Long Memory versus Structural Breaks in Modeling and Forecasting Realized Volatility

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Abstract

We explore the possibility of structural breaks in the realized volatility with the observed long-memory property for the daily Deutschemark/Dollar, Yen/Dollar and Yen/Deutschemark spot exchange rate realized volatility. The paper finds that the structural breaks can partly explain the persistence of realized volatility. We propose a VAR-RV-Break model that provides a superior predictive ability compared to most of the forecasting models when the future break is known. With unknown break dates and sizes, we find that the VAR-RV-I(d) long memory model, however, is a very robust forecasting method even when the true financial volatility series are generated by structural breaks.

JEL classification: C32, C52, C53, G10

Keywords: Realize volatility; Exchange rate; Long memory; Structural break; Fractional integration; Volatility forecasting

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1. Introduction

Conditional volatility and correlation modeling has been one of the most important areas of research in empirical finance and time series econometrics for the past two decades. Asset return volatility and correlation, henceforth volatility, are especially central to finance, as they are key inputs for asset and derivatives pricing, portfolio allocation, and risk measurement. Although daily financial asset returns are approximately unpredictable, return volatility is time-varying but highly predicable with persistent dynamics.[4] Furthermore, the dynamics of volatility is well modeled as a long memory process. An inherent problem for measuring, modeling and forecasting conditional volatility is that the volatility is unobservable or latent, which implies modeling must be indirect. Typically, measurements of conditional volatility are from parametric methods, such as GARCH models or stochastic volatility models for the underlying returns. These parametric volatility models, however, depend on specific distributional assumptions and are subject to misspecification problems.

Given the availability of intraday ultra-high-frequency price and quote data on assets, Andersen, Bollerslev, Diebold, and Labys (2003), henceforth ABDL, and Barndorff-Nielsen and Shephard (2001, 2002, 2004) introduce a consistent nonparametric estimate of the price volatility that has transpired over a given discrete interval, called *realized volatility*. They compute daily Deutschemark/Dollar, Yen/Dollar, and Deutschemark/Yen spot exchange rates realized volatilities simply by summing high-frequency finely sampled intraday squared and cross-products returns. By sampling intraday returns sufficiently frequently, the model-free realized volatility can be made arbitrarily close to underlying integrated volatility, the integral of instantaneous volatility over the interval of interest, which is a natural volatility measure.

[4] The findings suggest that volatility persistence is highly significant in daily data but will weaken as the data frequency decreases.

ABDL find logarithmic realized volatility could be modeled and accurately forecast using simple parametric fractionally integrated ARFIMA models. Their low-dimensional multivariate realized volatility model provides superior out-of-sample forecasts for both low-frequency and high-frequency movements in the realized volatilities compared to GARCH and related approaches. Nevertheless, many studies have pointed out that observed long memory may not only be generated by linearly fractional integrated process but also by: (1) cross-sectional aggregation of short memory stationary series (Granger and Ding, 1996); (2) mixture of numerous heterogeneous short-run information arrivals (Andersen and Bollerslev, 1997); (3) non-linear models, such as structural breaks (changes) or regime switches (Granger and Hyung, 2004; Choi and Zivot, 2007; Diebold and Inoue, 2001). In particular, it has been conjectured that persistence of asset return volatility may be overstated with the presence of structural changes.

In this paper, we focus on the possibility of structural breaks and trend in the realized volatility, with the observed long-memory property, for the Deutschemark/Dollar, Yen/Dollar and Yen/ Deutschemark spot exchange rate realized volatility obtained from ABDL. First, we test for long memory and estimate long memory models for the realized volatility series. We find strong evidence of long memory property in exchange rate realized volatility. Second, we test for and estimate a multiple mean break model based on Bai and Perron (1998, 2003)'s method. We find several common structural breaks within these three series. Using Beran and Ocker (2001)'s method, we cannot find evidence of flexible trend on volatilities. Third, we exam the evidence for long memory in the break adjusted data. We find a partial reduction of persistence in realized volatility after the removal of breaks. The evidence suggests that part of the long memory may be accounted for by the presence of structural breaks in the exchange rate volatility series.

Sun and Phillips (2003) point out that it is very difficult to separate low-frequency dynamics and high-frequency fluctuations, in particular when short-run fluctuations have high variance. Therefore, we propose an alternative short memory model, which adapts more quickly to the current volatility, with detected breaks information on realized volatility. Surprisingly, we find that our VAR-RV-Break model provides competitive forecasts compared to most of the forecasting models considered by ABDL if future break dates and sizes are known. The VAR-RV-I(*d*) model, however, is still the best forecasting model even when the true financial volatility series are generated by structural breaks and we have little knowledge about break dates and sizes. The Monte Carlo experiment also supports the findings.

The rest of the paper is organized as follows. Section 2 presents the long memory model and estimations. Section 3 presents empirical results using structural breaks model and examines the long memory estimations after adjusted breaks series. Section 4 reports the evaluation for forecasting. Section 5 concludes.

2. Realized Volatility and Long Memory Model

2.1. Realized Variance

ABDL utilize an empirical measure of daily return variability called realized volatility, which is easily computed from high-frequent intraday returns. By treating volatility as observed rather than latent, volatility modeling and forecasting using simple ARFIMA models is straightforward. We assume that an arbitrage-free logarithmic price $p_t = \log(P_t)$ process can be expressed as a continuous-time diffusion process in terms of the following stochastic differential equation without a jump term,

$$dp_t = \mu_t dt + \sigma_t dW_t \tag{1}$$

where μ_t is the predictable drift coefficient, σ_t is the instantaneous volatility of the logarithmic price process, and W_t is a standard Brownian motion. We denote the daily continuously compounded return as

$$r_t = p_t - p_{t-1} = \int_{t-1}^{t} \mu_s ds + \int_{t-1}^{t} \sigma_s dW_s \tag{2}$$

where $\int_{t-1}^{t} \sigma_s dW_s$ is a local martingale, and we denote the corresponding integrated variance (IV_t) as

$$IV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{3}$$

This natural measure of the inherent return variability, however, is not directly observable. Realized variance (RV_t) is computed by simply summing cross-products of intraday returns,

$$RV_t \equiv \sum_{i=1}^{1/h} r_{t-1+ih}^{(h)} \cdot r_{t-1+ih}^{(h)'} \equiv R'_{t,h} R_{t,h} \approx IV_t \tag{4}$$

where $r_t^{(h)} \equiv p_t - p_{t-h}$ is the intraday return, $R'_{t,h} \equiv (r_{t-1+h}^{(h)}, r_{t-1+2h}^{(h)}, \ldots.., r_t^{(h)})$, *h* is sample frequency[5] and *1/h* is assumed to be an integer. ABDL show that in the absence of measurement error in high frequency returns, realized variance is consistent for integrated variance as $h \to 0$. In practice, however, there is a lower bound on the sampling frequency because of market microstructure frictions features such as, discrete price, transactions costs, and bid-ask spreads at the very highest frequency.

2.2. Data

We use the same data as ABDL, which are spot exchange rates for the U.S. dollar, the Deutschemark, and the Japanese yen from December 1, 1986 through June 30, 1999.[6] They get thirty-minute prices from the linearly interpolated logarithmic average of the bid and ask quotes

[5] For example of the 30-minute intraday sample frequency from a 24-hour trading day (1440 minutes), *h* is 30/1440=1/48. There are 48 intraday returns.
[6] The raw data include all interbank DM/$ and Yen/$ bid/ask quotes shown on the Reuters FX screen provided by Olsen & Associates. These three currencies were the most actively traded in the foreign exchange market during the sample period.

for the two ticks immediately before and after the thirty-minute time stamps over the global 24-hour trading day. Thirty-minute returns are obtained from the first difference of the logarithmic prices. They exclude all the returns from Friday 21:00 Greenwich Mean Time (GMT) to Sunday 21:00 GMT and certain holiday periods to avoid weekend and holiday effects. The final data set consists of 3,045-days bivariate series of DM/$ and Yen/$ 30-minute returns over the sample period. The intraday return is denoted $r_t^{(h)}$, where $t = h, 2h, 3h, \ldots\ldots 47h, 1, 49h, \ldots\ldots, 3045$, and $h = 1/48 = 0.0208$. As in equation (4), realized volatility of DM/$ and Yen/$ will be the diagonal elements of $R_{t,h}^{'} R_{t,h}$. By absence of triangular arbitrage, the Yen/DM returns can be calculated directly from the difference between the DM/$ and Yen/$. Realized volatilities, also called realized standard deviations, which are calculated from the square root of the realized variance.

As shown in Table 1, the distributions of three realized volatility series are all right-skewed and fat-tailed. The distribution of logarithmic realized volatilities, however, are close to Gaussian as the logarithmic transformation reduces the impact of outliers, which provides strong evidence of the log-normality property for realized volatility. Last, the Ljung-Box statistics indicate strong serial correlation in all of the series.

[Insert Table 1 here]

2.3. Long Memory Model

Before conducting further modeling and forecasting, it is crucial to determine whether the time series is stationary or not. The distinction between I(0) and I(1) for the conditional mean, however, may be far too narrow. Long memory model that allows fractional orders of integration, I(*d*), provides more flexibility. For an I(0) process, shocks decay at an exponential rate; for an I(1) process, shocks have permanent effect; for an I(*d*) process, shocks dissipate at a slow hyperbolic rate. Long memory behavior in volatility has been well established, see for example, Ding, Granger, and Engle (1993), Baillie, Bollerslev and Mikkelsen (1996), and Andersen and Bollerslev (1997).

A time series process, y_t, with autocorrelation function ρ_k at lag *k*, is a long memory process when

$$\lim_{n\to\infty}\sum_{k=-n}^{n}\left|\rho_k\right| \to \infty \tag{5}$$

The spectral density $f(\omega)=(\sigma^2/2\pi)\sum_{k=-\infty}^{\infty}\rho_k e^{-ik\omega}$ tends to infinity at zero frequency, $f(0)=\infty$. In contrast, for a stationary process with short memory, the autocorrelation function is geometrically bounded, i.e. $\left|\rho_k\right| \le cm^{-k}$ with $0 < m < 1$. Its spectral density is $0 < f(0) < \infty$. There are several tests for long memory ranging from parametric to nonparametric methods. Granger and Joyeux (1980) and Hosking (1981) show that a long memory process for y_t can be modeled parametrically as a fractionally integrated process I(*d*), if

$$(1-L)^d(y_t-\mu)=\varepsilon_t \tag{6}$$

where *L* denotes the lag operator, *d* is fractional difference parameter, μ is the unconditional mean of y_t, and ε_t is independent and identically distributed with zero mean and finite variance. The fractional difference filter $(1-L)^d$ is defined as the binominal expansion

$$(1-L)^d = 1-dL+\frac{d(d-1)}{2!}L^2-\frac{d(d-1)(d-2)}{3!}L^3+\ldots=\sum_{k=0}^{\infty}\frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \tag{7}$$

where $\Gamma(k+1)$ is the Gamma function. A more flexible process called the ARFIMA (*p, d, q*) model[7] allows $(1-L)^d(y_t-\mu)$ to be auto autocorrelated:

[7] To obtain a stationary process, y_t must be differenced *d* times. The parameter *d* determines the long-term behavior, whereas *p* and *q* affect the short-term properties.

$$\phi(L)(1-L)^d(y_t-\mu)=\theta(L)\varepsilon_t \quad (8)$$

where $\phi(L)$ and $\theta(L)$ are autoregressive and moving average polynominals, respectively, with roots lie outside the unit circle. An ARFIMA process is non-stationary when $|d|$ > 0.5 and stationary when $|d|$ < 0.5. When 0 < *d* < 0.5, y_t is called stationary long memory. When -0.5 < *d* < 0, y_t is called intermediate memory and antipersistent. When *d* = 0, it is simply short memory.

2.4. SEMIFAR Model

To allow for the data-driven distinction of long memory, short memory, stochastic trends, and deterministic trends without any prior knowledge, we also consider a semiparametric fractional autoregressive (SEMIFAR) model proposed by Beran and Ocker (2001),

$$\phi(L)(1-L)^\delta((1-L)^m y_t - g(i_t))=\varepsilon_t \quad (9)$$

where δ is the long memory parameter, and $g(i_t)$ is a smooth trend function on [0,1] with $i_t = t/T$. y_t must be differenced to achieve stationarity by parameter $d=\delta+m$. m determines whether the trend should be estimated from the original data (when *m* = 0) or the first difference (when *m* = 1). When δ > 0, y_t is long memory. When δ < 0, y_t is antipersistent. When δ = 0, y_t has short memory.

2.5. Long Memory Estimation

According to the slow decay of autocorrelations in Figure 1, it is evident that the logarithmic realized volatility of the exchange rate series appears to have long memory dynamics. To estimate the long memory parameter *d*, we use the method of Geweke and Porter-Hudak

(1983), henceforth GPH, based on the simple linear regression of the log periodogram on a deterministic regression

$$\ln[I(\omega_j)] = c - d\ln[4\sin^2(\omega_j/2)] + u_j, \quad j = 1,.....n \tag{10}$$

where $I(\omega_j) = (1/2\pi)\left|\sum_{i=1}^{T} y_t \exp(i\omega_j t)\right|^2$ is the periodogram at frequency $\omega_j = 2\pi j/T$. The window size *n* depends on the sample size *T* ($n = T^{\alpha}$). Following Choi and Zivot (2007), we choose α = 0.7 and 0.8 as the desired periodogram ordinates. The least squares estimator *d* will be asymptotically normal with variance $\pi^2/6n$. There are several other methods of testing long memory time series, and we also use them as a robustness check. For a detailed discussion of long memory testing methods, see Baillie (1996), and Robinson (1995).

[Insert Figure 1 here]

The estimates of *d* for realized volatility are reported in Table 2, and the estimates of *d* for logarithmic realized volatility are reported in Table 3. Whether used nonparametric, parametric, or semiparametric methods, all of the estimates of *d* are in the range between 0.37 and 0.56, which confirms the long memory property in the (logarithmic) realized volatility.

[Insert Table 2 here]

[Insert Table 3 here]

3. Structural Break Model

3.1. Multiple Structural Break Model

It is well known that structural change and unit roots are easily confused (see Perron 1989; Zivot and Andrews 1992). Recently the confusion between long memory and structural change has been getting more and more attention. Granger and Ding (1996), Granger and Hyung (2004), and Choi and Zivot (2007) suggest that observed long memory property in the asset return volatility may be explained by the presence of structural breaks. To investigate this conjecture for realized volatility, we use the pure multiple mean break method proposed by Bai and Perron (1998, 2003), henceforth BP, to test this hypothesis. The *m* model (*m* + 1 regimes) is defined as

$$y_t = c_j + u_t,\quad t = T_{j-1}+1, T_{j-1}+2, ..., T_j \tag{11}$$

where $j = 1, 2, \ldots, m+1$, y_t is the logarithmic realized volatility, and c_j is the mean of the logarithmic realized volatility. The break points ($T_1, T_2, ..., T_m$) are treated as unknown. The error term u_t may be serial correlated and heteroskedastic. The estimation is based on the least-squares principle. The estimated break points ($\hat{T}_1, \hat{T}_2, ..., \hat{T}_m$) are obtained by solving

$\arg\min_{T_1,\ldots,T_m} S_T(T_1, T_2, ..., T_m)$ where

$$S_T(T_1, T_2, ..., T_m) = \sum_{j=1}^{m+1} \sum_{t=T_{i-1}+1}^{Ti} (y_t - c_j)^2 \tag{12}$$

Given the estimated break points, the corresponding estimates $\hat{c}_j(\hat{T}_1, \hat{T}_2, ..., \hat{T}_m)$ are obtained for each regime. We used several tests for structural change proposed in BP. Let $\sup F_T(l)$ denotes the F statistic for the null of no structural breaks versus an alternative hypothesis containing an arbitrary number of breaks, and let *M* denote the maximum number of breaks allowed. We set *M* = 5 with the trimming value = 0.15. Define the double maximum statistic

$UD_{\max} = \max_{1 \le l \le M} \sup F_T(l)$, and the weighted double max statistic $WD_{\max} = \max_{1 \le l \le M} w_l \sup F_T(l)$, where the marginal p-values are equal across values of l. The null hypothesis of both tests is no structural breaks against the alternative of an unknown number of breaks given some specific upper bound *M*. Sequential $\sup F_T(l+1|l)$ tests the null of l breaks versus the alternative $l+1$ breaks. To determine the number of breaks, we first use $UD_{\max}$ and $WD_{\max}$ to determine if at least one break occurred. If there is evidence for structural change, we select the number of structural breaks using $\sup F_T(l+1|l)$. To allow for a penalty factor for the increased dimension of a model, the above procedure may be complemented by selecting the number of breaks by minimizing a Bayesian Information Criterion (BIC) and a modified Schwarz Criterion (LWZ).

3.2. Multiple Structural Break Estimation

Table 4 displays values of all the tests used to determine the number of breaks for the logarithmic realized volatility series. The *UD*max and *WD*max tests point to the presence of multiple breaks for all series. The $\sup F_T(l)$ tests reject the null hypothesis of no breaks versus the alternative of an unknown number of breaks for the all series. For DM/$, the $\sup F_T(l+1|l)$ is significant at 1% level when l = 4, which suggests 5 breaks. BIC suggests 5 breaks as well while LWZ suggests 2 breaks. Therefore, we choose 5 breaks for DM/$. For Yen/$, $\sup F_T(l+1|l)$ is significant when l = 3 but not significant when l = 4, which suggest 4 breaks. We follow BIC to choose the 5 breaks for Yen/$. For Yen/DM, $\sup F_T(l+1|l)$ suggest 4 breaks as well as BIC. Hence 4 breaks should be chosen for Yen/DM.

[Insert Table 4 here]

In Table 4 we also report the estimates of the break dates with their respective 90% confidence intervals. The break dates estimated for DM/$ and Yen/$ are very similar, which suggests common break dates for the process: May 1989, March – May 1991, March 1993, June – August 1995, and May – July 1997. The estimates of the mean parameters ($\hat{c}_j$) for regimes (m + 1) are also provided on the bottom of Table 4. Figure 2 presents the graphs for the logarithmic realized volatility and the estimated $\hat{c}$ value. We suggest that the mean breaks are coincided with historical financial or currency crisis. For example, the highest volatility regime of DM/$ exchange rate occurred between two breaks: March 1991 and March 1993 and the second highest volatility regime occurred between two breaks: June 1992 and May 1994. Both regimes can be attributed to the Exchange Rate Mechanism (ERM) crisis of 1992-93 in Europe. Asian financial crisis occurred in 1997 caused the breaks in July 1997 for DM/$, May 1997 for Yen/$, and May 1997 for Yen/DM. Since the breaks in May 1997, Yen/$ and Yen/DM have turned to their highest volatility regimes among the whole sample.

[Insert Figure 2 here]

3.3. Long Memory Estimation After Adjusting for Structural Breaks

The sixth column in Table 3 shows the long memory parameter estimates for the three series after adjustment for the estimated structural breaks. The parameter d is estimated using the residual series $y_t - \hat{c}_j$. All estimates of d are lower after using break-adjusted series, especially in Yen/DM series. The long memory parameter decreases from 0.5694 to 0.4606, from 0.479 to 0.4271, from 0.4229 to 0.3852, from 0.5673 to 0.3839, and from 0.4685 to 0.3833 for GPH (α = 0.7), GPH (α = 0.8), Whittle, ARFIMA, and SEMIFAR methods, respectively. Figure 3 displays the autocorrelation function for the adjusted volatility series. Compared to Figure 1 for the autocorrelation before adjustment for breaks, it is evident that the persistence of volatility has been reduced after removing the estimated breaks. The presence of structure breaks, however, can not totally explain the persistence of exchange rate realized volatility. The other reasons for the long memory might be (1) aggregation of intraday squared return series; (2) mixture of numerous heterogeneous short-run information arrivals (Andersen and Bollerslev, 1997).

[Insert Figure 3 here]

In contrast to our two-step approach, Hsu (2005) proposes a one-step approach which estimates the long memory parameter directly from the data with unknown mean changes. He also finds a smaller long memory parameter for G7 inflation rates. On the other hand, from Figure 2, there might be an upward trend in the volatility series, especially in Yen/DM series, which has most persistence. We use the SEMIFA model with flexible trend by Beran and Ocker (2001) mention in Section 2.5 to test this possibility. The results for the estimated trend are shown in Figure 4. We see that the trend is not statistically significant. It is worth noting that Beran and Ocker' (BO) method is an alternative to the BP model. The BP model gives abrupt change whereas the BO model admits a smoother flexible trend. Our results show that the realized volatility series fit the BP model better than the BO model.

[Insert Figure 4 here]

3.4. Monte Carlo Simulation for Long Memory Process

We discussed previously that structural change is easily confused with long memory. Granger and Hyung (2004) point out that there exists another perplexity: a long memory model without breaks may cause breaks to be detected spuriously by standard estimation methods. To illustrate this phenomenon, we generate six long memory series with d = 0.1, 0.2, 0.3, 0.35, 0.4, 0.45, respectively, with mean: -0.5, standard deviation: 0.4, and sample size: 3,045. These series, which are similar to our sample logarithmic realized volatility, are shown in Figure 5. Table 5 shows results for the structural break tests of BP for the different data-generating processes (DGPs). The results suggest a positive relationship between the number of breaks and the value of d as found in Granger and Hyung (2004). This reveals the fact that a long memory/fractionally integrated process itself contains some portion of a permanent shock, which often appears as a break in some situations.[8] The important implication from this Monte Carlo evidence is that the long memory DGP provides a good parsimonious alternative of in-sample fit for the true structural-break DGP when we have little knowledge for the past break dates and size.[9]

[Insert Figure 5 here]

[Insert Table 5 here]

[8] Currently there is no formal test available for multiple structural changes in the I(d) process with unknown number of breaks. It will be interesting for the future research.
[9] This property, which is trivial here, will become much more important when we discuss the long memory and structural breaks for out-of-sample forecasting in Section 5.

4. Forecast Evaluation and Simulation

4.1. I(*d*) versus Breaks Model

Many models have been provided for forecasting asset return volatility and the success of a volatility model lies in its out-of-sample forecasting power. For example, ABDL propose a trivariate VAR-RV-I(*d*) (fractionally integrated Gaussian vector autoregressive-realized volatility),

$$\Phi(L)(1-L)^d(Y_t-\mu)=\varepsilon_t \tag{13}$$

where Y_t is (3×1) vector of logarithmic realized exchange rate volatilities; μ is unconditional mean and ε_t is a vector white noise process. They fix the value of *d* for each series at 0.401, which is also close to our long memory estimates in Table 3. They choose the orders of 5 for the lag polynomials in $\Phi(L)$ to being equal to five days, or one week. They compare with the volatility forecasts from several popular models, and they find that their VAR-RV-I(*d*) model produces superior out-of-sample one-step-ahead and ten-step-ahead forecasts.

Granger and Joyeux (1980), however, point out that long memory model would not necessarily produce clearly superior short-run forecasts, which is of interest in financial forecasting. Although we only get partial reduction of persistence after removing the exchange rate volatility breaks, it is still worth trying short memory model for break-adjusted series on short-run forecasts (i.e. one-step-ahead forecasts), in which the forecast converges fast at an exponential rate. Hence we propose and assess the alternative -- VAR-RV-Break model,

$$\Phi(L)(Y_t^*-\mu)=\varepsilon_t \tag{14}$$

where Y_t^* is the vector of logarithmic realized exchange rate volatilities after mean break adjustment. Although the Bayesian information criteria select a fourth-order VAR, we use a fifth-order model to compare our result to those in ABDL.[10] Forecasts are obtained by estimating rolling models. We estimate initially over the first 2449 observations, December 2, 1986 to December 1, 1996, and using the in-sample parameter estimates,[11] one-day-ahead forecasts are made for the next day, say day 2450. The process is then rolled forward 1 day, deleting the first observation and adding on the 2450 observation, the model is re-estimated and the second forecast is made for 2451. The rolling method is repeated until 3045, the end of the out-of-sample forecast period. We get 596 one-step-ahead predictions in the out-of-sample period, which is from December 2, 1996 to June 30, 1999.

[Insert Figure 6 here]

4.2. Known versus Unknown Breaks Information

Meanwhile, we consider two assumptions. First, we assume that the future (out-of-sample) break dates and sizes are known. In the true real-time forecasts by financial practitioners, breaks are possible to be noticed and adjusted by human judgment. For example, when Bank of Japan announced that they would abandon their zero-interest rate policy, we knew to some extent that structural break in Japanese bonds market's volatility would happen. In this case, when the break happened, we quickly adjust the forecasts based on the given out-of-sample breaks dates and means. Second, we assume that we have little knowledge about out-of-sample breaks and sizes. For example, we did not know there was a structural break occurred in 1984 for the US real output volatility until recently. In this case, we do not do any adjustments even though we make systematic forecasts errors. Indeed, we have BP method to detect the structural breaks when we

[10] In Figure 6, partial autocorrelations suggest fifth-order AR model. We also evaluate model by VAR(4). The results are similar to VAR(5).
[11] We choose this in-sample period to compare our result to those in ABDL.

roll over the out-of-sample period, but we still need to wait a sufficient time to estimate it. Andrews (1993) suggests a restricted interval [0.15, 0.85] instead of the full interval for trimming to avoid the much reduced power of test statistics.

4.3. Forecast Evaluation and Comparison

Figure 7 displays the DM/$, Yen/$, and DM/Yen realized volatility along with the corresponding one-day-ahead VAR-RV-Break forecasts when the future breaks information are known. It appears that our forecasts capture movement of the realized volatilities well. To determine which model provides more information about the future value, we use the encompassing regression[12] by Mincer and Zarnowitz (1969),

$$\mathrm{vol}_{t+1,i} = \beta_0 + \beta_1 \mathrm{vol}_{t+1|t,i}^{VAR-RV-Break} + \beta_2 \mathrm{vol}_{t+1|t,i}^{Model} + \varepsilon_t \tag{15}$$

where we denote our benchmark VAR-RV-Break model prediction of future volatility by $\mathrm{vol}_{t+1|t}^{VAR-RV-Break}$, and future volatility prediction from other candidate methods by $\mathrm{vol}_{t+1|t}^{Model}$. The alternative models are all selected by ABDL and described as follows. First, the VAR-RV-I(*d*) model (13) is the main model proposed by ABDL. Second, the VAR-ABS model is fractionally integrated vector autoregressive using daily absolute returns instead of realized volatility. Third, the GARCH model pioneered by Engle (1982) and Bollerslev (1986) describes short-memory conditional volatility via maximum likelihood procedure as a linear function of past squared forecast errors. Based on 2,449 daily in-sample returns, ABDL use the GARCH (1,1) estimates with AR polynomial for DM/$, Yen/$, and DM/Yen being 0.986, 0.968, and 0.99, respectively. Fourth, the RiskMetrics model from J. P. Morgan is widely used by practitioners. ABDL use the

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[12] This is a regression-based method where the prediction is unbiased only if β_0 =0 and β_1 =1. When there are more than one forecasting models, additional forecasts are added to the right-hand-side to check for incremental explanatory power. The first forecast is said to subsume information in other forecasts if these additional forecasts do not significantly increase the R^2.

RiskMetrics daily variances and covariances using exponentially weighted moving averages of the cross products of daily returns by a smoothing factor λ=0.94 .[13] Fifth, the fractionally integrated exponential GARCH (FIEGARCH)[14] (1,*d*,0) by Bollerslev and Mikkelsen (1996) is a variant of FIGARCH model by Baillie, Bollerslev, and Mikkelsen (1996). The last one is the high-frequency FIEGARCH model using the "deseasonalized"[15] and "filtered"[16] 30-minutes returns.

[Insert Figure 7 here]

For the robustness check, we also present the popular out-of-sample forecast evaluation, relative mean squared error (MSE),

$$\frac{\sum (vol_{t+1} - vol_{t+1|t}^{Model})^2}{\sum (vol_{t+1} - vol_{t+1|t}^{Break})^2} \tag{16}$$

where the denominator is the benchmark model mean squared forecast error and the numerator is the candidate methods mean squared forecast error. If the relative MSE is less than one, the candidate model forecast is determined to have performed better than the benchmark. The results

[13] RiskMetrics is a special form of integrated GARCH (IGARCH) in which the intercept is fixed at zero and the coefficient for the squared returns (λ) is 0.94. λ could be interpreted as a persistence parameter. When λ is closer to one, more weight is put on the previous period's estimate relative to the current period's observation, which means it is more persistent.

[14] FIEGARCH has volatility persistence shorter than IGARCH but longer than GARCH. Bollerslev and Mikkelsen (1996) found that FIGARCH outperforms GARCH and IGARCH and FIEGARCH is better than FIGARCH for S&P 500 returns.

[15] The deseasonalization is from the fact that the intraday volatility has obvious "seasonal" components related to the opening and closing hours of exchange worldwide. This intraday patterns damage the estimation of traditional volatility models from the raw high-frequency returns. Following ABDL, we get the seasonal factor by averaging the individual squared returns in the various intra-day intervals. And then we can construct the seasonal adjusted high frequency returns.

[16] To decrease the impact of the serial correlation in high frequency asset returns from different market microstructure frictions, following ABDL, we use simple first order AR "filter" to the high-frequency returns before estimating FIEGARCH model.

are presented in Table 6. Our VAR-RV-Break model out-of-sample forecasts perform as well as ABDL's VAR-RV-I(*d*) model, and outperform most of the rest of the models.

[Insert Table 6 here]

4.4. Evaluation Results

First, the regression R^2 from VAR-RV-Break model is similar to that from VAR-RV-I(*d*) model and is higher than most of the rest models. Second, we can not reject the hypothesis that β_0 = 0 and β_1 = 1 in the VAR-RV-Break model using *t* tests while we reject the hypothesis that β_0 = 0 and/or β_2 = 1 for all the other models except the VAR-RV-I(*d*) model. Third, in the encompassing regression that includes both the break model and an alternative forecast, the estimates for β_1 are closer to unity and the estimates for β_2 are closer to zero. Fourth, including an alternative forecast method has little contribution to increasing R^2. Finally, most of the relative MSEs are bigger than one, which means that VAR-RV-Break model has the smaller MSE than that in other forecasts.

The results in Table 6 show the superior forecasting ability for the VAR-RV-Break model in which the future break dates and sizes are known in the out-of-sample period. This result is consistent with Hyung, Poon and Granger (2006), in which they investigate the S&P 500 return volatility. Pesaran, Pettenuzzo, and Timmermann (2006) provide a Bayesian estimation and prediction procedure allowing breaks occurring over the forecast horizon. They find their method, which has similar inference with known break information (prior), leads to better out-of-sample forecasts than alternative models. Without the additional information in detecting out-of-sample breaks, unsurprisingly, the prediction ability of the VAR-RV-Break would be deteriorated, which is shown in Table 7. The degree of deterioration depends on the numbers and sizes of the out-of-sample breaks. For the DM/$ series, the VAR-RV-Break model still outperforms all the models except the VAR-RV-I(*d*) model because the out-of-sample break is not large as shown in Figure 2. But for the Yen/$ and Yen/DM, which show larger breaks, the VAR-RV-Break model's prediction ability becomes inferior to the other models (except VAR-ABS). In this case, the VAR-RV-I(*d*) would be the best forecasting model.

[Insert Table 7 here]

4.5. Forecast Simulation for Break and Long Memory Models

For the robustness check about the comparison of the VAR-RV-Break and the VAR-RV-(*d*) out-of-sample forecasts, we simulate a DGP of structural break. We first generate an AR(1) process with 3000 observations, which is pure short memory process with AR(1) coefficient: 0.41, and unconditional variance: 0.16. And then we break it to six periods by four ad hoc breaks shown in Figure 8.A. Each period's range and mean are as follows: Period 1[1:700; 0.5], Period 2[701:1500; -1.3], Period 3[1501:2000; -0.5], Period 4[2001:2300; -0.5], Period 5[2301:2700; -1.2], and Period 6[2701:3000, 0.7] where Period 1 to Period 3 for 2000 observations are in-sample period and Period 4 to Period 6 for 1000 observations are out-of-sample period. The data and break's parameters and ranges are chosen to mimic the logarithmic exchange rate realized volatility data.

[Insert Figure 8 here]

For the AR-Break model, we perform one-step-ahead forecasts simply based on the true short memory DGP with AR(1) coefficient: 0.41. Again, we consider two situations. First, when the out-of-sample breaks are known, we adjust the mean for the forecast evaluation. If the breaks information is not known, we don't do any adjustment. For the AR-I(d) model, we use in-sample data (Figure 8.A P1 to P3) to estimate the long memory parameter and the AR(1) coefficient based on SEMIFAR model as mentioned before.[17] We get d = 0.2697 and AR(1) = 0.2137. We therefore conduct one-step-ahead forecasts using the same SEMIFAR model. The one step ahead forecasts are produced using rolling window, which is the same method in Section 4.1. Figure 8.B shows the result for Period 4 in which the out-of-sample break has not occurred. Whether breaks are known or not, the break model performs a little bid better than I(d) model. Surprisingly, in Period 5 and 6 after breaks occurred, the I(d) model still accurately predicts while the break model deteriorates substantially when the breaks are unknown.

Table 8 contains the root mean squared error (MSE) and relative mean squared error to evaluate known break model, unknown break model, and I(d) model's out-of-sample forecasting. The root MSE is

$$[\sum (y_{t+1} - y_{t+1|t}^{Model})^2 / N]^{1/2} \tag{17}$$

where y_{t+1} is true DGP and $y_{t+1|t}$ is the one-step-ahead forecast. N is the number of forecasts. The relative MSE is computed as

$$\frac{\sum (y_{t+1} - y_{t+1|t}^{I(d)})^2}{\sum (y_{t+1} - y_{t+1|t}^{Break})^2} \tag{18}$$

When relative MSE equal one, I(d) and break model perform the same. When relative MSE is smaller than one, I(d) model forecasts better than break model. When relative MSE is larger than one, break model performs better. In Period 4, the relative MSE for known break and unknown break model are 1.02, which implies that break models perform slightly better than I(d) model. In period 5, 6, and the whole sample, the relative MSE for known break model and I(d) model is

[17] The number of lagged values to use for prediction is 50. k=100000 is chosen to determine the Fourier frequencies to use in evaluating the theoretical spectrum of a ARFIMA model.

1.117, 1.589, and 1.222, respectively. Unsurprisingly, the known break model forecast better in true break process than I(d) model. Compared with the results in Table 6 for the exchange rate realized volatility data, the relative MSE are 0.98, 1.02, and 1.04, respectively, which means the break model's forecasting is not as good as that in simulated samples here. This is reasonable because the true series in Table 6 is not pure AR-Break process while it is true AR-Break process here. For the unknown break model and I(d) model, its relative MSE in Period 5, 6 and the whole ample is 0.301, 0.155, and 0.257, respectively. The I(d) model performs far better than the unknown break model.

[Insert Table 8 here]

Table 9 shows the forecasts evaluation for ten steps ahead. That is, we compute the root MSE by

$$[\sum (y_{t+10} - y_{t+10|t}^{Model})^2 / N]^{1/2} \tag{19}$$

Basically we can see the similar pattern in the 10-step-ahead forecast evaluation. The known break model is better than I(d) model, and I(d) model is better than unknown break model. It is worth noting that when we compare the root MSE results in Table 8 and 9, I(d)'s 10-step-ahead forecast worsens more than break model's. For the I(d) model, its whole out-of-sample root MSE increases from 0.453 to 0.634 when it forecasts from one-step-ahead to ten-step-ahead. In contrast to the known break model, its whole out-of-sample root MSE only increases from 0.410 to 0.441 when it forecasts from one-step-ahead to ten-step-ahead. Although the long memory is expected to have better multi-step-ahead forecasts compared to other models, the findings here can not support this hypothesis.

[Insert Table 9 here]

In summary, even though the DGP is pure mean break series without any long memory, we still can get very good out-of-sample forecast performance using simple AR-I(d) model. This result shows that long memory/fractional integrated model will still be the best forecasting model when the true financial volatility series are generated by structural breaks and we have little knowledge about these breaks information.

5. Conclusions

Realized volatility constructed by intraday high-frequency data improves its out-of-sample forecasts ability compared with traditional volatility models. ABDL (2003)'s VAR-RV-I(d) model beats popular GARCH-type models. The main reason is because that the former model, which exploits the intraday volatility information, provides a relative accurate and fast-adapting estimate of current volatility while the latter model, depending on slowly decaying past squared returns, adapts only gradually to the current volatility shocks. In light of this property, the forecasts could be improved further by constructing a short memory model and incorporating breaks information on realized volatility instead of long memory model.

In this paper, we investigate the alternative in-sample model and out-of-sample forecasts on the realized volatility. We explore the existence and of structural changes in realized volatility for the DM/$, Yen/$ and Yen/ DM spot exchange rate realized volatility. First, our analysis has found strong evidence of long memory behavior in exchange rate realized volatility. Second, we test for and estimate a multiple mean breaks model; and we find several common structural breaks within the three series. Third, after adjusting the realized volatility series for the estimated breaks, we find a partial reduction of persistence in the realized volatility. The evidence suggests that long memory is partly caused by the presence of structural breaks. Fourth, the Monte Carlo simulation reports that the long memory model could spuriously produce multiple structure breaks. This evidence justifies the usage of long memory model for the in-sample DGP of pure structural breaks.

We propose the VAR-RV-Break model based on detected in-sample breaks with short memory dynamics. Our short-memory-Break model is superior among most of the current forecasting methods if the future break dates and sizes are known. With little knowledge about out-of-sample break dates and size, the parsimonious long memory model, however, is still a robust forecasting method even when the true financial volatility series are generated by structural breaks.

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Table 1. Daily Realized Volatility Distributions

	Mean	S.D.	Skewness	Kurtosis	Q(20)
			Volatility		
DM/$	0.616	0.269	2.111	11.55	6095.6
Yen/$	0.661	0.331	3.323	33.72	6523.2
Yen/DM	0.618	0.279	2.985	32.56	12443.9
			Logarithmic Volatility		
DM/$	-0.562	0.386	0.308	3.49	8627.2
Yen/$	-0.51	0.43	0.217	3.65	9150.1
Yen/DM	-0.565	0.406	0.101	3.38	18402.3

1. The sample is from Dec 1, 1986 to June 30, 1999.
2. The top panel is the distribution of realized standard deviation, $(\text{realized variance})^{1/2}$.
3. The bottom panel is the distribution of logarithmic realized standard deviation.
4. Ljung-Box test statistics for twentieth order serial correlation, Q(20).

Table 2. Realized Volatility Long Memory Parameters before Adjustment

Tests	Series	d	AR(1)	MA(1)
GPH (α = 0.7)	DM/$	0.4356	N/A	N/A
	Yen/$	0.4356	N/A	N/A
	Yen/DM	0.5376	N/A	N/A
GPH (α = 0.8)	DM/$	0.3952	N/A	N/A
	Yen/$	0.3999	N/A	N/A
	Yen/DM	0.4582	N/A	N/A
Whittle	DM/$	0.3489	N/A	N/A
	Yen/$	0.3931	N/A	N/A
	Yen/DM	0.4160	N/A	N/A
ARFIMA (p,d,q)	DM/$	0.3489	0	0
	Yen/$	0.39	0	0
	Yen/DM	0.4143	0	0
SEMIFAR (p,d,0)	DM/$	0.3444	0	0
	Yen/$	0.3859	0	0
	Yen/DM	0.4096	0	N/A

1. GPH test is based on Geweke and Porter-Hudak (1983).
2. Whittle's method is based on a frequency domain maximum likelihood estimation of a process i.e. equation (8).
3. ARFIMA model is based on Beran (1995). $\phi(L)(1-L)^{\delta}[(1-L)^{m} y_t - \mu] = \theta(L)\varepsilon_t$ where -0.5 < d < 0.5. The integer m is the number of times that y must be differenced to achieve stationarity, and the long memory parameter is given by $d = \delta + m$. The method uses BIC to choose the short memory parameters p and q. When m = 0, μ is the expectation of y_t ; when m = 1, μ is the slope of linear trend component in y_t .
4. SEMIFAR (Semiparametric Fractional Autoregressive) model is based on Beran and Ocker (2001). $\phi(L)(1-L)^{\delta}[(1-L)^{m} y_t - g(i_t)] = \varepsilon_t$. By using a nonparametric kernel estimate of $g(i_t)$ instead of constant term μ . The method uses BIC to choose the short memory parameter p.

Table 3. Estimations for Long and Short Memory Parameters

		Log Realized Volatility Before Adjustment			Log Realized Volatility After Adjustment		
		d	AR(1)	MA(1)	d	AR(1)	MA(1)
GPH (α = 0.7)	DM/$	0.4617 (0.0404)	N/A	N/A	0.3558 (0.0404)	N/A	N/A
	Yen/$	0.4503 (0.0404)	N/A	N/A	0.3921 (0.0404)	N/A	N/A
	Yen/DM	0.5694 (0.0404)	N/A	N/A	0.4606 (0.0404)	N/A	N/A
GPH (α = 0.8)	DM/$	0.4219 (0.0269)	N/A	N/A	0.3648 (0.0269)	N/A	N/A
	Yen/$	0.4292 (0.0269)	N/A	N/A	0.4009 (0.0269)	N/A	N/A
	Yen/DM	0.479 (0.0269)	N/A	N/A	0.4271 (0.0269)	N/A	N/A
Whittle	DM/$	0.3816	N/A	N/A	0.3693	N/A	N/A
	Yen/$	0.4146	N/A	N/A	0.3975	N/A	N/A
	Yen/DM	0.4229	N/A	N/A	0.3852	N/A	N/A
ARFIMA	DM/$	0.3817 (0.0142)	0	0	0.3671 (0.0142)	0	0
	Yen/$	0.4107 (0.0142)	0	0	0.3945 (0.0142)	0	0
	Yen/DM	0.5673 (0.0316)	0.28 (0.05)	0.49 (0.02)	0.3839 (0.0142)	0	0
SEMIFAR	DM/$	0.3778 (0.0142)	0	N/A	0.367 (0.0142)	0	N/A
	Yen/$	0.4096 (0.0142)	0	N/A	0.3976 (0.0142)	0	N/A
	Yen/DM	0.4685 (0.0212)	-0.1018 (0.03)	N/A	0.3833 (0.0142)	0	N/A

1. The numbers in the parentheses indicate standard errors.
2. GPH, Whittle, ARFIMA, and SEMIFAR models are explained in the detail below the Table 2.

Table 4. Multiple Structural Changes Test Results

Statistics \ Series	DM/$	Yen/$	Yen/DM
	Tests		
$\sup F_T(1)$	52.58	133.36	305.13
$\sup F_T(2)$	55.64	76.86	246.36
$\sup F_T(3)$	42.37	54.06	188.79
$\sup F_T(4)$	36.92	50.74	149.35
$\sup F_T(5)$	33.94	42.29	110.46
$UD\max$	55.64**	133.36**	305.13**
$WD\max$	84.95**	133.36**	323.48**
$\sup F_T(2\|1)$	36.23	17.88**	131.56
$\sup F_T(3\|2)$	12.83	7.19	71.95
$\sup F_T(4\|3)$	16.29	26.37	21.31
$\sup F_T(5\|4)$	16.29	8.57	0
	Numbers of Changes Selected		
BIC	**5**	**5**	**4**
LWZ	2	2	2
Sequential	4	2	4
	Multiple Structural Changes Dates Estimation		
$\hat{T}_1$	1989.5.11 [89.2.14-89.11.1]	1989.5.11 [88.12.21-89.9.6]	1989.11.21 [89.11.7-89.12.15]
$\hat{T}_2$	1991.3.18 [90.9.24-91.8.20]	1991.5.16 [91.3.20-91.10.2]	1992.6.10 [92.3.4-92.10.20]
$\hat{T}_3$	1993.3.5 [92.12.7-93.5.14]	1993.3.30 [92.11.25-93.5.21]	1994.5.4 [94.3.24-94.6.1]
$\hat{T}_4$	1995.8.24 [95.6.15-95.11.29]	1995.6.15 [94.8.24-96.6.19]	1997.5.8 [97.4.17-97.5.28]
$\hat{T}_5$	1997.7.10 [97.4.9-89.10.27]	1997.5.7 [97.4.3-97.6.17]	
	Estimations of Mean for Each Regime		
$\hat{c}_1$	-0.662 (0.015)	-0.686 (0.017)	-0.924 (0.012)
$\hat{c}_2$	-0.472 (0.017)	-0.457 (0.018)	-0.5 (0.013)
$\hat{c}_3$	-0.334 (0.016)	-0.687 (0.017)	-0.342 (0.015)
$\hat{c}_4$	-0.553 (0.015)	-0.446 (0.017)	-0.659 (0.012)
$\hat{c}_5$	-0.770 (0.017)	-0.580 (0.018)	-0.205 (0.014)
$\hat{c}_6$	-0.585 (0.017)	-0.205 (0.018)	

1. * indicates 5% significance level
2. ** indicates 1% significance level
3. In bracket are the 90% confidence intervals
4. In parentheses are standard errors
5. Number of Changes Selected From Sequential Method is based on 1% level

Table 5. Estimated Spurious Breaks for Long Memory Simulation

	Breaks exist or not		Number of Breaks Selected			
d	*Ud*max	*Wd*max	$\sup F_T(l+1\mid l)$	BIC	LWZ	Sequential
0.1	No	No	0	0	0	0
0.2	Yes	Yes	0	2	0	1
0.3	Yes	Yes	3	4	2	3
0.35	Yes	Yes	3	3	3	3
0.4	Yes	Yes	2	4	2	2
0.45	Yes	Yes	3	4	3	3

1. Six different long memory parameters DGP based on Monte Carlo Simulation for 3045 observations.
2. Structural breaks tests are based on Bai and Perron (1998, 2003).
3. The tests are based on 1% significance level.

Table 6. Out-of-Sample Forecast Evaluation When Future Breaks Are Known

	β_0	β_1	β_2	R^2	Rel MSE
DM/$					
VAR-RV-Break	0.036 (0.048)	0.978 (0.091)	--	0.246	--
VAR-RV-I(d)	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV-I(d)	0.021 (0.049)	0.366 (0.332)	0.628 (0.327)	0.250	0.98
VAR-RV-Break + VAR-ABS	0.037 (0.046)	0.980 (0.102)	-0.009 (0.096)	0.246	3.86
VAR-RV-Break + Daily GARCH	-0.041 (0.060)	0.907 (0.120)	0.189 (0.137)	0.249	1.23
VAR-RV-Break + Daily RiskMetrics	-0.004 (0.047)	0.906 (0.119)	0.139 (0.098)	0.250	1.22
VAR-RV-Break + Daily FIEGARCH	0.046 (0.052)	0.987 (0.109)	-0.024 (0.100)	0.246	1.38
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.066 (0.059)	0.369 (0.207)	0.689 (0.217)	0.274	1.08
Yen/$					
VAR-RV-Break	-0.030 (0.106)	1.090 (0.144)	--	0.330	--
VAR-RV-I(d)	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV-I(d)	-0.024 (0.101)	0.603 (0.564)	0.490 (0.662)	0.331	1.02
VAR-RV-Break + VAR-ABS	-0.058 (0.109)	1.044 (0.148)	0.166 (0.136)	0.331	3.01
VAR-RV-Break + Daily GARCH	-0.103 (0.141)	0.734 (0.131)	0.432 (0.263)	0.348	1.03
VAR-RV-Break + Daily RiskMetrics	-0.048 (0.112)	0.842 (0.102)	0.245 (0.134)	0.340	1.13
VAR-RV-Break + Daily FIEGARCH	-0.279 (0.209)	0.384 (0.260)	0.962 (0.484)	0.385	0.95
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.395 (0.252)	-0.007 (0.375)	1.656 (0.734)	0.380	1.06
DM/Yen					
VAR-RV-Break	-0.047 (0.096)	1.097 (0.132)	--	0.353	--
VAR-RV-I(d)	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV-I(d)	-0.054 (0.099)	0.483 (0.452)	0.650 (0.548)	0.357	1.04
VAR-RV-Break + VAR-ABS	-0.044 (0.094)	1.107 (0.148)	-0.028 (0.140)	0.353	3.60
VAR-RV-Break + Daily GARCH	-0.021 (0.082)	0.816 (0.135)	0.235 (0.167)	0.365	1.16
VAR-RV-Break + Daily RiskMetrics	-0.029 (0.089)	0.860 (0.117)	0.199 (0.121)	0.362	1.21
VAR-RV-Break + Daily FIEGARCH	-0.063 (0.106)	0.978 (0.118)	0.141 (0.143)	0.355	1.01
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.228 (0.156)	0.232 (0.294)	1.197 (0.530)	0.407	1.08

1. In parentheses are standard errors.

Table 7. Out-of-Sample Forecast Evaluation When Future Breaks Are Unknown

	β_0	β_1	β_2	R^2	Rel MSE
DM/$					
VAR-RV-Break	0.057 (0.050)	0.879 (0.088)	--	0.214	--
VAR-RV-I(d)	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV-I(d)	0.018 (0.050)	0.057 (0.179)	0.933 (0.196)	0.249	0.95
VAR-RV-Break + VAR-ABS	0.065 (0.047)	0.895 (0.103)	-0.056 (0.104)	0.214	3.73
VAR-RV-Break + Daily GARCH	-0.002 (0.060)	0.814 (0.131)	0.160 (0.161)	0.216	1.19
VAR-RV-Break + Daily RiskMetrics	0.029 (0.046)	0.814 (0.131)	0.115 (0.116)	0.216	1.18
VAR-RV-Break + Daily FIEGARCH	0.069 (0.051)	0.890 (0.111)	-0.029 (0.108)	0.214	1.33
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.072 (0.059)	0.140 (0.189)	0.888 (0.210)	0.267	1.05
Yen/$					
VAR-RV-Break	-0.040 (0.127)	1.419 (0.218)	--	0.262	--
VAR-RV-I(d)	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV-I(d)	-0.000 (0.120)	-0.042 (0.151)	1.111 (0.146)	0.329	0.67
VAR-RV-Break + VAR-ABS	-0.156 (0.137)	1.242 (0.212)	0.577 (0.155)	0.282	1.98
VAR-RV-Break + Daily GARCH	-0.153 (0.147)	0.688 (0.129)	0.686 (0.203)	0.327	0.68
VAR-RV-Break + Daily RiskMetrics	-0.096 (0.132)	0.841 (0.137)	0.471 (0.108)	0.319	0.74
VAR-RV-Break + Daily FIEGARCH	-0.360 (0.185)	0.458 (0.146)	1.084 (0.295)	0.387	0.62
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.399 (0.190)	-0.371 (0.260)	1.961 (0.450)	0.384	0.70
DM/Yen					
VAR-RV-Break	-0.052 (0.138)	1.486 (0.248)	--	0.227	--
VAR-RV-I(d)	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV-I(d)	-0.030 (0.131)	-0.087 (0.175)	1.190 (0.096)	0.355	0.56
VAR-RV-Break + VAR-ABS	-0.106 (0.142)	1.259 (0.269)	0.484 (0.143)	0.247	1.93
VAR-RV-Break + Daily GARCH	0.004 (0.123)	0.655 (0.162)	0.519 (0.110)	0.325	0.62
VAR-RV-Break + Daily RiskMetrics	-0.041 (0.132)	0.762 (0.177)	0.485 (0.085)	0.324	0.65
VAR-RV-Break + Daily FIEGARCH	-0.140 (0.148)	0.795 (0.194)	0.631 (0.107)	0.303	0.54
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.188 (0.133)	-0.261 (0.192)	1.608 (0.295)	0.407	0.58

1. In parentheses are standard errors.

Table 8. One-Step-Ahead Out-of-Sample Forecast Evaluation for Simulations

		AR-Break Model		AR-I(d) Model
		Known Break	Unknown Break	
Period 4	Root MSE	0.407	0.407	0.411
	Relative MSE	1.02	1.02	1
Period 5	Root MSE	0.422	0.812	0.446
	Relative MSE	1.117	0.301	1
Period 6	Root MSE	0.398	1.272	0.502
	Relative MSE	1.589	0.155	1
Whole Out-of-Sample	Root MSE	0.41	0.894	0.453
	Relative MSE	1.222	0.257	1

Table 9. Ten-Step-Ahead Out-of-Sample Forecast Evaluation for Simulations

		AR-Break Model		AR-I(d) Model
		Known Break	Unknown Break	
Period 4	Root MSE	0.447	0.447	0.448
	Relative MSE	1.03	1.03	1
Period 5	Root MSE	0.452	0.825	0.574
	Relative MSE	1.608	0.484	1
Period 6	Root MSE	0.418	1.283	0.837
	Relative MSE	4.013	0.425	1
Whole Out-of-Sample	Root MSE	0.441	0.912	0.634
	Relative MSE	2.09	0.488	1

1. Root Mean Squared Error (MSE) is calculated as $[\sum (y_{t+10} - y_{t+10|t}^{Model})^2 / N]^{1/2}$.

2. Relative MSE is calculated as $\sum (y_{t+1} - y_{t+1|t}^{I(d)})^2 \Big/ \sum (y_{t+1} - y_{t+1|t}^{Break})^2$

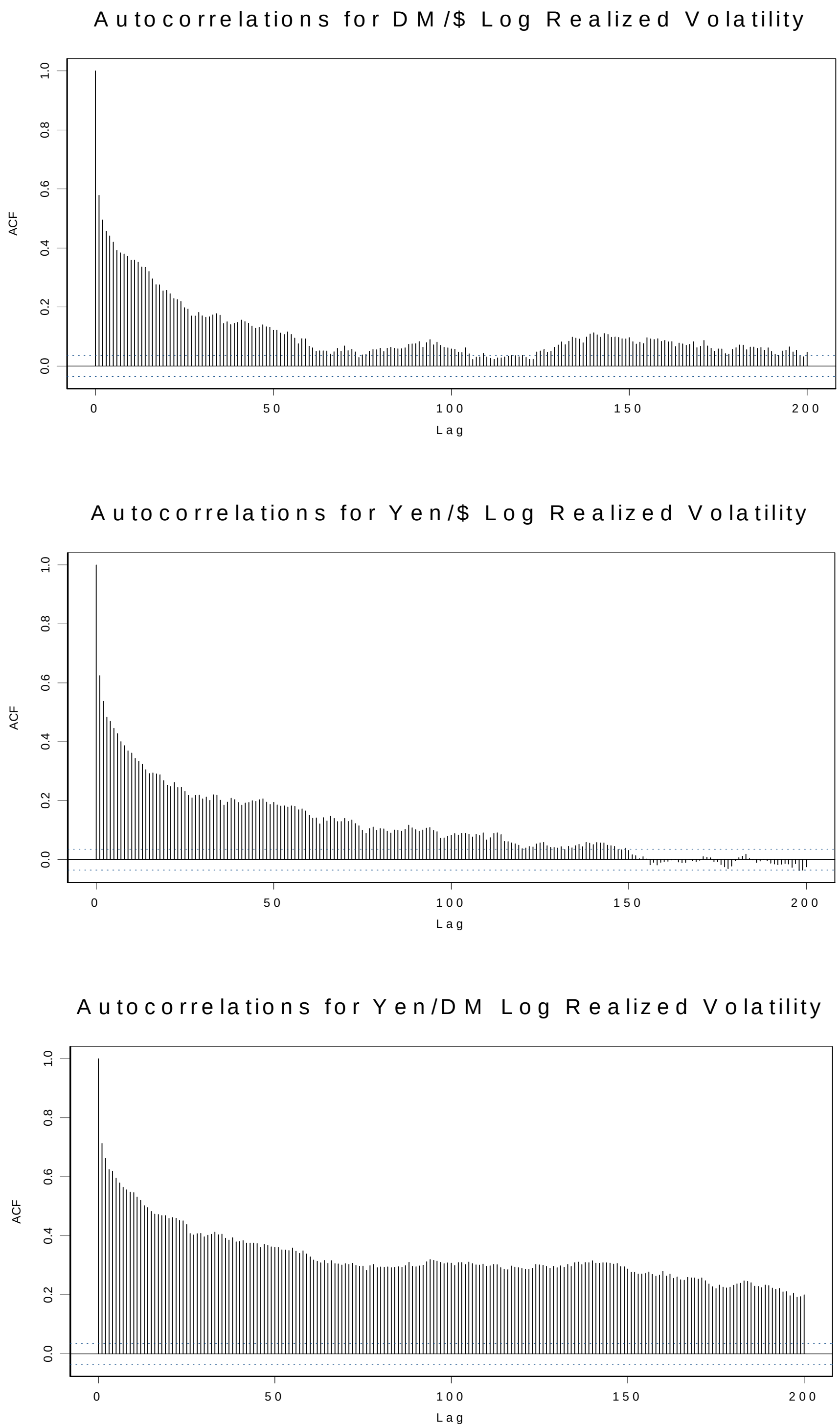


Figure 1. Autocorrelations for Log Realized Volatility

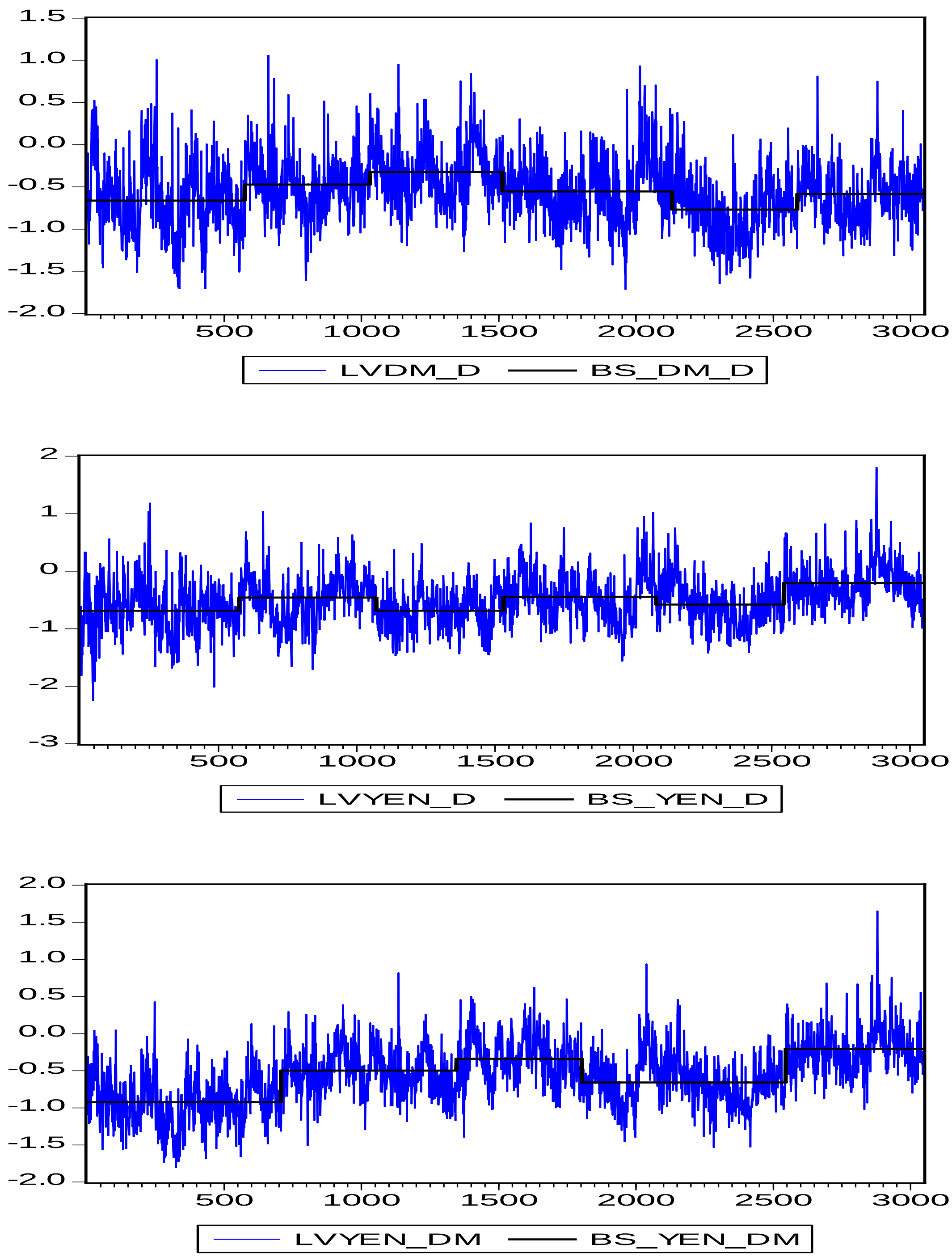


Figure 2. Estimated Structural Breaks Means and Dates for Daily Exchange Rate Log Realized Volatility
(1986.12.2 – 1999.6.30)

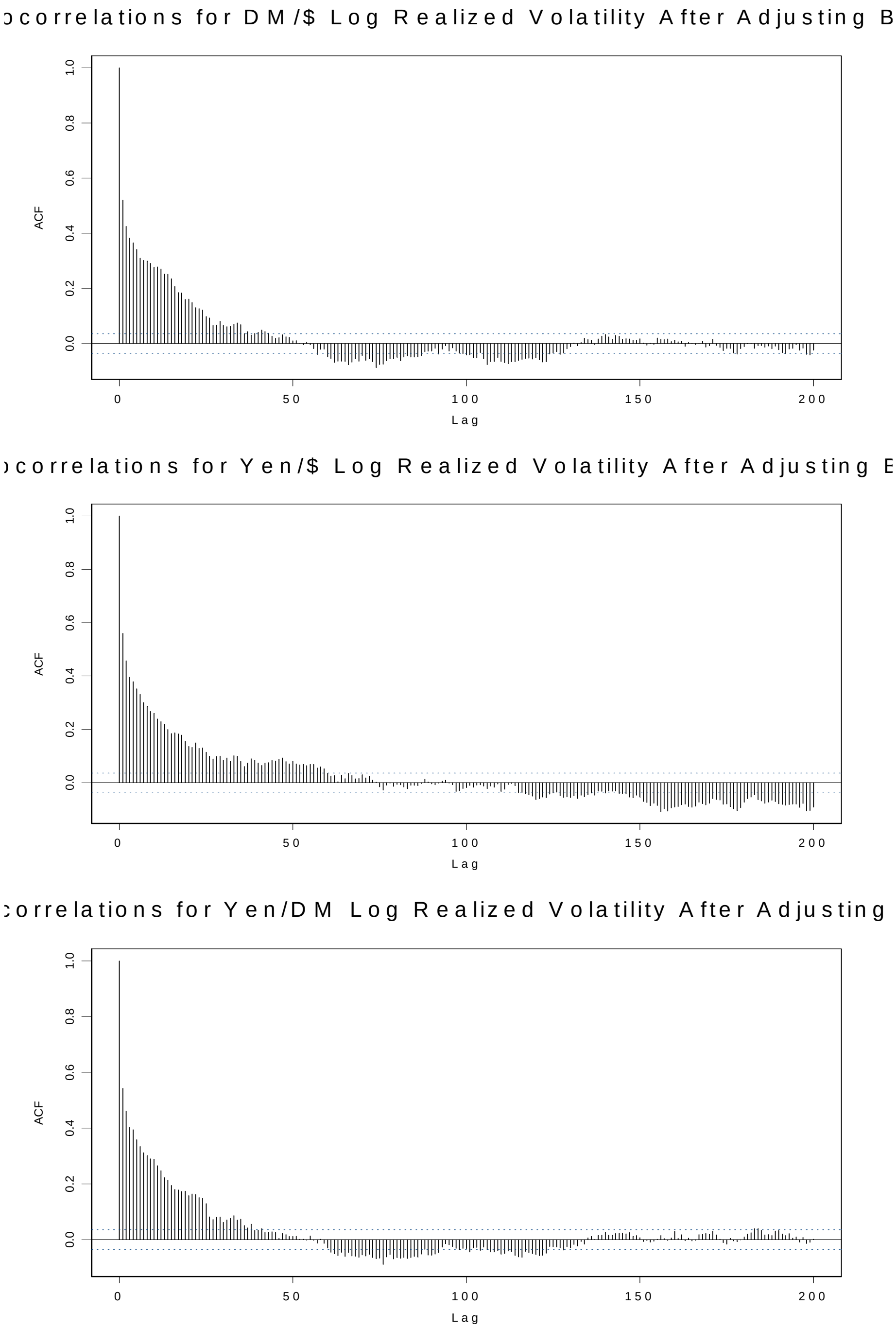
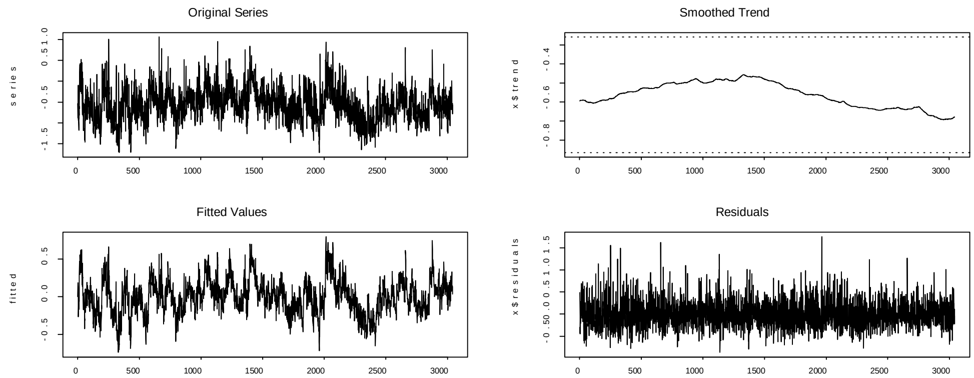
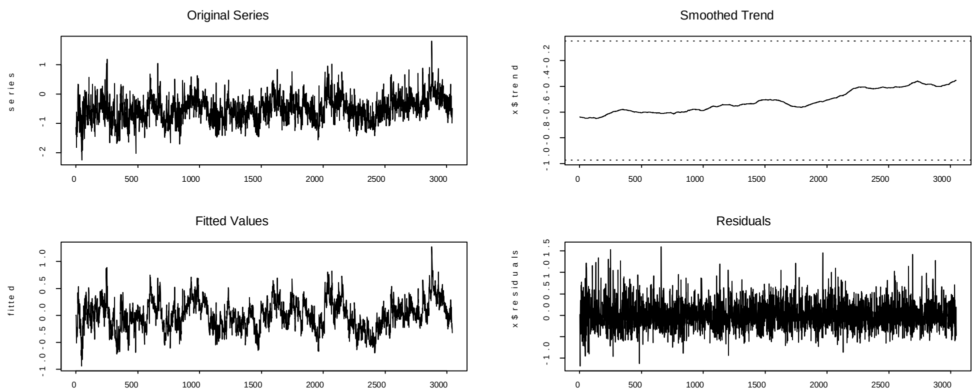


Figure 3. Autocorrelations for Log Realized Volatility After Adjusting for Structural Breaks

DM/$ Log Realized Volatility



Yen/$ Log Realized Volatility



Yen/DM Log Realized Volatility

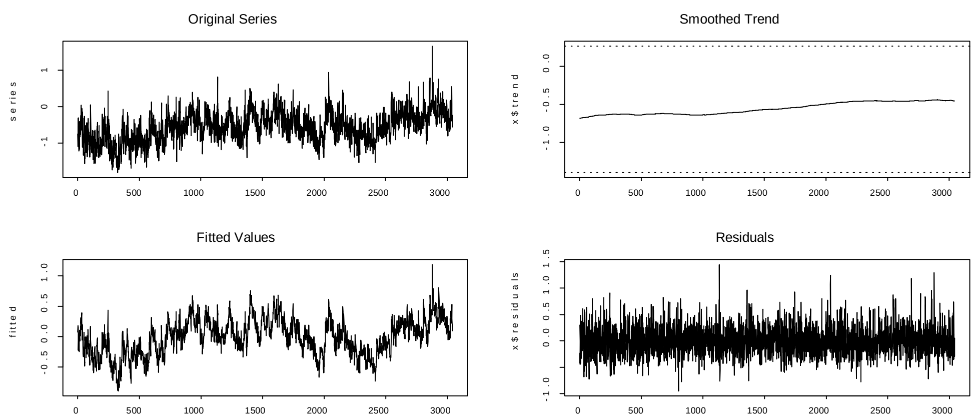


Figure 4. Semiparametric Fractional Autoregressive Model Decomposition

Notes: Based on Beran, Feng and Ocker's method (1998)

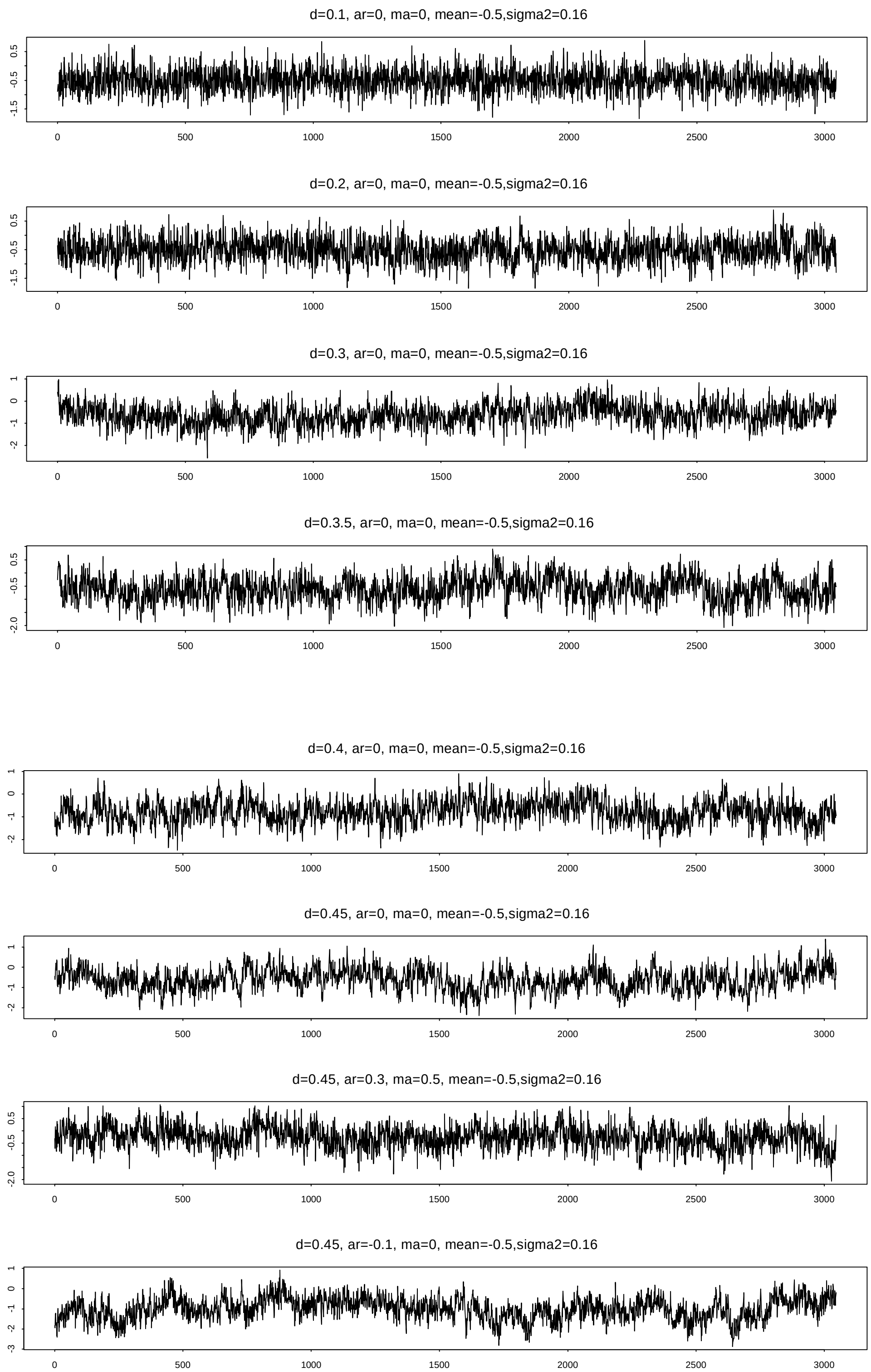


Figure 5. Monte Carlo Simulation for Long Memory Processes

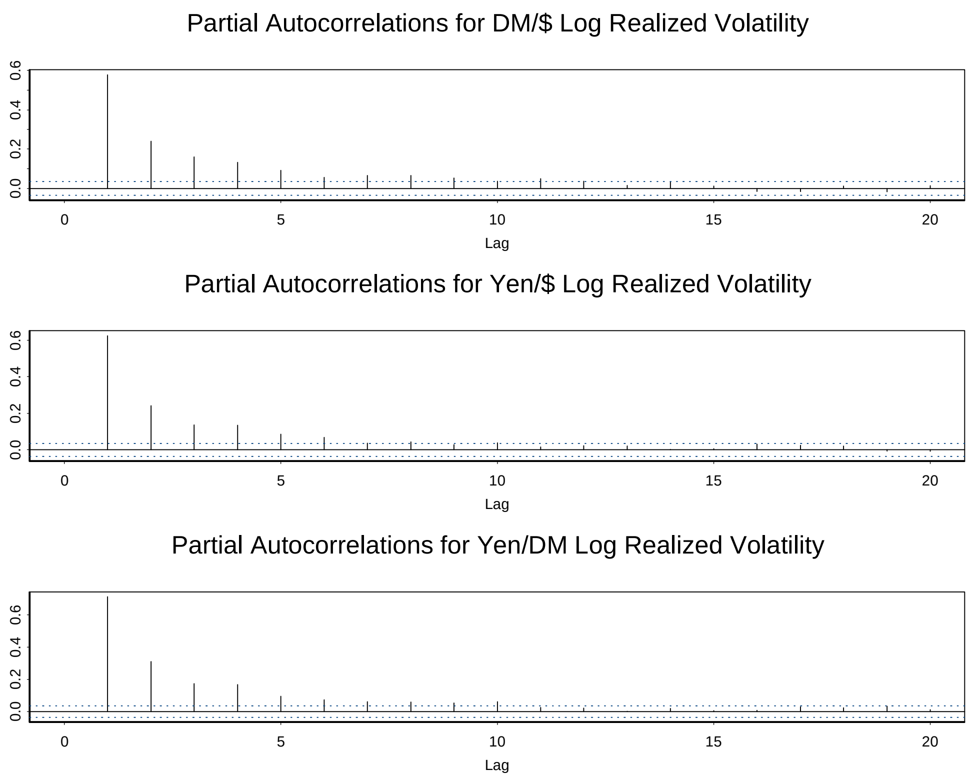


Figure 6. Partial Autocorrelation Function for Log Realized Volatility

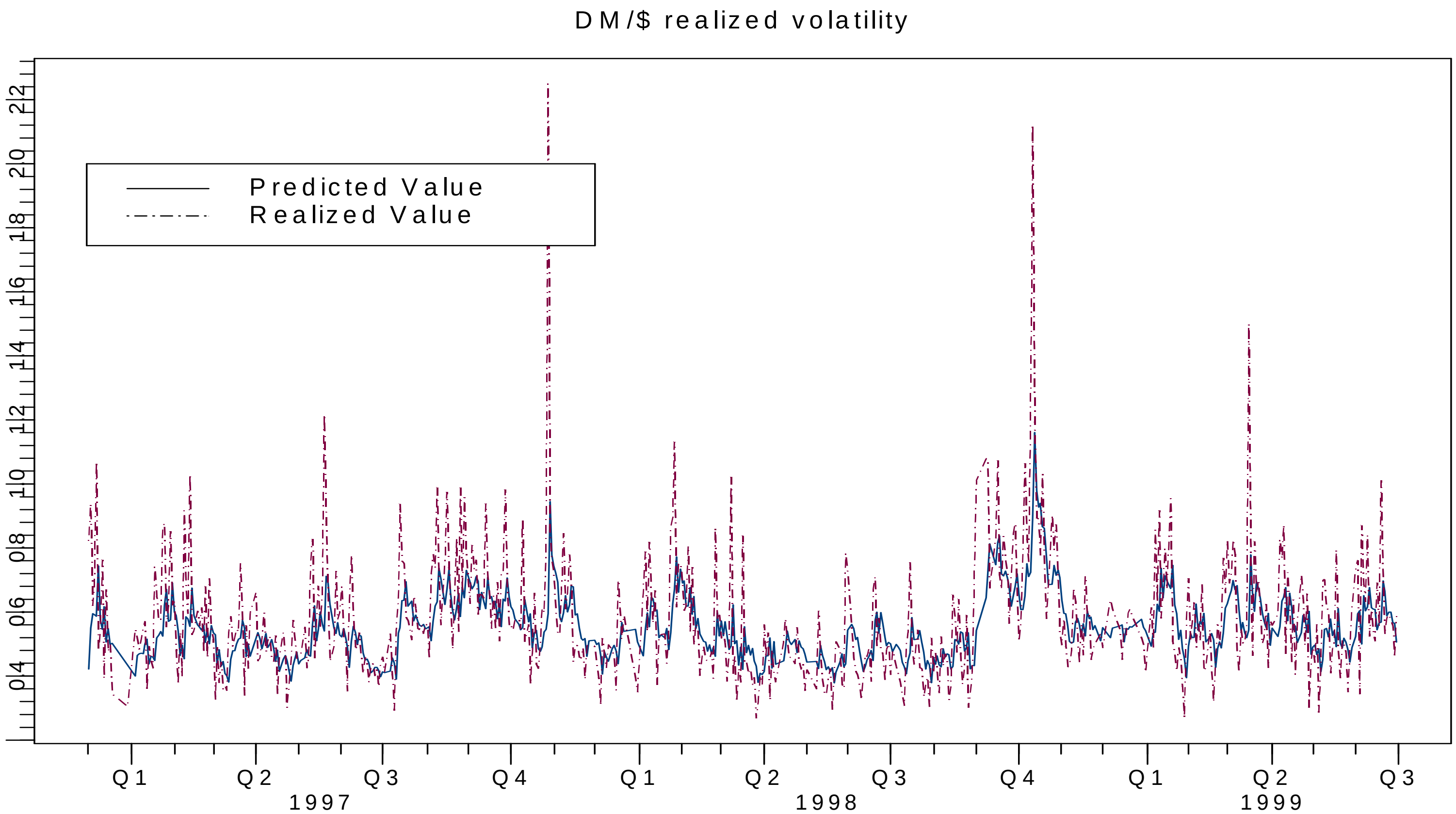


Figure 7.A. Realized Volatility and Out-of-Sample VAR-RV-Break Forecasts

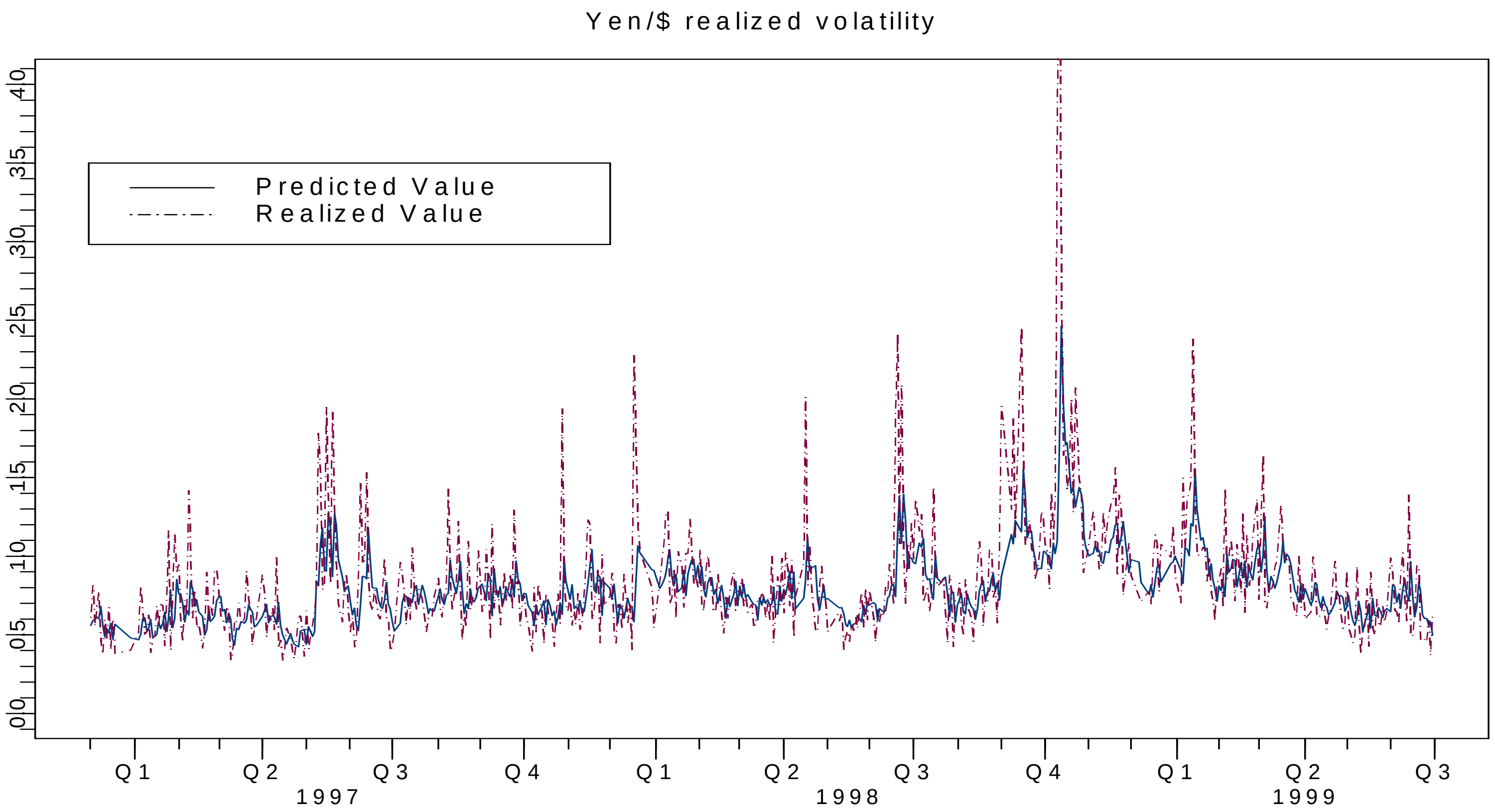


Figure 7.B. (Continued)

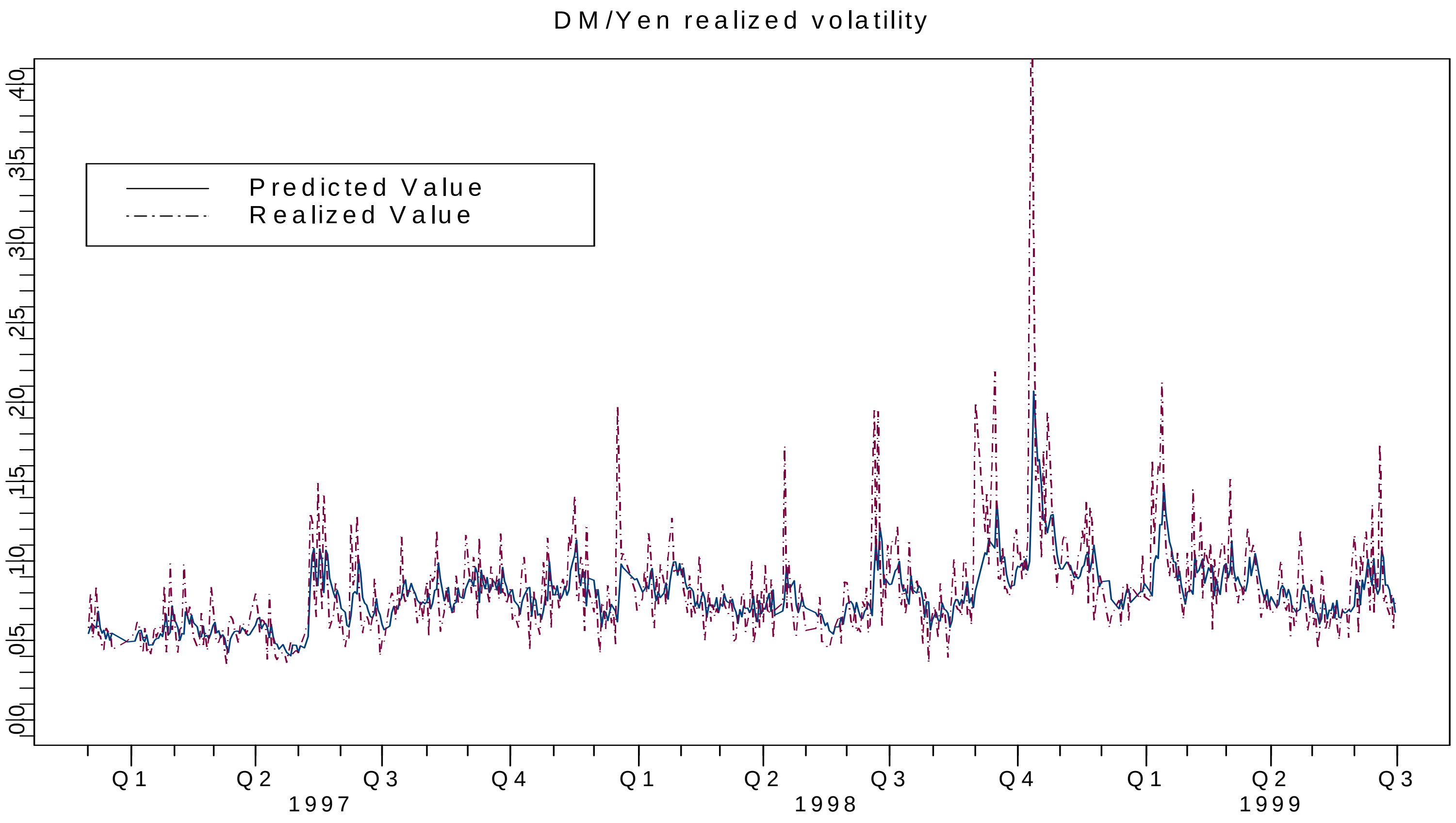
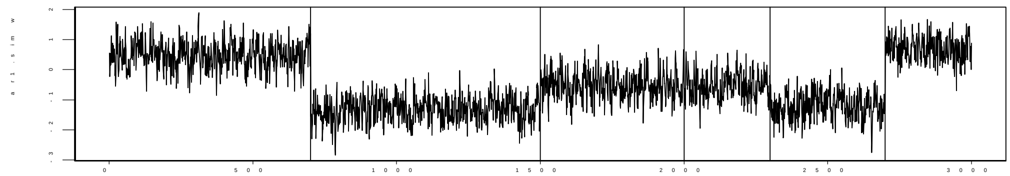
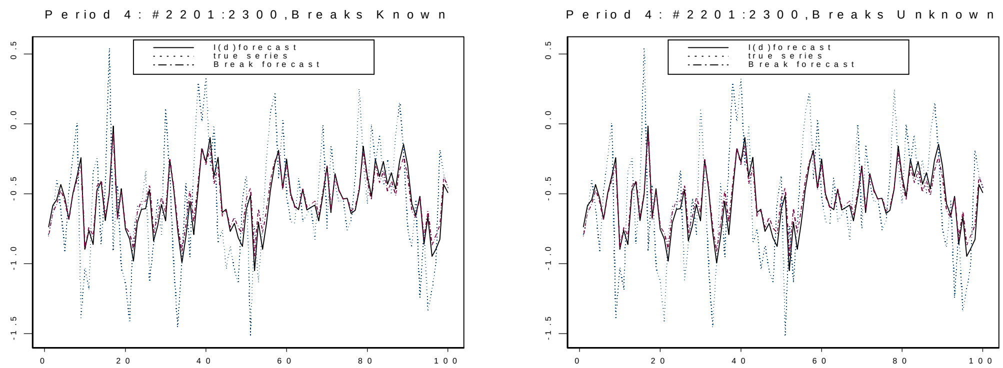


Figure 7.C. (Continued)

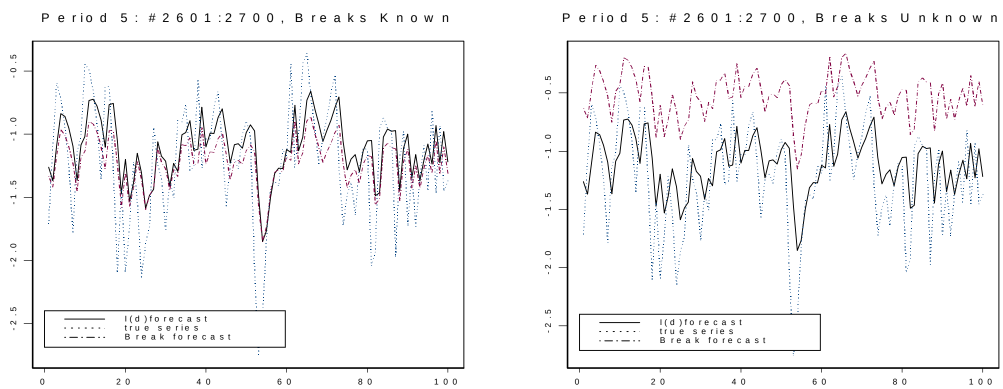
A. Simulated Mean Breaks Series



B.



C.



D.

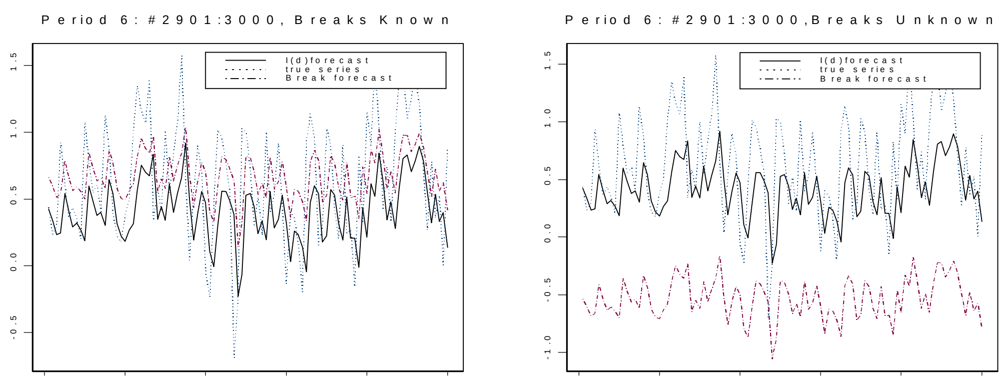


Figure 8. Out of Sample Forecast Evaluation from Simulation