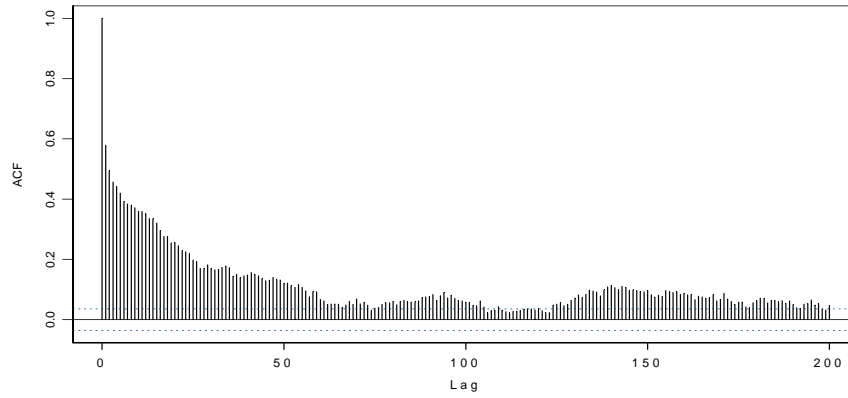
		<b>One-Step Ahead</b>		
		AR-Break Model		AR-I(d) Model
		Known Break	Unknown Break	
Period 4	Root MSE	0.407	0.407	0.411
	Relative MSE	1.02	1.02	1
Period 5	Root MSE	0.422	0.812	0.446
	Relative MSE	1.117	0.301	1
Period 6	Root MSE	0.398	1.272	0.502
	Relative MSE	1.589	0.155	1
Whole Out-of-Sample	Root MSE	0.41	0.894	0.453
	Relative MSE	1.222	0.257	1

		<b>Ten-Step-Ahead</b>		
		AR-Break Model		AR-I(d) Model
		Known Break	Unknown Break	
Period 4	Root MSE	0.447	0.447	0.448
	Relative MSE	1.03	1.03	1
Period 5	Root MSE	0.452	0.825	0.574
	Relative MSE	1.608	0.484	1
Period 6	Root MSE	0.418	1.283	0.837
	Relative MSE	4.013	0.425	1
Whole Out-of-Sample	Root MSE	0.441	0.912	0.634
	Relative MSE	2.09	0.488	1

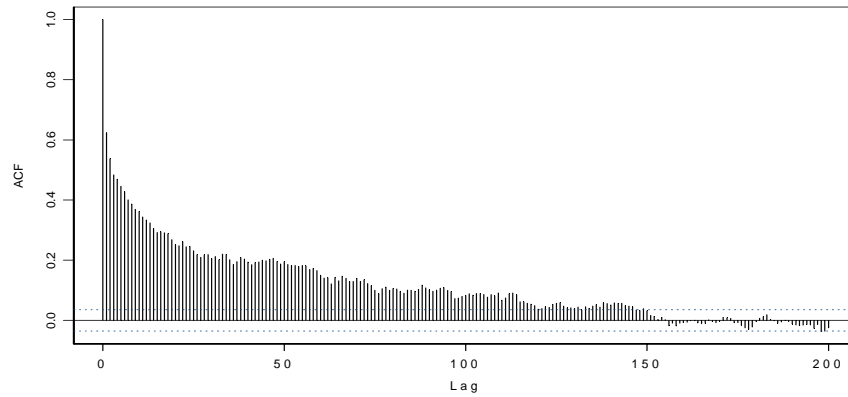
1. Root Mean Squared Error (MSE) is calculated as  $[\sum (y_{t+10} - y_{t+10|t}^{Model})^2 / N]^{1/2}$ .

2. Relative MSE is calculated as  $\frac{\sum (y_{t+1} - y_{t+1|t}^{I(d)})^2}{\sum (y_{t+1} - y_{t+1|t}^{Break})^2}$

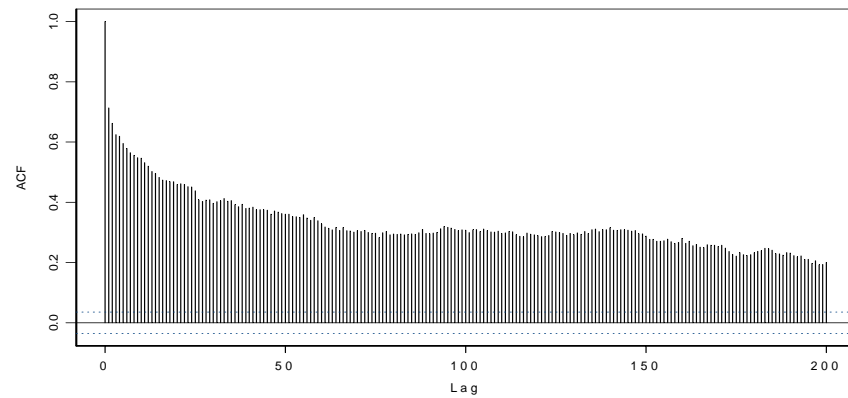
Autocorrelations for DM/\$ Log Realized Volatility



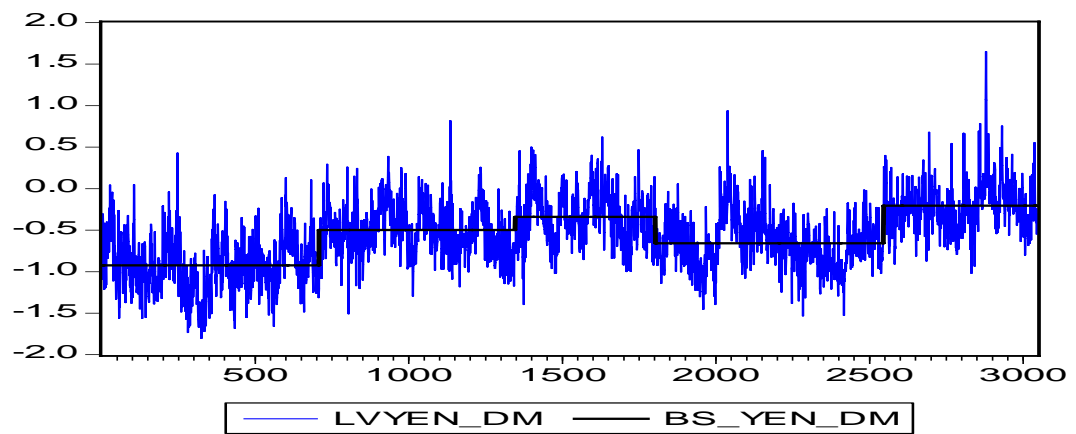
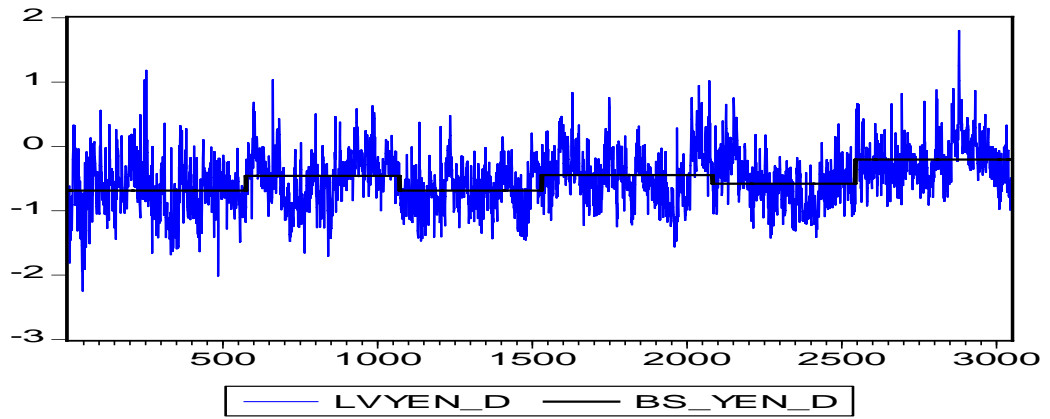
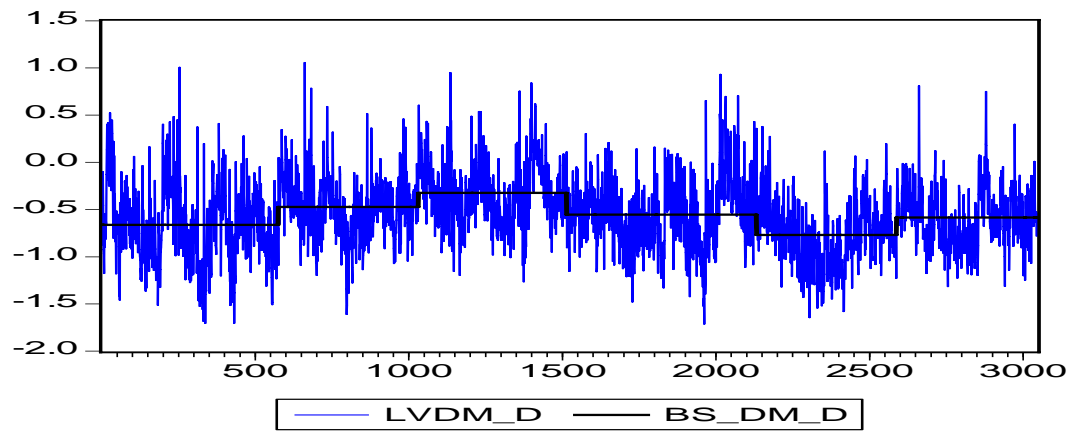
Autocorrelations for Yen/\$ Log Realized Volatility



Autocorrelations for Yen/DM Log Realized Volatility

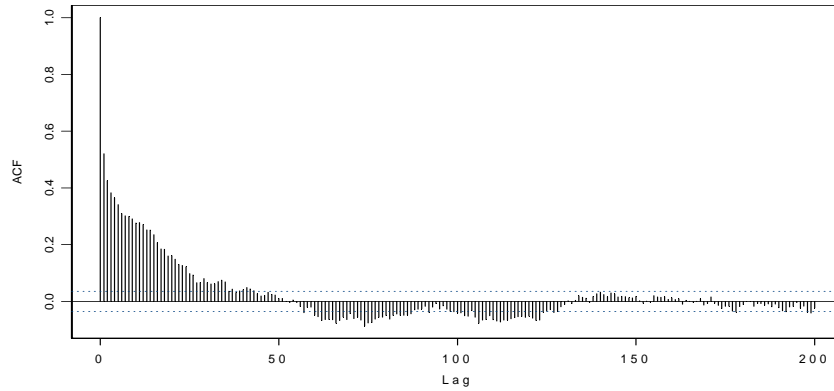


**Figure 1. Autocorrelations for Log Realized Volatility**

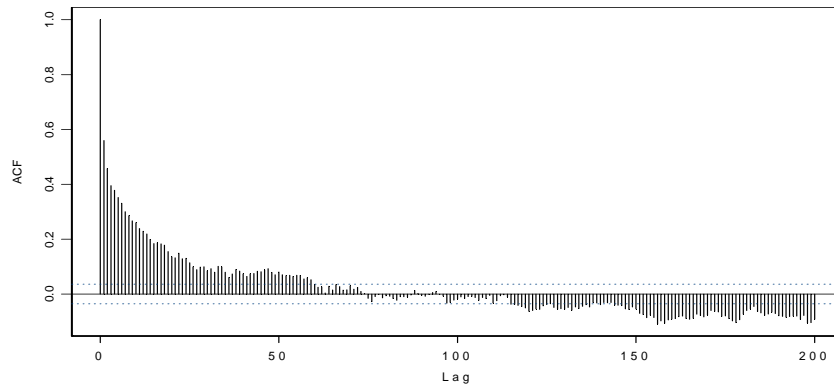


**Figure 2. Estimated Structural Breaks Means and Dates for Daily Exchange Rate Log Realized Volatility (1986.12.2 – 1999.6.30)**

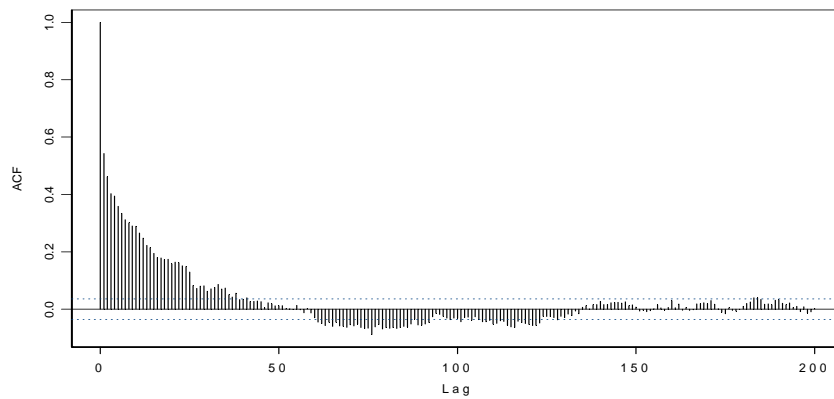
Correlations for DM/\$ Log Realized Volatility After Adjusting B



Correlations for Yen/\$ Log Realized Volatility After Adjusting E

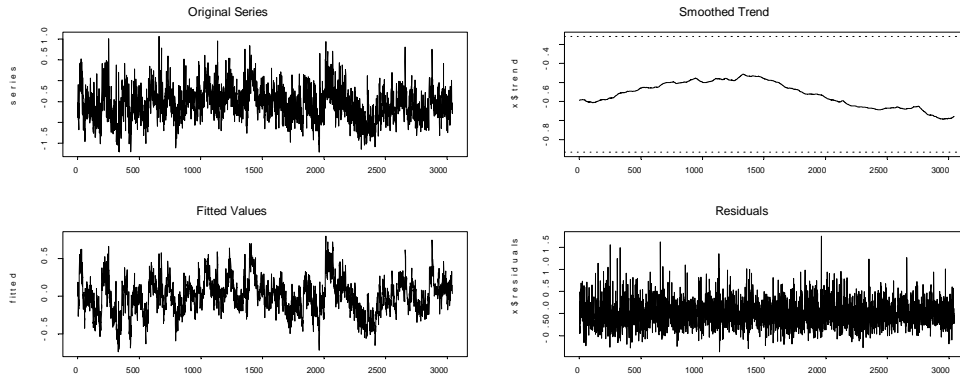


Correlations for Yen/DM Log Realized Volatility After Adjusting

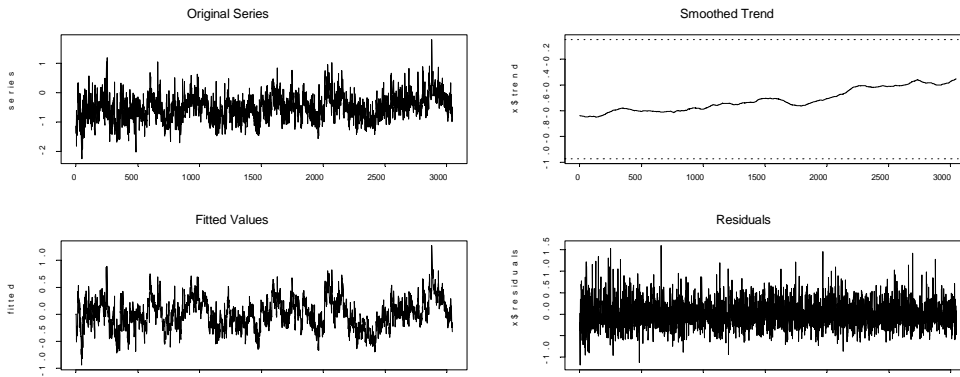


**Figure 3. Autocorrelations for Log Realized Volatility After Adjusting for Structural Breaks**

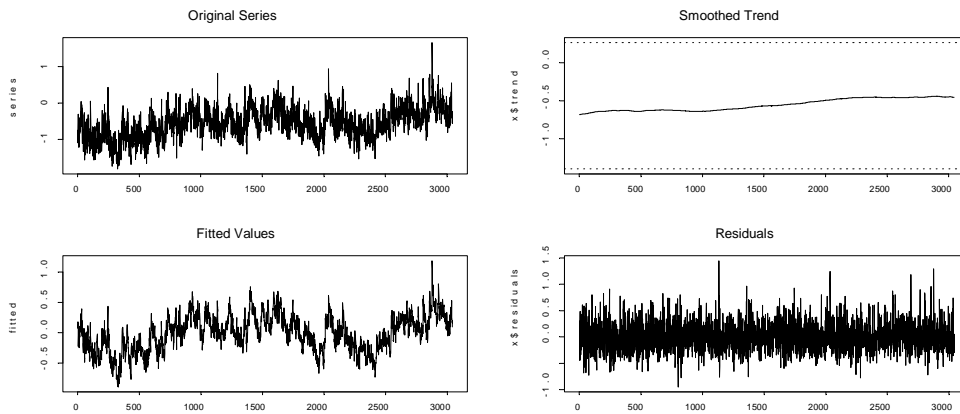
### DM/\$ Log Realized Volatility



### Yen/\$ Log Realized Volatility

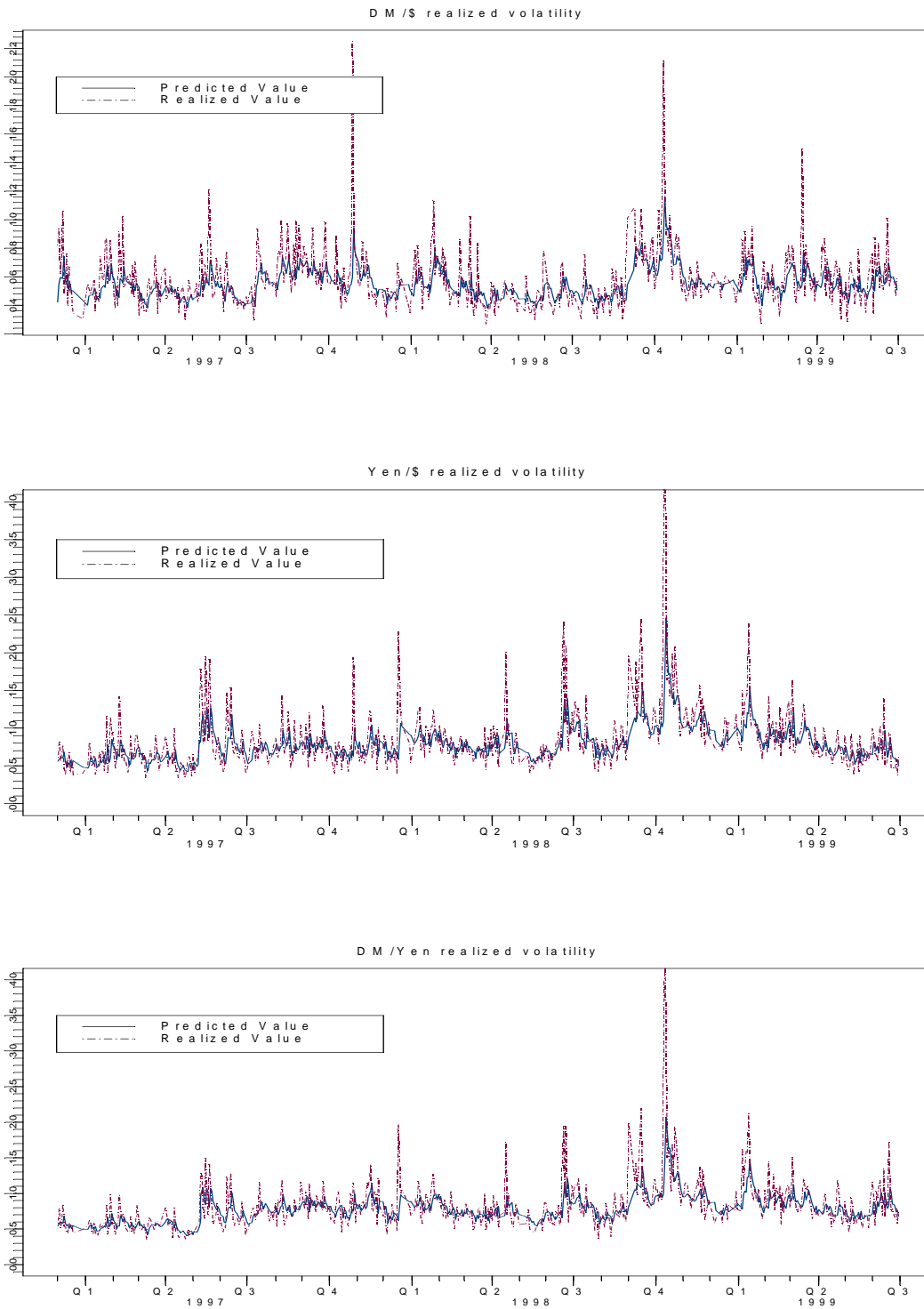


### Yen/DM Log Realized Volatility



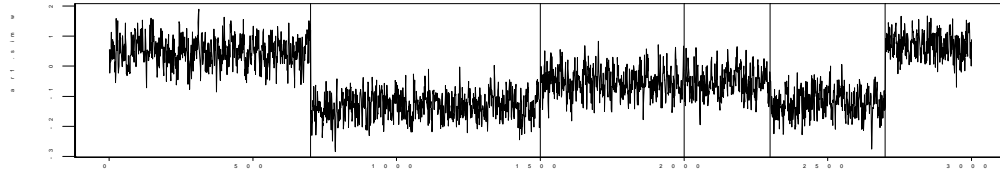
**Figure 4. Semiparametric Fractional Autoregressive Model Decomposition**

*Notes:* Based on Beran and Ocker's method (2001)

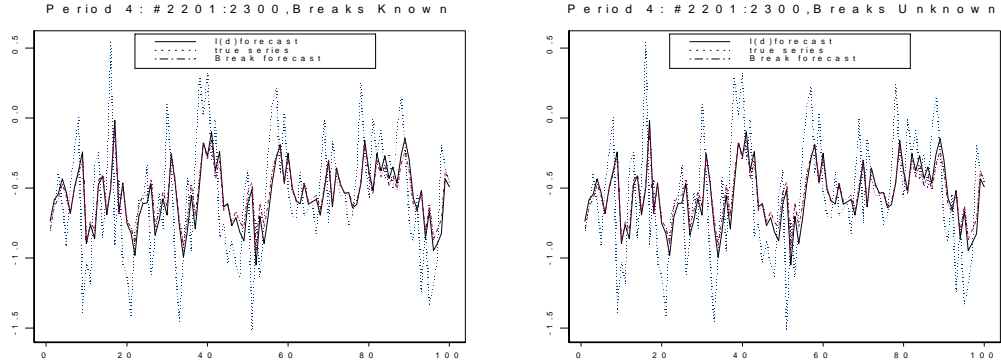


**Figure 5. Realized Volatility and Out-of-Sample VAR-RV-Break Forecasts**

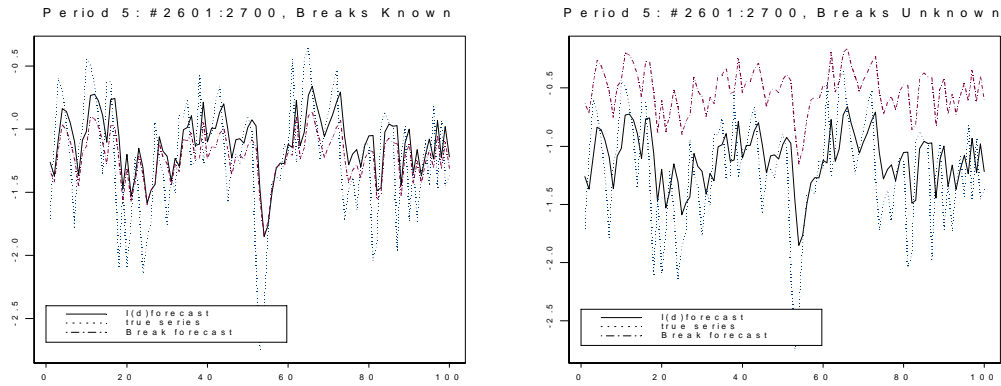
### A. Simulated Mean Breaks Series



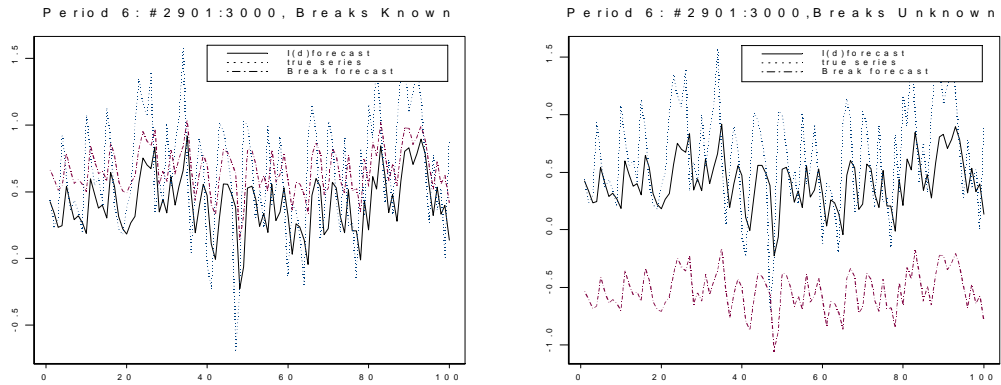
### B.



### C.



### D.



**Figure 6. Out of Sample Forecast Evaluation from Simulation**