Introduction to ARCH and GARCH Models

ARCH (AutoRegressive Conditional Heteroskedasticity) models were proposed by Engle in 1982.

GARCH (Generalized ARCH) models proposed by Bollerslev in 1986.

Engle received the Nobel price in 2003. The GARCH model framework is considered as one of the most important contributions in empirical finance over the last 20 years.
Nobel Prize citation: "for methods of analyzing economic time series with time-varying volatility (ARCH)"

Champion Pairs Skater Too!
Other Acronyms for ARCH

- Anything Can Really Happen
- Another Risk Can't Hurt
- All Reality Comes Here
- Applied Research Can Help
- Another Rather Crazy Hypothesis
- Almost Right Conjected Heuristic
- Almost Real Crummy Hogwash
- Applied Research on Changing Histories
- A Real Chinese Hero
- A Rice Cake History
- A Real Clever Heuristic
- Another Really Cute Hunch
- Ahh, Robert... Call Home!
- A Really Cool Hat
- And Robert Can Hit
- All Risks Compensate Highly

The ARCH Family

Bolerslev (2008) identified over 150 different ARCH models. Here are some of the most common:

- GJR-GARCH
- TARCH
- STARCH
- AARCH
- NARCH
- MARCH
- SWARCH
- SNPARCH
- APARCH
- TAYLOR-SCHWERT
- FIGARCH
- FIEGARCH
- Component
- Asymmetric Component
- SQGARCH
- CESGARCH
- Student t
- GED
- SPARCH
Stylized Facts of Daily Asset Returns

Microsoft Returns

S & P 500 Returns

Volatility clustering

Microsoft Squared Returns

S & P 500 Squared Returns

Microsoft Absolute Returns

S & P 500 Absolute Returns

Sample Autocorrelations of Daily Returns

Microsoft Returns

S&P 500 Returns

Microsoft Squared Returns

S & P 500 Squared Returns

Microsoft Absolute Returns

Microsoft Absolute Returns

Dependence in volatility
Stylized Facts for Monthly Asset Returns

Microsoft Returns

S&P 500 Returns

Less volatility clustering

Microsoft Squared Returns

S&P 500 Squared Returns

Less volatility dependence

Summary statistics

```
> summaryStats(msft.ts)
> summaryStats(sp500.ts)
> normalTest(msft.ts, method="jb")
> normalTest(sp500.ts, method="jb")
```

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean</th>
<th>Med</th>
<th>Min</th>
<th>Max</th>
<th>Std. Dev</th>
<th>Skew</th>
<th>Kurt</th>
<th>QB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSFT</td>
<td>0.0016</td>
<td>0.0000</td>
<td>-0.3012</td>
<td>0.1957</td>
<td>0.0253</td>
<td>-0.2457</td>
<td>11.66</td>
<td>13693</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.0001</td>
<td>0.0005</td>
<td>0.2017</td>
<td>0.0009</td>
<td>0.0113</td>
<td>1.186</td>
<td>32.50</td>
<td>16048</td>
</tr>
<tr>
<td></td>
<td>Monthly Returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSFT</td>
<td>0.0336</td>
<td>0.0336</td>
<td>-0.3661</td>
<td>0.4934</td>
<td>0.1145</td>
<td>0.1845</td>
<td>4.004</td>
<td>9.922</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.0082</td>
<td>0.0122</td>
<td>-0.2003</td>
<td>0.1250</td>
<td>0.0459</td>
<td>-0.8977</td>
<td>5.180</td>
<td>65.75</td>
</tr>
</tbody>
</table>

Notes: Sample period is 03/14/86 - 06/30/03 giving 4365 daily observations.
Simulating an ARCH(1) Model

> sim.arch1 <-
+ simulate.garch(model=list(a.value=1,arch=0.8),
+                 n=500, rseed=124)
> names(sim.arch1)
[1] "et"     "sigma.t"

c = 0 (by default), a.value = a₀, arch = a₁, eₜ = εₜ and sigma.t = σₜ (conditional standard deviation).

Simulated ARCH(1) with a₁ = 0.8

- Squared values
- Absolute values
- Conditional Volatility
Sample Autocorrelations from ARCH(1)

Returns

Squared Returns

Absolute Returns

Distribution of Simulated ARCH(1) Data

Sample Quantiles:
\[
\begin{align*}
\text{min} & \quad 1Q & \quad \text{median} & \quad 3Q & \quad \text{max} \\
-1.11 & -0.1068 & -0.006157 & 0.0915 & 1.292
\end{align*}
\]

Sample Moments:
\[
\begin{align*}
\text{mean} & \quad \text{std} & \quad \text{skewness} & \quad \text{kurtosis} \\
-0.0168 & 0.229 & -0.3925 & 8.732
\end{align*}
\]

Number of Observations: 500

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Simulated GARCH(1,1) Data

Simulated GARCH(1,1) with $a_1 = 0.1, b_1 = 0.7$

Squared values

Absolute values

Conditional Volatility

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Sample Autocorrelations from GARCH(1,1)

Returns

Squared Returns

Absolute Returns

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Distribution of Simulated GARCH(1,1) Data

Sample Quantiles:
\[
\begin{align*}
\text{min} & \quad -0.6763 \\
1Q & \quad -0.1587 \\
\text{median} & \quad -0.01016 \\
3Q & \quad 0.1342 \\
\text{max} & \quad 0.6103
\end{align*}
\]

Sample Moments:
\[
\begin{align*}
\text{mean} & \quad -0.01029 \\
\text{std} & \quad 0.2296 \\
\text{skewness} & \quad -0.05683 \\
\text{kurtosis} & \quad 3.102
\end{align*}
\]

Number of Observations: 500

Modified Box-Ljung Statistic

\[
> \text{autocorTest}\left(\text{seriesMerge}(\text{msft.ts}^2, + \quad \text{abs(\text{msft.ts})), lag.n = 5}\right)
\]

Test for Autocorrelation: Ljung-Box
Null Hypothesis: no autocorrelation

Test Statistics:
\[
\begin{align*}
\text{MSFT.1} & \quad 562.1016 \\
\text{MSFT.2} & \quad 463.8208
\end{align*}
\]

\[
\begin{align*}
p\text{-value} & \quad 0.0000 \\
p\text{-value} & \quad 0.0000
\end{align*}
\]

Dist. under Null: chi-square with 5 degrees of freedom
Total Observ.: 4365
Testing for ARCH effects in daily MSFT returns

Engle’s LM Test

> archTest(msft.ts, lag.n = 5)

Test for ARCH Effects: LM Test

Null Hypothesis: no ARCH effects

Test Statistics:

<table>
<thead>
<tr>
<th>MSFT</th>
<th>Test Stat</th>
<th>p.value</th>
<th>Easily reject H0: no ARCH effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Stat</td>
<td>377.9377</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Dist. under Null</td>
<td>chi-square with 5 degrees of freedom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Observ.:</td>
<td>4365</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Less evidence for ARCH effects in monthly returns

Estimate GARCH(1,1) for MSFT

> msft.g11 = garch(msft.ts ~ 1, ~ garch(1, 1))

Iteration 0 Step Size = 1.00000 Likelihood = 2.32406
Iteration 0 Step Size = 2.00000 Likelihood = 2.32445
Iteration 0 Step Size = 4.00000 Likelihood = 2.23425
Iteration 1 Step Size = 1.00000 Likelihood = 2.32489
Iteration 1 Step Size = 2.00000 Likelihood = 2.32497
Iteration 1 Step Size = 4.00000 Likelihood = 2.32402
Iteration 2 Step Size = 1.00000 Likelihood = 2.32506
Iteration 2 Step Size = 2.00000 Likelihood = 2.32500

Convergence R-Square = 4.710438e-006 is less than tolerance = 0.0001
Convergence reached.

Conditional mean formula: msft ~ 1
Conditional variance formula: ~ garch(1,1)
GARCH Object

```r
class(msft.g11)  # print, summary, plot, predict, simulate
    [1] "garch"

names(msft.g11)
[1] "residuals" "sigma.t"  "df.residual" "coef"  "model"
[6] "cond.dist" "likelihood" "opt.index" "cov"  "prediction"

msft.g11$coef

coef(msft.g11)

msft.g11$asympt.sd

summary(msft.g11)

Call: garch(formula.mean = msft.ts ~ 1, formula.var = ~ garch(1, 1))

Conditional Distribution: gaussian

Estimated Coefficients:

| Value    | Std.Error | t value | Pr(>|t|) |
|----------|-----------|---------|---------|
| C 0.00187904290 | 3.256e-004 | 5.771   | 8.430e-009 |
| A 0.00002803065 | 3.419e-006 | 8.199   | 4.441e-016 |
| ARCH(1) 0.09044383310 | 5.924e-003 | 15.267  | 0.000e+000 |
| GARCH(1) 0.86581806057 | 1.016e-002 | 85.187  | 0.000e+000 |

Note: a1 + b1 = 0.09 + 0.87 = 0.96

Half life of shock to volatility: ln(0.5)/ln(0.96)=15.5 days
```
Examining a garch object

```r
> names(msft.g11$model)
[1] "c.which" "c.value" "MA" "AR" "arch"
[6] "garch" "a.which" "a.value" "info" "power.value"
> msft.g11$model
-- Constants in mean ---
  value which
0.001879 1
-- Constants in variance---
  value which
0.00002803 1
----- ARCH --------------
  value which
lag 1 0.09044 1
----- GARCH --------------
  value which
lag 1 0.8658 1
```

Revising a garch model

```r
# revise model starting values
> g11.model = msft.g11$model
> g11.model$garch$value=0.9
> g11.model$arch$value=0.1

# reestimate with new values
msft.g11.new = garch(series = msft.ts, model = g11.model)
Iteration 0 Step Size = 1.00000 Likelihood = 2.31871
Iteration 3 Step Size = 8.00000 Likelihood = 2.32465
...
Convergence R-Square = 9.360924e-006 is less than tolerance
= 0.0001
> cbind(coef(msft.g11), coef(msft.g11.new))
[,1]          [,2]
 C 0.00190253165 0.00195889793
A 0.00002755037 0.00002730452
ARCH(1) 0.08994106397 0.08967032459
GARCH(1) 0.86751369929 0.86806428699
> cbind(msft.g11$likelihood, msft.g11.new$likelihood)
[,1]    [,2]
[1,] 10148.91 10148.9
```

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The basic GARCH model assumes normal errors.
If the errors are not normal but the Gaussian likelihood is used then the resulting estimator is called the QMLE
Under general conditions, the QMLE is still consistent and asymptotically normal but the variance must be adjusted

\[
> \text{summary}(\text{msft.gll, method=“QMLE”})
\]
\[
> \text{vcov}(\text{msft.gll, method=“QMLE”})
\]

---

**Conditional Mean and Variance**

**General Conditional Mean Specification**

\[
E_{t-1}[y_t] = c + \sum_{j=1}^{r} \phi_j y_{t-j} + \sum_{k=1}^{s} \theta_k \epsilon_{t-k} + \sum_{l=0}^{L} \beta_l z_{t-l}
\]

**General Conditional Variance Specification**

\[
\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 + \sum_{l=0}^{L} \delta_l^2 z_{t-l}
\]

\[
\text{args(garch)}
\]
\[
\text{function(formula.mean = ~ arma(0, 0),}
\]
\[
\text{formula.var = ~ garch(0, 0),}
\]
\[
\text{series.start = 1, x = NULL, x.start = 1, xlag = 0,}
\]
\[
\text{z = NULL, z.start = 1, …}
\]

**Note:** Exogenous variables should be directly specified in \text{formula.mean} and \text{formula.var}
GARCH Model Diagnostics

- test if a coefficient is equal to zero
  - Standard t-statistics and p-value

- test if distribution of residuals is normal
  - `normalTest()`
  - Jarque-Bera test & Shapiro-Wilk test

- test if squared residuals are correlated
  - `autocorTest()`
  - Box-Pierce & modified Box-Pierce (or Ljung-Box)

- test if any ARCH effects remain in residuals
  - `archTest()`

Garch model diagnostics: `summary()`

```r
> summary(msft.g11)
...
Normality Test: Standardized residuals are non-normal
 ----------------------------------------------------------
Jarque-Bera P-value Shapiro-Wilk P-value
1752 0 0.9782 2.367e-025

Ljung-Box test for standardized residuals:
 ----------------------------------------------------------
Statistic P-value Chi^2 d.f.
17.62 0.1278 12

Ljung-Box test for squared standardized residuals:
 ----------------------------------------------------------
Statistic P-value Chi^2 d.f.
4.792 0.9646 12

Lagrange multiplier test: No autocorrelation or ARCH effects in residuals
 ----------------------------------------------------------
TR^2 P-value F-stat P-value
4.768 0.9653 0.4339 0.9918
```
> plot(msft.g11)

Make a plot selection (or 0 to exit):

1: plot: All
2: plot: Series and Conditional SD
3: plot: Series with 2 Conditional SD Superimposed
4: plot: ACF of the Observations
5: plot: ACF of Squared Observations
6: plot: Cross Corr. between Squared Series and Series
7: plot: Residuals
8: plot: Conditional Standard Deviations
9: plot: Standardized Residuals
10: plot: ACF of Standardized Residuals
11: plot: ACF of Squared Standardized Residuals
13: plot: QQ-Plot of Standardized Residuals

Selection:

> plot(msft.g11, which.plot = 1)
Garch model diagnostics: residuals

```r
> plot(msft.g11, which.plot = 10) > plot(msft.g11, which.plot = 12)
```

![ACF of Squared Std. Residuals](image1)

![QQ-Plot of Standardized Residuals](image2)

GARCH Model Selection

```r
> msft.a1 = garch(msft.ts~1, ~garch(1,0))
> msft.a2 = garch(msft.ts~1, ~garch(2,0))
> msft.a3 = garch(msft.ts~1, ~garch(3,0))
> msft.a4 = garch(msft.ts~1, ~garch(4,0))
> msft.a5 = garch(msft.ts~1, ~garch(5,0))
> msft.g11 = garch(msft.ts~1, ~garch(2,1))
> msft.g12 = garch(msft.ts~1, ~garch(1,2))
> compare.mgarch(msft.a1, msft.a2, msft.a3, msft.a4, msft.a5, msft.g11, msft.g12, msft.g21, msft.g22)
```

<table>
<thead>
<tr>
<th></th>
<th>msft.a1</th>
<th>msft.a2</th>
<th>msft.a3</th>
<th>msft.a4</th>
<th>msft.a5</th>
<th>msft.g11</th>
<th>msft.g12</th>
<th>msft.g21</th>
<th>msft.g22</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>-19977</td>
<td>-20086</td>
<td>-20175</td>
<td>-20196</td>
<td>-20211</td>
<td>-20290</td>
<td>-20290</td>
<td>-20292</td>
<td>-20288</td>
</tr>
<tr>
<td>BIC</td>
<td>-19958</td>
<td>-20060</td>
<td>-20143</td>
<td>-20158</td>
<td>-20166</td>
<td>-20264</td>
<td>-20264</td>
<td>-20260</td>
<td>-20249</td>
</tr>
<tr>
<td>Likelihood</td>
<td>9992</td>
<td>10047</td>
<td>10092</td>
<td>10104</td>
<td>10113</td>
<td>10149</td>
<td>10150</td>
<td>10151</td>
<td>10150</td>
</tr>
</tbody>
</table>
Compare ARCH(5) to GARCH(1,1)

GARCH(1,1) volatilities are smoother and more persistent than ARCH(5)

Conditional Volatility from GARCH(1,1)

Conditional Volatility from ARCH(5)

Rolling Analysis of GARCH Models

```r
roll.garch <- function(data)
{
  my.mod = garch(data~1, ~garch(1,1), trace=F)
  return(list(a0 = coef(my.mod)[2],
              a1 = coef(my.mod)[3],
              b1 = coef(my.mod)[4],
              asymp.sd = my.mod$asymp.sd))
}

Rolling analysis is performed with FinMetrics function `roll`

# rolling GARCH(1,1) estimation for MSFT
# 2 years of daily data, rolled ahead by 1 month
> roll.fit.msft <- roll(FUN="roll.garch", data=msft.ts,
                         width=500, incr=20)
```
Rolling Analysis of GARCH(1,1)

Persistence parameter: $a_1 + b_1$

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Garch predict() method

```r
> msft.g11.pred = predict(msft.g11, n.predict = 250)
> class(msft.g11.pred)
[1] "predict.garch"
> names(msft.g11.pred)
[1] "series.pred"  "sigma.pred"  "asymp.sd"
> plot(msft.g11.pred)
```

Volatility mean reversion

Mean reversion in volatility forecast

Predicted Volatility from GARCH(1,1) for Microsoft

```r
msft.g11$asymp.sd
[1] 0.0254
```

Out-of-sample conditional volatility forecast: predict()

In-sample conditional volatility: sigma.t()
Volatility Term Structure

\begin{verbatim}
> term.dat = 1:200
> msft.volTermStruc = cumsum(predict(msft.g11, n.predict=200)$sigma.pred^2)/term.dat
> tsplot(msft.volTermStruc)
\end{verbatim}

Note: constant variance implies linear volatility term structure

Testing for Asymmetry

\begin{verbatim}
> SignBias.fit = OLS(ehat2~tslag(S.minus), data=tmp.ts)
> NegativeSizeBias.fit = OLS(ehat2~tslag(S.minus*ehat), data=tmp.ts)
> PositiveSizeBias.fit = OLS(ehat2~tslag(S.plus*ehat), data=tmp.ts)
> All.fit =
+ OLS(ehat2~S.minus+tslag(S.minus*ehat)+tslag(S.plus*ehat), data=tmp.ts)
\end{verbatim}

<table>
<thead>
<tr>
<th>Asset</th>
<th>coeff(\sigma_t^2, r_{t-1})</th>
<th>Sign Bias</th>
<th>Negative Size Bias</th>
<th>Positive Size Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microsoft</td>
<td>-0.0315</td>
<td>-0.4417 (0.6587)</td>
<td>-6.816 (0.000)</td>
<td>3.174 (0.001)</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-0.198</td>
<td>9.457 (0.011)</td>
<td>-11.185 (0.000)</td>
<td>1.356 (0.175)</td>
</tr>
</tbody>
</table>

Notes: p-values are in parentheses.

Clear evidence for leverage effects
Exponential GARCH Model (EGARCH)

\[ h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \frac{|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j h_{t-j} \]

\[ h_t = \log(\sigma_t^2), \sigma_t^2 = \exp(h_t) \]

- good news effect \( \varepsilon_{t-i} : (1 + \gamma_i)\varepsilon_{t-i} \)
- bad news effect \( \varepsilon_{t-i} : (1 - \gamma_i)|\epsilon_{t-i}| \)

Note: Conditional variance is always positive due to exponential transformation

\[ \text{garch}(\text{hp.s~1, ~egarch}(1,1), \text{leverage}=T) \]

Threshold GARCH Model (TGARCH)

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i S_t \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \]

\[ S_t = \begin{cases} 
1 & \text{if } \varepsilon_i < 0 \\
0 & \text{if } \varepsilon_i \geq 0 
\end{cases} \]

- good news effect \( \varepsilon_{t-i} : a_i \varepsilon_{t-i}^2 \)
- bad news effect \( \varepsilon_{t-i} : (a_i + \gamma_i)\varepsilon_{t-i}^2 \)
- usually, \( \gamma_i > 0 \)

\[ \text{garch}(\text{hp.s~1, ~tgarch}(1,1)) \]
\[ \sigma_t^d = a_0 + \sum_{i=1}^{p} a_i \left( |\epsilon_{t-i}| + \gamma_i \epsilon_{t-i} \right)^d + \sum_{j=1}^{q} b_j \sigma_{t-j} \]

\(d\) is a positive exponent (can be fixed or estimated from data).

- The coefficient of the leverage effects is denoted by \(\gamma\).
- \(d\) can be fixed (\(~garch(p,q,d)\)) or estimated by ML method (\(~garch(p,q)\)).

\[
\text{garch}(hp.s\sim1, \sim pgarch(1,1,1), \text{leverage=T})
\]

---

**Estimation of Asymmetric GARCH models**

- \(\text{msft.egarch} = \text{garch}(\text{msft.ts}\sim1, \sim \text{egarch}(1,1), \text{leverage=T})\)
- \(\text{msft.tgarch} = \text{garch}(\text{msft.ts}\sim1, \sim \text{tgarch}(1,1))\)
- \(\text{msft.pgarch.2} = \text{garch}(\text{msft.ts}\sim1, \sim \text{garch}(1,1), \text{leverage=T})\)
- \(\text{msft.pgarch.1} = \text{garch}(\text{msft.ts}\sim1, \sim \text{pgarch}(1,1,1), \text{leverage=T})\)

<table>
<thead>
<tr>
<th>Model</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(b_1)</th>
<th>(\gamma_1)</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGARCH</td>
<td>-0.7273</td>
<td>0.2144</td>
<td>0.9247</td>
<td>-0.2417</td>
<td>-20265</td>
</tr>
<tr>
<td></td>
<td>[0.1064]</td>
<td>[0.0594]</td>
<td>[0.0459]</td>
<td>[0.0758]</td>
<td></td>
</tr>
<tr>
<td>TGARCH</td>
<td>3.01e^{-5}</td>
<td>0.0564</td>
<td>0.8581</td>
<td>0.0771</td>
<td>-20291</td>
</tr>
<tr>
<td></td>
<td>[1.02e^{-5}]</td>
<td>[0.0141]</td>
<td>[0.0342]</td>
<td>[0.0306]</td>
<td></td>
</tr>
<tr>
<td>PGARCH 2</td>
<td>2.87e^{-5}</td>
<td>0.0858</td>
<td>0.8672</td>
<td>-0.2104</td>
<td>-20290</td>
</tr>
<tr>
<td></td>
<td>[9.27e^{-6}]</td>
<td>[0.0206]</td>
<td>[0.0313]</td>
<td>[0.0579]</td>
<td></td>
</tr>
<tr>
<td>PGARCH 1</td>
<td>0.0010</td>
<td>0.0921</td>
<td>0.8876</td>
<td>-0.2307</td>
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<td>[0.0238]</td>
<td>[0.0401]</td>
<td>[0.0813]</td>
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</table>
News Impact curves of GARCH processes

Functional relationship between conditional variance at time $t$ and error term at time $t-1$, holding constant information dated $t-2$ and earlier and with all lagged conditional variances evaluated at unconditional variance

$GARCH(1,1)$
\[
\sigma_t^2 = A + a_1 \left( |\varepsilon_{t-1}| + \gamma \varepsilon_{t-1} \right)^2 \\
A = a_0 + b_1 \bar{\sigma}^2
\]

$TGARCH(1,1)$
\[
\sigma_t^2 = A + (a_1 + \gamma \varepsilon_{t-1}) \varepsilon_{t-1}^2 \\
A = a_0 + b_1 \bar{\sigma}^2
\]

$PGARCH(1,1,1)$
\[
\sigma_t^2 = A + 2\sqrt{A}a_1 \left( |\varepsilon_{t-2}| + \gamma \varepsilon_{t-1} \right) + a_1^2 \left( |\varepsilon_{t-1}| + \gamma \varepsilon_{t-1} \right)^2 \\
A = (a_0 + b_1 \bar{\sigma})^2
\]

$EGARCH(1,1)$
\[
\sigma_t^2 = A \exp \left\{ a_1 \left( |\varepsilon_{t-1}| + \gamma \varepsilon_{t-1} \right) \bar{\sigma} \right\} \\
A = \bar{\sigma} \exp \left\{ a_0 \right\}
\]

Estimated News Impact Curves

Asymmetric $GARCH(1,1)$ Models for Microsoft

TARCH(1,1) and PGARCH(1,2,1) almost same

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Estimation of Non-Gaussian GARCH(1,1) Models

\[ msft.g1lt = \text{garch}(msft.ts-1, \sim\text{garch}(1,1), \text{cond.dist}="t") \]
\[ msft.tgarch.t = \text{garch}(msft.ts-1, \sim\text{tgarch}(1,1), \text{cond.dist}="t") \]

<table>
<thead>
<tr>
<th>Model</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \beta_1 )</th>
<th>( \gamma_1 )</th>
<th>( \nu )</th>
<th>BIC</th>
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<td>7.070</td>
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</tbody>
</table>

Normal GARCH BIC = -20264
Normal TARCH BIC = -20294

QQ-plot of Standardized Residuals from TGARCH with t-errors

QQ-Plot of Standardized Residuals

Nice and linear QQ-plot
Comparison of GARCH Model Forecasts

Volatility Forecasts for Microsoft from Competing GARCH Models

GARCH
GARCH(1)
PGARCH(2)
PGARCH(1,1)
TGARCH
TGARCH(1)