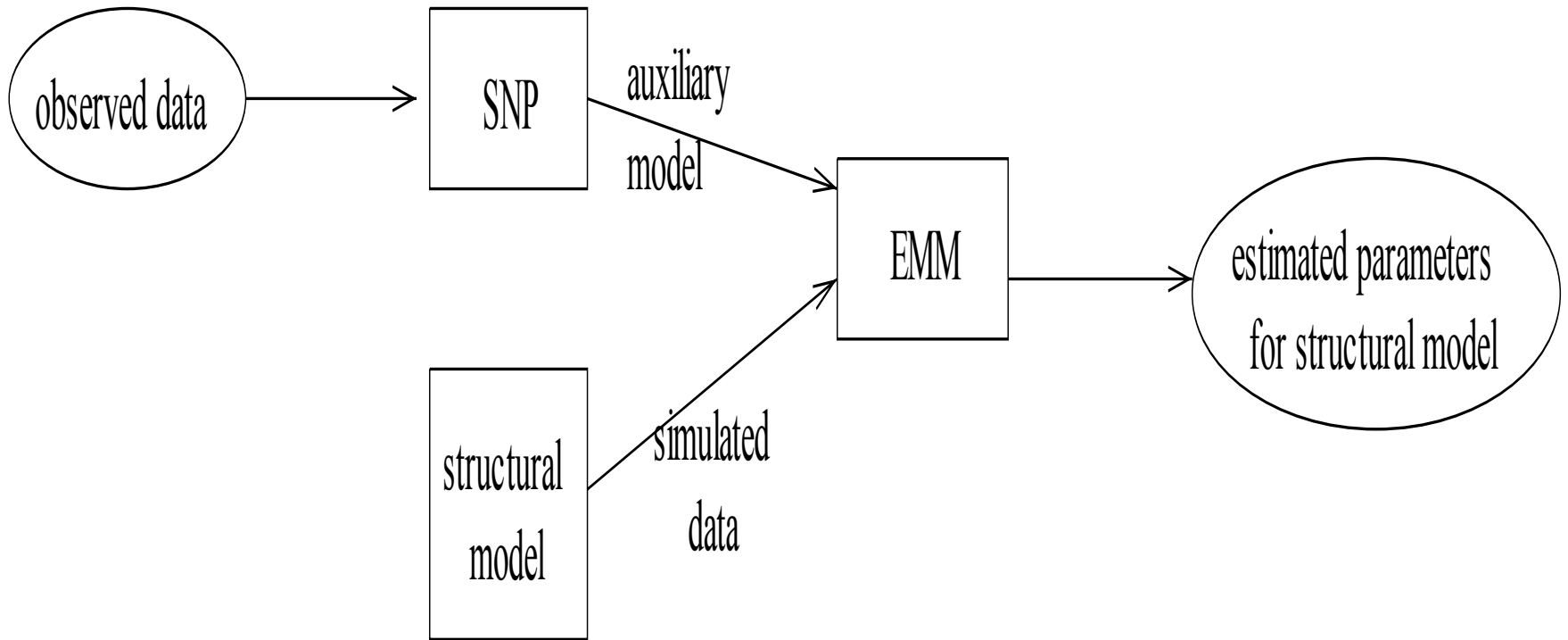


# EMM Flow Diagram



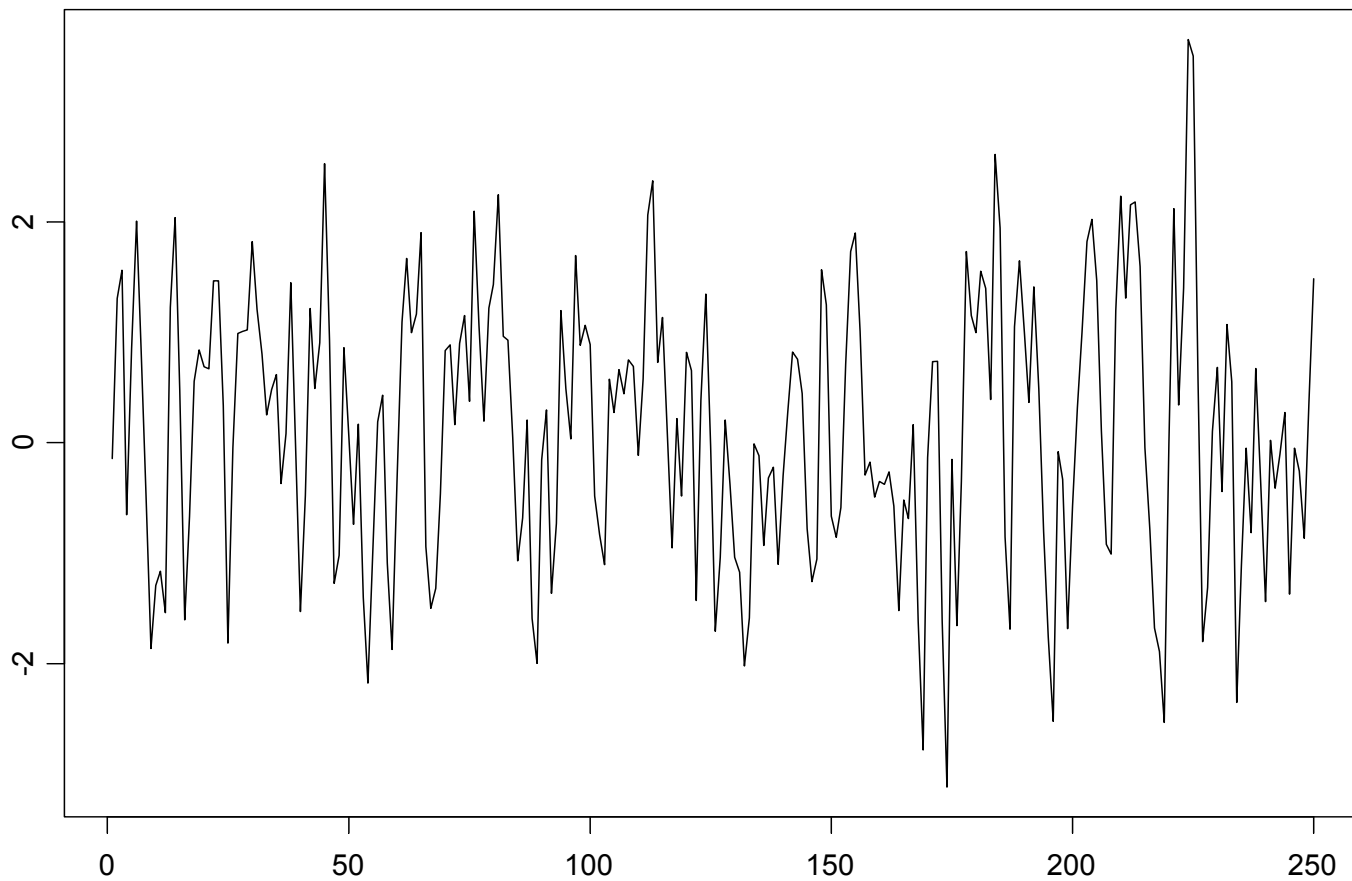
# Simulate MA(1) Data Using `gensim` Function

```
MA1.gensim <- function(rho, n.sim=100, n.var=1, n.burn=25,
                      aux=NULL) {
  # simulate from MA(1) model
  #  $y(t) = e(t) - \theta * e(t-1)$ ,  $e(t) \sim \text{iid } N(0, \sigma^2)$ 
  # rho = (theta, sigma2) '
  # aux is a list with components
  # innov = standard normals used for simulation
  # start.innov = standard normals used for start up values

  ans = arima.sim(model = list(ma=rho[1]),
                 innov = aux$innov*sqrt(rho[2]),
                 start.innov = aux$innov.start*sqrt(rho[2]))
  ans
}
```

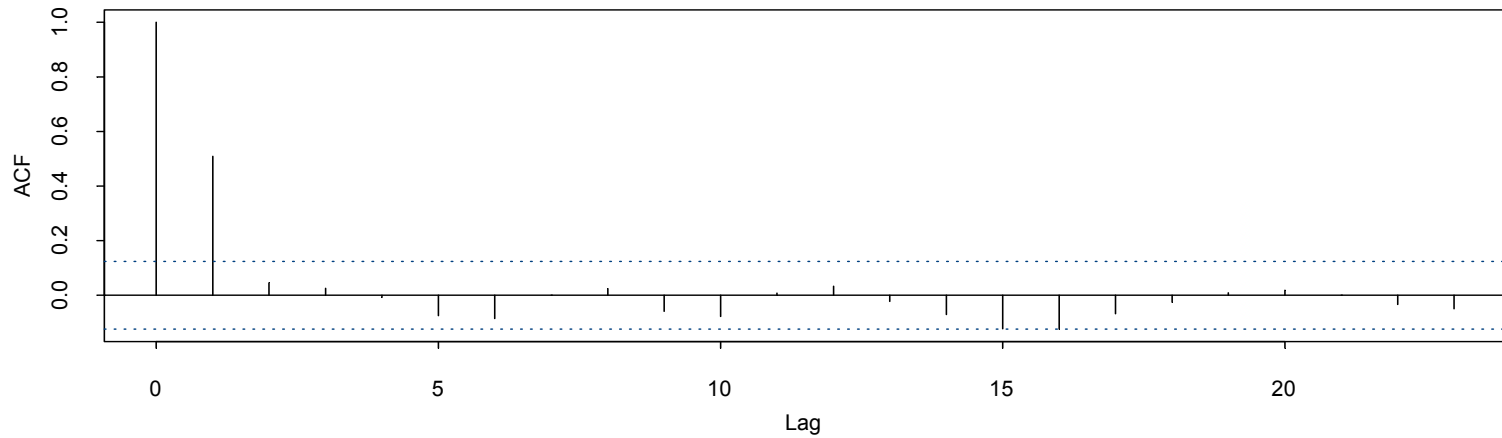
# MA(1) Data

$\theta = -0.75$ ,  $\sigma = 1$

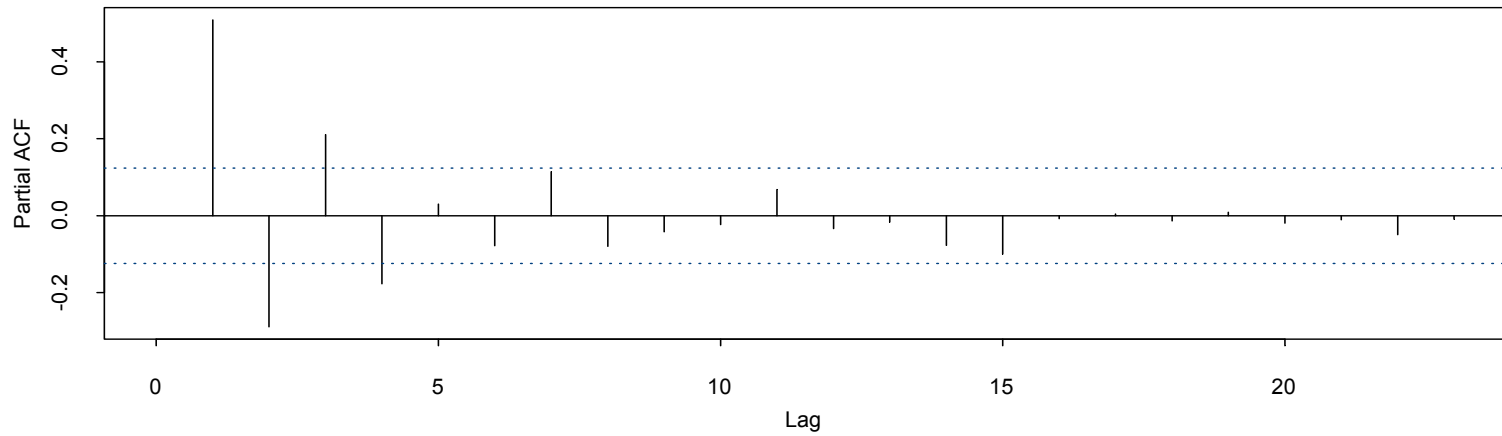


# MA(1) Data

Series : MA1.sim



Series : MA1.sim



# MLE of MA(1)

```
> MA1.data = MA1.sim - mean(MA1.sim)
> mle.fit = arima.mle(MA1.data,model=list(ma=-0.5))
> mle.fit
```

Coefficients:

```
MA : -0.76773
```

```
> mle.fit$sigma2
[1] 0.8854201
```

```
> sqrt(mle.fit$var.coef)
      ma(1)
ma(1) 0.04052596
```

# Fit AR(3) Auxiliary Model

```
> ar3.fit = SNP(data=MA1.sim, model=SNP.model(ar=3))
> class(ar3.fit)
[1] "SNP"
> summary(ar3.fit)
Model: Gaussian VAR
```

Conditional Mean Coefficients:

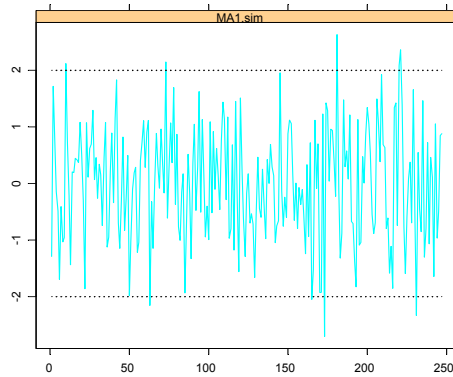
	mu	ar(1)	ar(2)	ar(3)
coef	-0.0039	0.7205	-0.4307	0.2114
(std.err)	0.0517	0.0626	0.0672	0.0630
(t.stat)	-0.0749	11.5170	-6.4077	3.3549

Conditional Variance Coefficients:

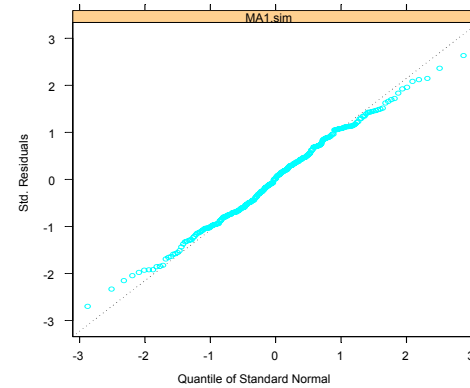
	sigma2
coef	0.8052
(std.err)	0.0424
(t.stat)	19.0124

# Auxiliary Model Diagnostics

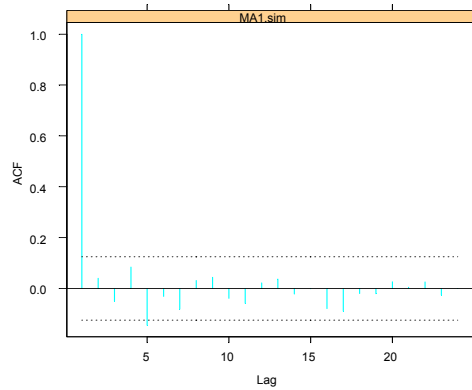
Std. Residuals versus Time



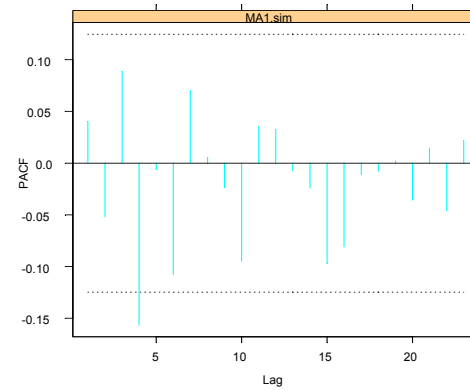
Normal Q-Q Plot



Std. Residual ACF



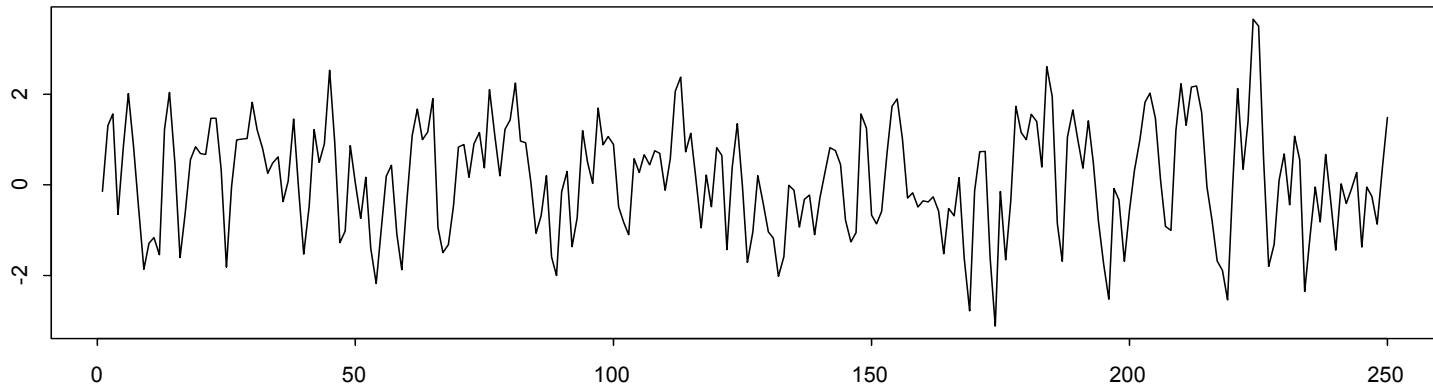
Std. Residual PACF



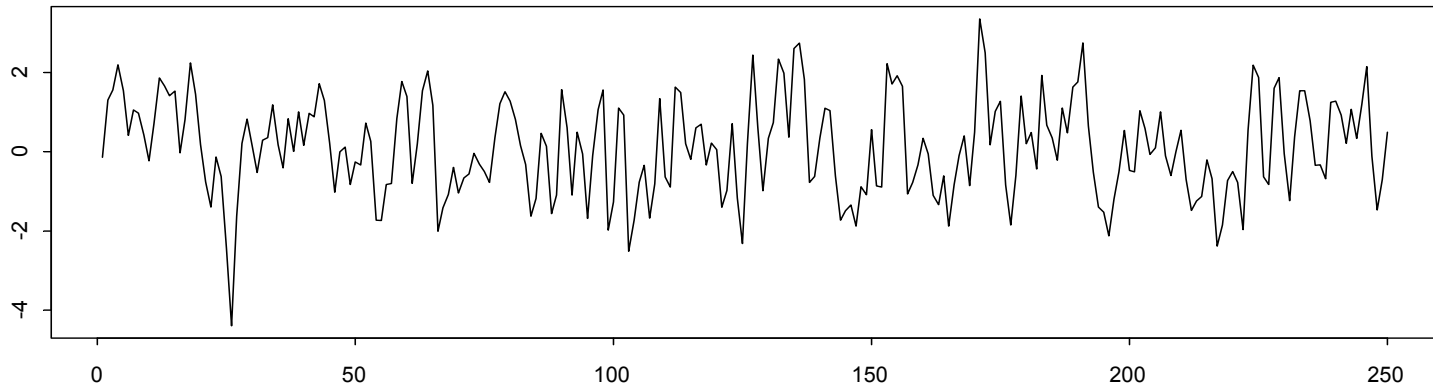
# Simulation from Auxiliary Model

```
ar3.sim = simulate(ar3.fit)
```

actual MA(1) data



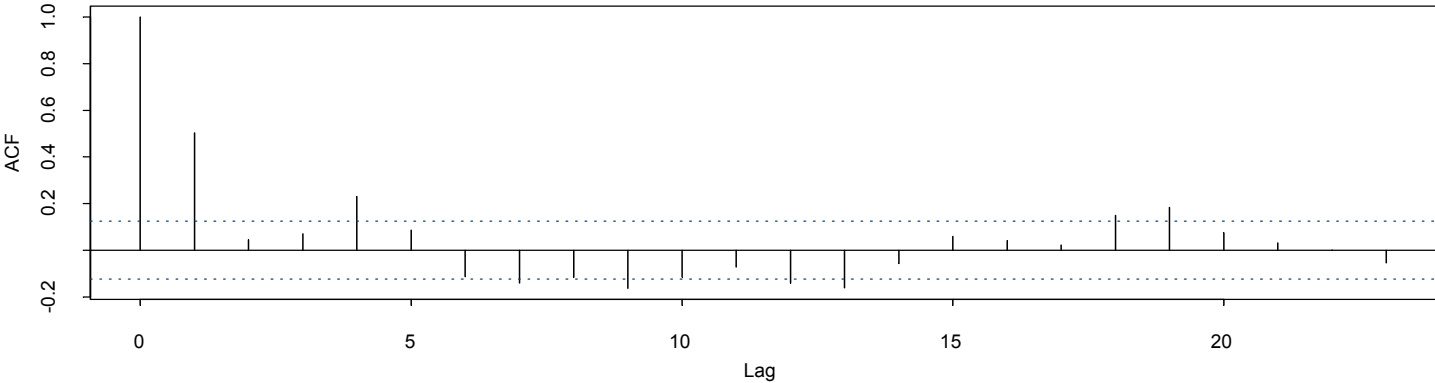
simulated data from fitted AR(3) model



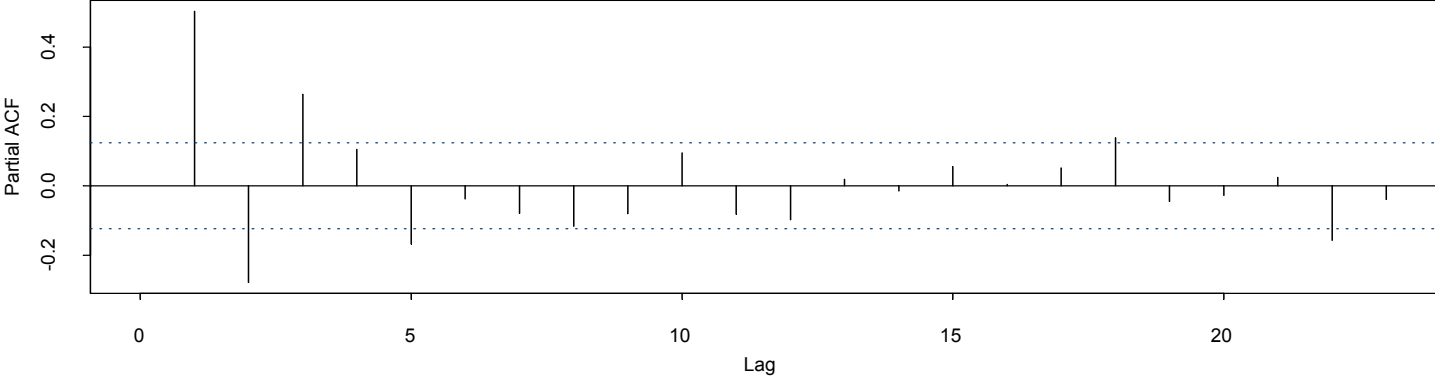


# Diagnostics from Simulated Auxiliary Model

Series : ar3.sim



Series : ar3.sim



# EMM Fit of MA(1) Model

```
# create inputs to MA1.gensim
> set.seed(345)
> z = rnorm(10000)
> z.start = rnorm(100)
> MA1.aux = list(innov = z, innov.start = z.start)

# fit MA(1) model using EMM
> EMM.fit = EMM(ar3.fit,coef=c(-0.5,1),
+ control = EMM.control(n.burn=100,n.sim=10000),
+ gensim.fn="MA1.gensim2",gensim.language="SPLUS",
+ gensim.aux=MA1.aux)

> class(EMM.fit)
[1] "EMM"
```

# EMM Fit of MA(1) Model

```
> EMM.fit
```

```
Call:
```

```
EMM(score = ar3.fit, coef = c(-0.5, 1), control =  
EMM.control(n.burn = 100, n.sim = 10000), gensim.fn =  
"MA1.gensim2",  
gensim.language = "SPLUS", gensim.aux = MA1.aux)
```

```
Coefficients:
```

	Value	Std. Error	95% Conf. Int.	
theta	-0.7812	0.11832	-1.1729	-0.6298
sigma	0.9486	0.04586	0.8205	1.0170

```
Final optimization:
```

```
Convergence: relative function convergence
```

```
Iterations: 8
```

```
Normalized objective at final iteration: 0.9392
```

```
P-value on 3 df is 0.816
```

# EMM Diagnostics

## EMM Diagnostics

	Score	Std. Error	t-ratio
mu	1.15746942	1.241852	0.932051340
phi.1	0.03844991	1.267703	0.030330384
phi.2	0.28548072	1.359517	0.209986938
phi.3	0.17188357	1.197335	0.143555120
sigma2	0.01452084	1.545968	0.009392714

# Fit SNP 200100 Model

```
> fit.200100 = SNP(tb3mo,model=SNP.model(ar=2),n.drop=6)
> summary(fit.200100)
```

Model: Gaussian VAR

Conditional Mean Coefficients:

	mu	ar(1)	ar(2)
coef	0.0006	1.0911	-0.0969
(std.err)	0.0036	0.0107	0.0109
(t.stat)	0.1661	101.9655	-8.8810

Conditional Variance Coefficients:

	sigma
coef	0.0974
(std.err)	0.0006
(t.stat)	153.7107

# Fit SNP 204100 Model

```
> fit.204100 = expand(fit.200100, arch=4)
> summary(fit.204100)
```

Model: Gaussian ARCH

Conditional Mean Coefficients:

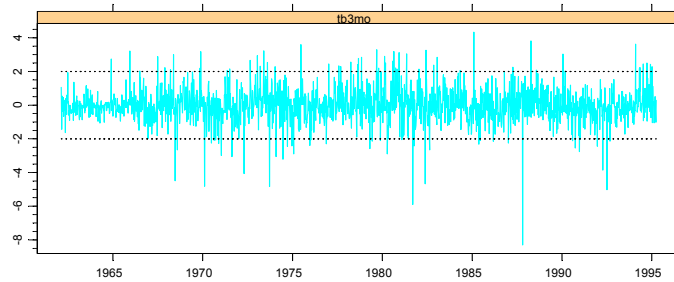
	mu	ar(1)	ar(2)
coef	0.0009	1.0189	-0.0215
(std.err)	0.0008	0.0188	0.0190
(t.stat)	1.2491	54.3261	-1.1321

Conditional Variance Coefficients:

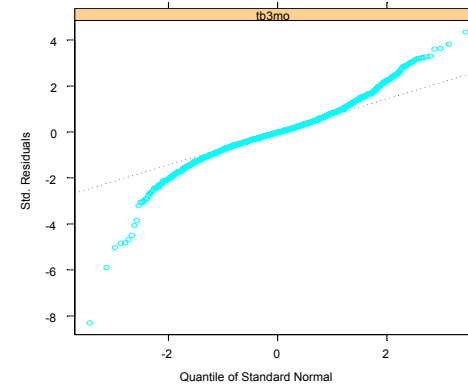
	s0	arch(1)	arch(2)	arch(3)	arch(4)
coef	0.0211	0.4324	0.1358	0.3126	0.2879
(std.err)	0.0007	0.0257	0.0191	0.0224	0.0231
(t.stat)	28.5737	16.8110	7.1039	13.9848	12.4683

# Diagnostics for SNP 204100

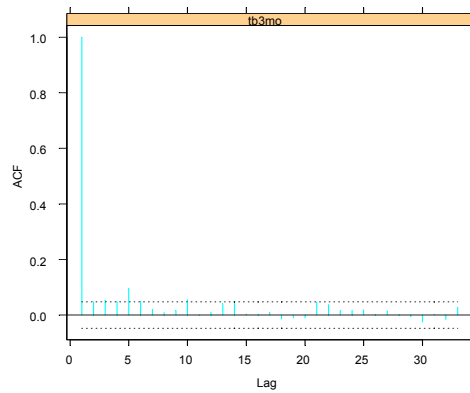
Std. Residuals versus Time



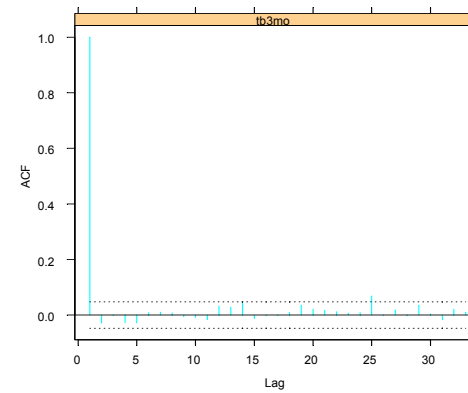
Normal Q-Q Plot



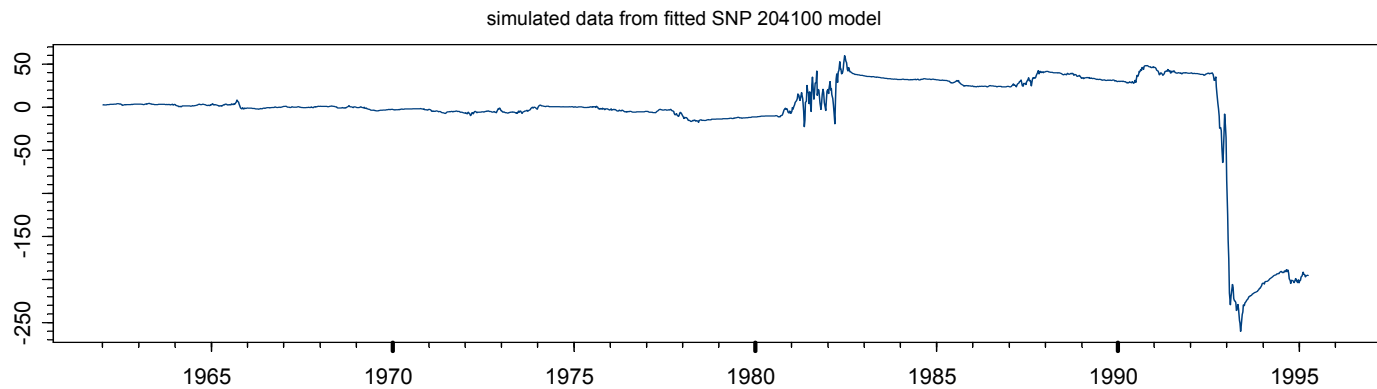
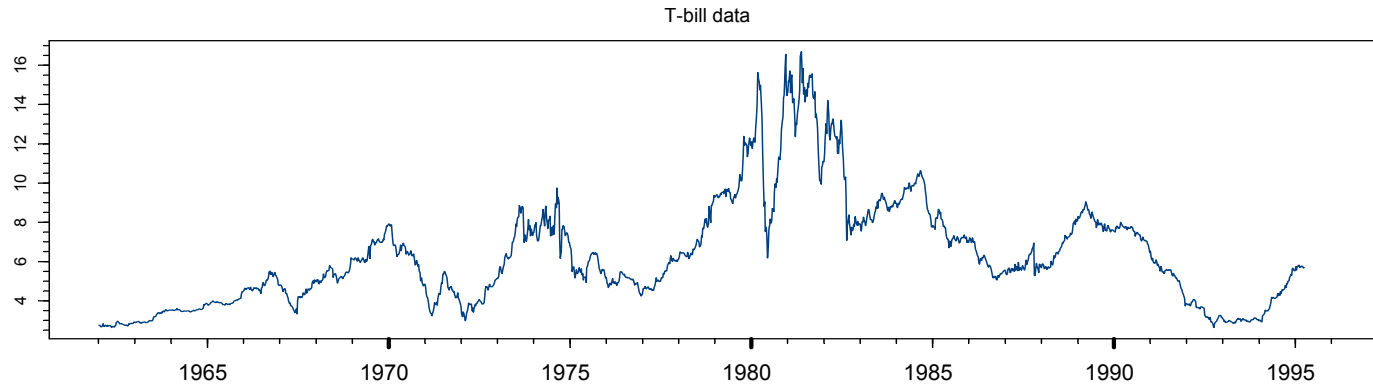
Std. Residual ACF



Std. Residual<sup>2</sup> ACF



# Simulation from SNP 204100





# Fit SNP 211100 Model

```
> fit.211100 = expand(fit.200100, arch=1, garch=1)
> summary(fit.211100)
```

Model: Gaussian GARCH

Conditional Mean Coefficients:

	mu	ar(1)	ar(2)
coef	-0.0014	1.0280	-0.0307
(std.err)	0.0009	0.0230	0.0230
(t.stat)	-1.5642	44.7881	-1.3380

Conditional Variance Coefficients:

	s0	arch(1)	garch(1)
coef	0.0023	0.2037	0.8383
(std.err)	0.0001	0.0085	0.0050
(t.stat)	27.0769	23.9992	167.7730

# Fit SNP 211140 Model

```
> fit.211140 = expand(fit.211100, zPoly=4)
> summary(fit.211140)
```

Model: Semiparametric GARCH

Hermite Polynomial Coefficients:

	z <sup>0</sup>	z <sup>1</sup>	z <sup>2</sup>	z <sup>3</sup>	z <sup>4</sup>
coef	1.0000	0.0754	-0.2025	-0.0065	0.0249
(std.err)		0.0242	0.0163	0.0051	0.0025
(t.stat)		3.1098	-12.4038	-1.2911	10.1074

Conditional Mean Coefficients:

	mu	ar(1)	ar(2)
coef	-0.0065	1.0425	-0.0482
(std.err)	0.0016	0.0251	0.0252
(t.stat)	-4.0240	41.5080	-1.9124

Conditional Variance Coefficients:

	s0	arch(1)	garch(1)
coef	0.0021	0.2362	0.8367
(std.err)	0.0001	0.0138	0.0069
(t.stat)	15.2416	17.1264	121.0534

# Fit SNP 211141 Model

```
> fit.211141 = expand(fit.211140,xPoly=1)
> fit.211141
```

Model: Nonlinear  
Nonparametric

Conditional Mean Coefficients:  
mu ar(1) ar(2)  
-0.0083 1.0382 -0.0469

Hermite Polynomial

Coefficients:

	x^0	x^1
z^0	1.0000	0.3545
z^1	0.0961	0.0787
z^2	-0.1522	0.0291
z^3	-0.0075	-0.0086
z^4	0.0222	0.0007

Conditional Variance Coefficients:

	s0	arch(1)	garch(1)
	0.0028	0.2034	0.8303

Information Criteria:

	BIC	HQ	AIC	logL
	-1.4444	-1.4593	-1.468	2553.258

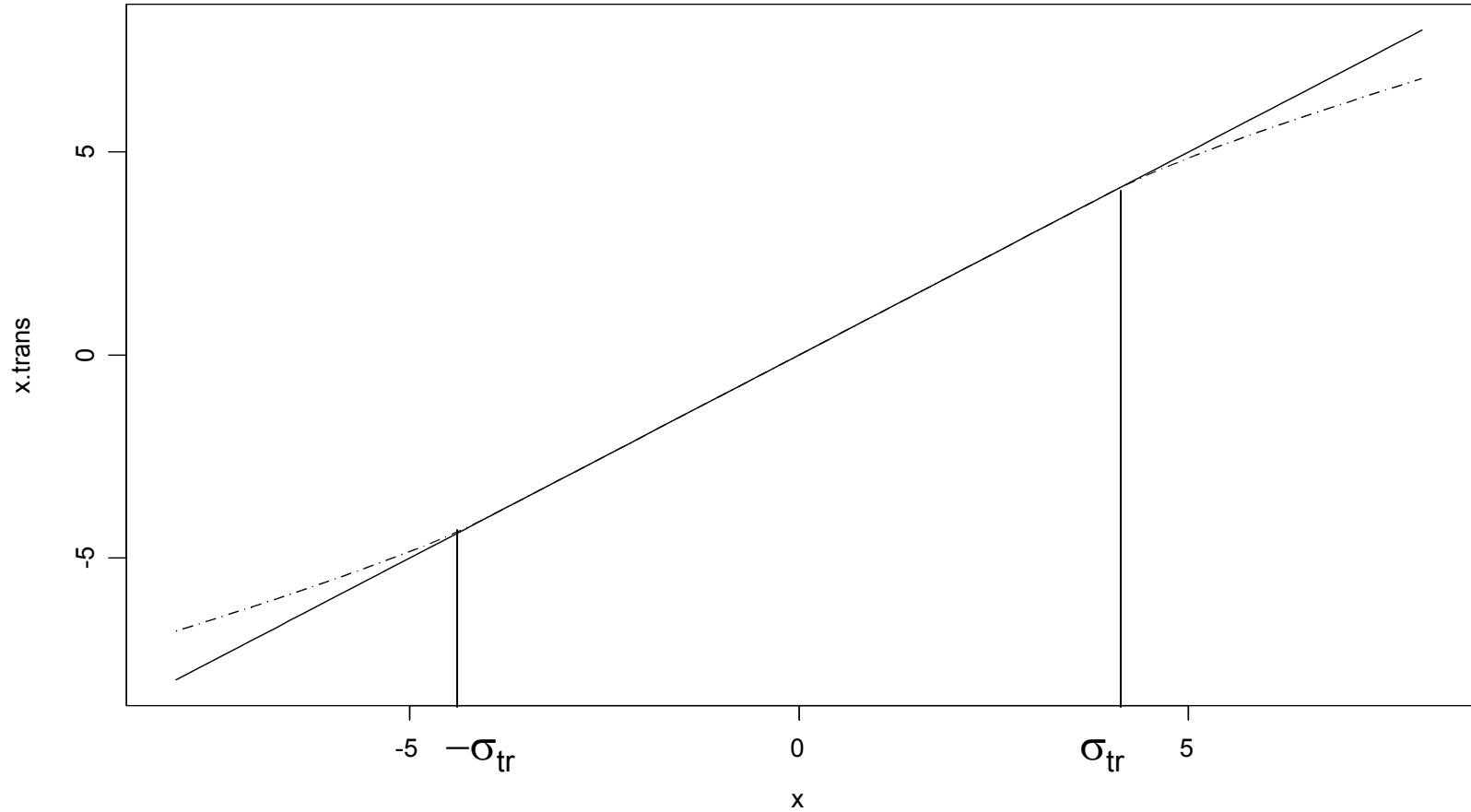
# SNP Tuning Parameters

Parameter	SPLUS argument	Interpretation
$L_u$	ar	VAR lag
$L_r$	arch	ARCH lag
$L_g$	garch	GARCH lag
$K_z$	zPoly	Degree of z in P(z,x)
$K_x$	xPoly	Degree of x in P(z,x)
$L_p$	lagP	Lags in x part of Polynomial P(z,x)

# Taxonomy of SNP Models

Parameter Setting	Auxiliary model for $y_t$
$L_u=0, L_g=0, L_r=0, L_p \geq 0, K_z=0, K_x=0$	iid Gaussian
$L_u > 0, L_g=0, L_r=0, L_p \geq 0, K_z=0, K_x=0$	Gaussian VAR
$L_u > 0, L_g=0, L_r=0, L_p \geq 0, K_z > 0, K_x=0$	Semiparametric VAR
$L_u \geq 0, L_g=0, L_r > 0, L_p \geq 0, K_z=0, K_x=0$	Gaussian ARCH
$L_u \geq 0, L_g=0, L_r > 0, L_p \geq 0, K_z > 0, K_x=0$	Semiparametric ARCH
$L_u \geq 0, L_g > 0, L_r=0, L_p \geq 0, K_z=0, K_x=0$	Gaussian GARCH
$L_u \geq 0, L_g > 0, L_r=0, L_p \geq 0, K_z > 0, K_x=0$	Semiparametric GARCH
$L_u \geq 0, L_g \geq 0, L_r \geq 0, L_p > 0, K_z > 0, K_x > 0$	Nonlinear nonparametric

# Log Spline Transformation



# Fit SNP 204100 with Spline Transformation

```
> fit.204100.s =  
+ SNP(tb3mo,model=SNP.model(ar=2,arch=4),n.drop=6,  
+ control=SNP.control(xTransform="spline",inflection=4))  
> fit.240100.s
```

Model: Gaussian ARCH

Conditional Mean Coefficients:

mu	ar(1)	ar(2)
0.0009	1.0189	-0.0215

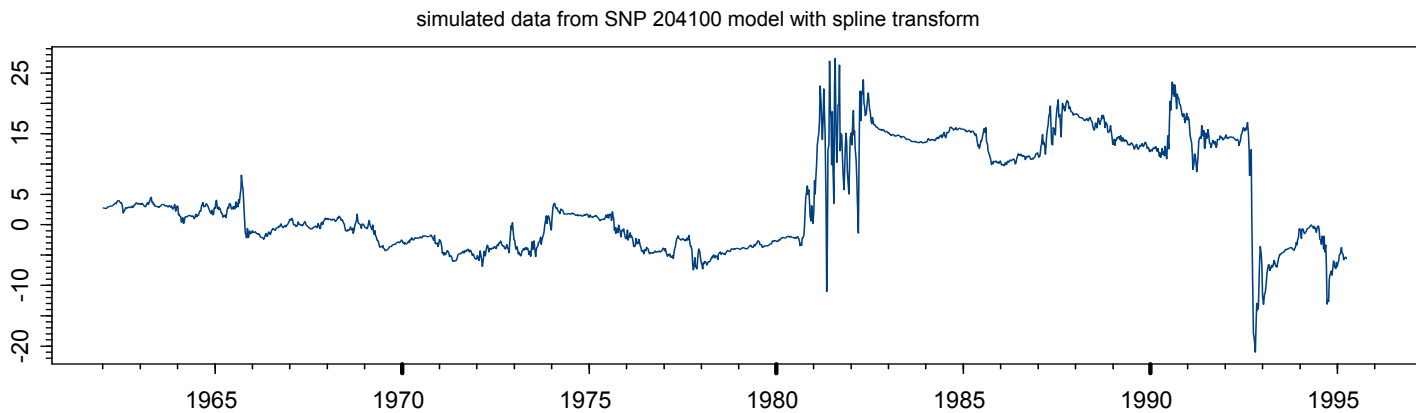
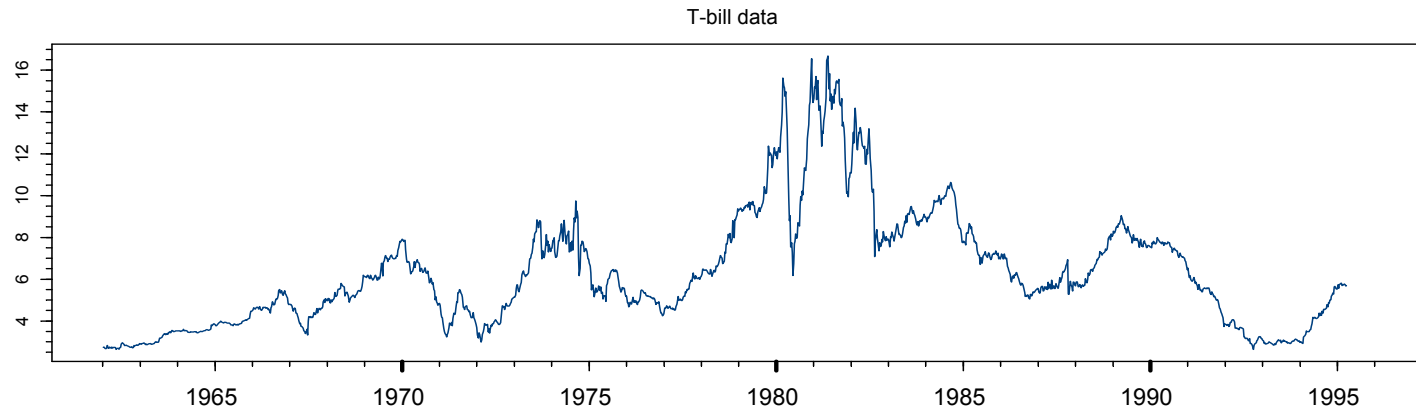
Conditional Variance Coefficients:

s0	arch(1)	arch(2)	arch(3)	arch(4)
0.0211	0.4324	0.1358	0.3126	0.2879

Information Criteria:

BIC	HQ	AIC	logL
-1.3418	-1.3498	-1.3544	2349.84

# Simulation from SNP 204100 with Spline Transformation





# SNP: Automatic Model Selection

```
> fit.auto = SNP.auto(tb3mo,n.drop=6,  
+ control=SNP.control(xTransform="spline",  
+ inflection=4),  
+ arMax=4,zPolyMax=8,xPolyMax=4,lagPMax=4)
```

Initializing using a Gaussian model ...

Expanding the order of VAR: 1234

Expanding toward GARCH model ...

Expanding the order of z-polynomial:

12345678

Expanding the order of x-polynomial: 1234

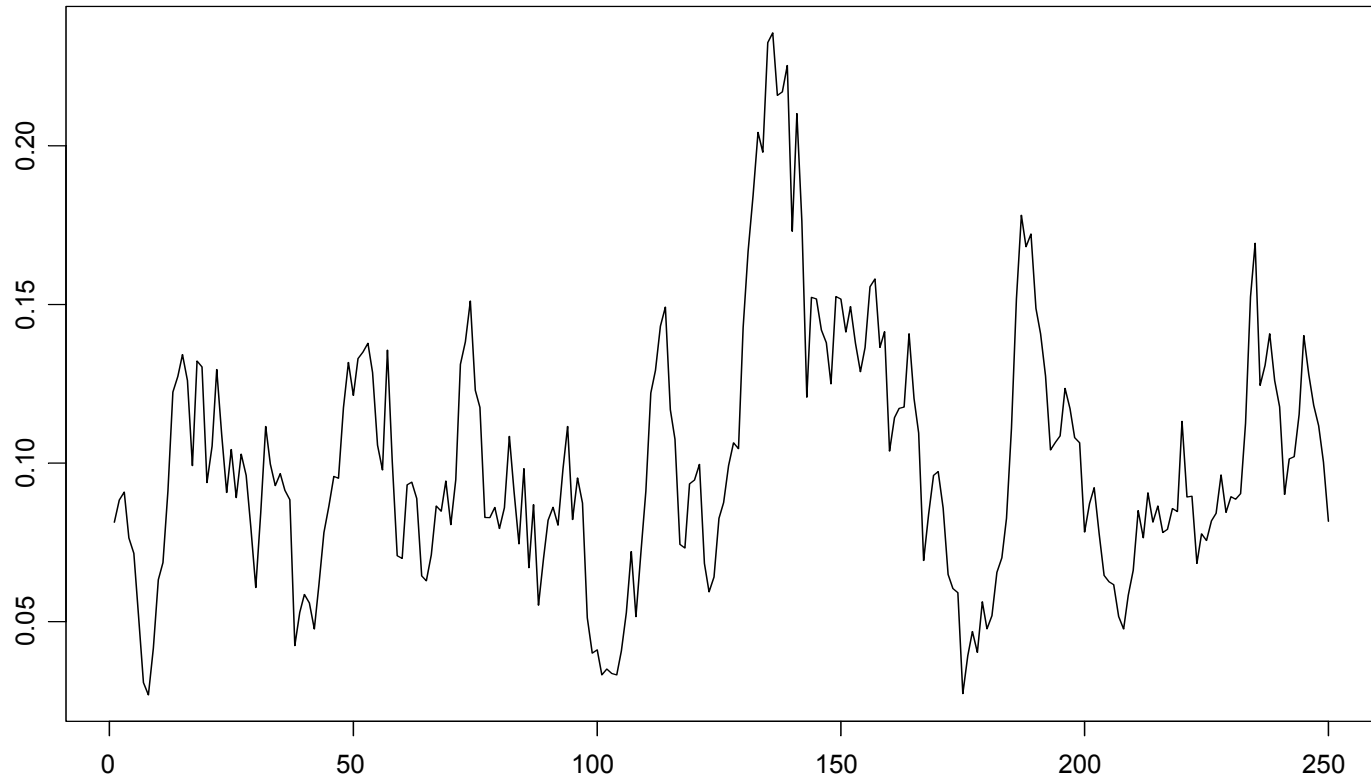
# Simulation from CIR Model

```
# auxiliary parameters for simulator
> cir.aux = euler.pcode.aux(ndt = 100, seed=0,
+ lbound=0, ubound=100, x0=0.1,
+ drift.expr=expression(kappa*(theta-X)),
+ diffuse.expr=expression(sigma*sqrt(X)),
+ rho.names=c("kappa", "theta", "sigma"))

# model parameters
> rho.cir <- c(0.1,0.08,0.06)
> n.sim <- 250; n.burn <- 25; ndt <- 100

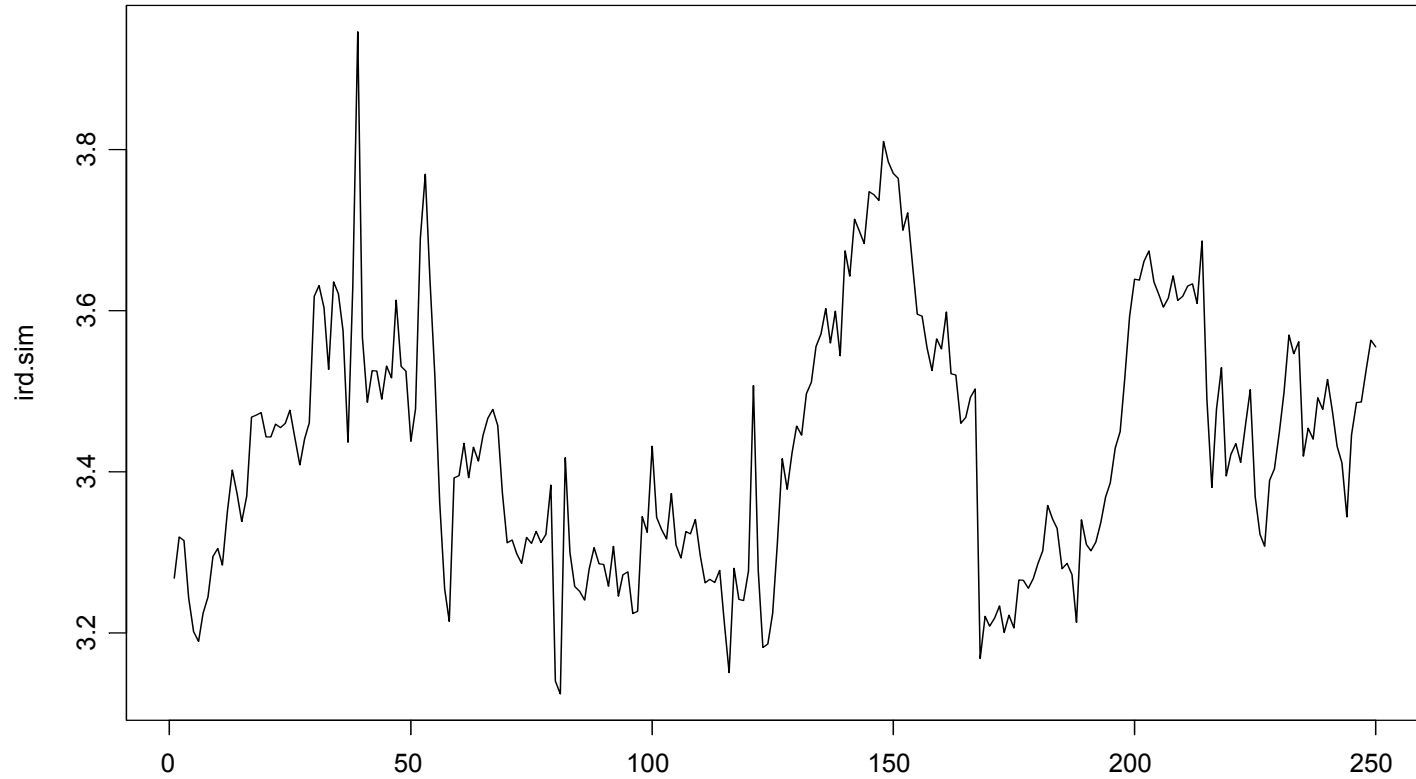
# Simulate using Euler's method
> cir.sim <- euler1d.pcode.gensim(rho=rho.cir,
+ n.sim=250,n.burn=n.burn,aux=cir.aux)
```

# Simulation from CIR Model





# Simulated 2 Factor Model



# Fit SNP Model to Simulated CIR data

```
> SNP.auto.cir = SNP.auto(cir.sim, arMax=8, n.drop=9,  
+ control=SNP.control(xTransform="spline"))
```

```
Initializing using a Gaussian model ...
```

```
Expanding the order of VAR: 12345678
```

```
Expanding toward GARCH model ...
```

```
Expanding the order of z-polynomial: 12345678
```

```
Expanding the order of x-polynomial: 1234
```

```
> SNP.auto.cir
```

```
Model: Gaussian VAR
```

```
Conditional Mean Coefficients:
```

```
      mu  ar(1)  
0.0075 0.8886
```

```
Conditional Variance Coefficients:
```

```
      sigma  
0.4478
```

# EMM Fit of CIR Model

```
> cir.nsim <- 50000
> set.seed(0)
> n.burn <- 25; ndt <- 100
> cir.z <- rnorm(ndt*(n.burn + cir.nsim))

> EMM.pcode.fit.cir <- EMM(SNP.auto.cir,
+ coef = c(0.1,0.1,0.1), est = c(1,1,1),
+ control = EMM.control(n.burn = n.burn, n.sim = cir.nsim),
+ gensim.fn = "euler1d.pcode.gensim",
+ gensim.language = "SPLUS",
+ gensim.aux = euler.pcode.aux(ndt = ndt, z = cir.z,
+                               lbound=0.0, ubound=100,
+                               drift.expr=expression(kappa*(theta-X)),
+                               diffuse.expr=expression(sigma*sqrt(X)),
+                               rho.names=c("kappa", "theta", "sigma")))
```

# EMM Fit of CIR Model

```
> EMM.pcode.fit.cir
```

```
Coefficients:
```

	Value	Std. Error	95% Conf. Int.
kappa	0.10942	0.03296	0.07263 0.23203
theta	0.10353	0.01013	0.08597 0.12097
sigma	0.05426	0.00378	0.04728 0.06173

```
Final optimization:
```

```
Convergence: absolute function convergence
```

```
Iterations: 15
```

```
Normalized objective at final iteration: 5.899e-007
```