Amath 546/Econ 589
Introduction to Credit Risk Models

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Reading

- QRM chapter 8, sections 1-4.

How Credit Risk is Different from Market Risk

- Market risk can typically be measured directly because equities trade frequently in active markets
  - Market volatility, VaR and ES can be computed from return distributions estimated from observed data
  - Market risk events are often short-horizon events

- Credit risk, the risk from holding defaultable loan-type securities, typically cannot be measured directly
  - Credit securities often do not trade frequently in active markets
  - Credit events such as defaults or ratings downgrades are rare
  - Defaults across different credit instruments are often correlated
  - Credit events are often long horizon events

Traditional Approaches to Credit Risk Measurement

- Focus on estimating the probability of default (PD) rather than on the magnitude of potential losses given default (LGD)

- Failure is typically bankruptcy filing, default or liquidation (not ratings downgrade)

- Three common types of models to estimate PD
  - Expert systems, rating systems and credit scoring models

Expert Systems: Subjective Evaluation of Credit Risk

5 C of credit expert systems:

1. Character - measure of firm reputation

2. Capital - equity contribution and leverage

3. Capacity - ability to repay

4. Collateral - capital to support losses if default occurs

5. Cycle conditions - macroeconomic factors that influence credit risk exposure

Comments:

- Evaluation of 5 C's is typically done by human "experts"

- Some quantitative expert systems use data-mining techniques based on the information gathered from the 5 Cs

Rating Systems

- External credit ratings provided by firms such as Fitch, Moody's and S&P give an assessment of PD

- Banks are required to have internal ratings of the credit quality of their loans

  - Much diversity in internal credit rating methodologies

Credit Scoring Models

- Most commonly used methodology by banks for credit decisions like credit card applications and mortgages

- Typical statistical methodologies
  - Logistic regression
  - multiple discriminant analysis

- Idea is to create a score variable (e.g., FICA score) based on balance sheet data to determine credit quality

Modern Approaches to Credit Risk Measurement

Two main approaches based on modern asset pricing theory to estimate default probabilities:

- Options theoretic (structural) approach of Merton (1973)

- Reduced form statistical approach utilizing intensity-based models to estimate hazard rates

Default probabilities and losses given defaults obtained from either approach are used as inputs into a risk model to estimate VaR or ES of a loss distribution

Options Theoretic Structural Models: Merton's Model

- Equity in a levered firm (a firm with both debt and equity financing) is a call option on the firm's assets with a strike price equal to the debt repayment amount.

- At maturity of the option (maturity of firm's liability payments) the firm's market value may be above or below the value of debt

  - If above, then firm's shareholders exercise the option to repurchase the firm's assets by repaying debt

  - If below, then the firm defaults on debt and the option to repurchase the firm's assets expires

- PD until expiration is equal to the likelihood that the option will expire out-of-the money

- Merton uses the BS formula to value equity as a call option

$$
\begin{aligned}
E &= C(A, \sigma_A, r, T, B) \\
A &= \text{market value of firm's assets} \\
\sigma_A &= \text{annual volatility of firm's assets} \\
r &= \text{annual risk-free rate} \\
T &= \text{maturity date of debt} \\
B &= \text{face value of firm's debt}
\end{aligned}
$$

Problem: Typically don't know A or σ_A but values can be iteratively inferred from $E = C(A, \sigma_A, r, T, B)$ and

$$\sigma_E = \left(\frac{A}{E}\right)\left(\frac{\partial E}{\partial A}\right)\sigma_A$$

- Values of A, σ_A and B are combined to determine a firm's current Distance to Default, which is calculated as the number of standard deviations between current asset values and the debt repayment amount

- Merton converts Distance to Default into a PD estimate using the assumption that A follows a log-normal distribution

- Defaults in the Merton model occur after a gradual decline of A to the default point due to the log-normal assumption of the BS model
  - PD declines to zero as time to maturity declines

Improvements to Merton's model

- KMV (Kealhofer, McQuown and Vasicek) model
  - PD is calibrated using a huge proprietary database of historical defaults
- JP Morgan's CreditMetrics
  - PD is calibrated using credit migration data (e.g., credit rating transition matrices)

Reduced Form Intensity-Based Models

- In contrast to structural models, reduced form models do not specify the economic process leading to default

- Default is modeled in a purely statistical way as a so-called "point process" (e.g., a jump process)
  - Defaults occur randomly with a probability determined by the intensity or hazard function

- Emphasis is on determining (1) PD at time t (conditional on there being no default prior to time t) and (2) LGD rate (1 - recovery rate)

- Methodology is empirical using observable risky debt prices and credit spreads as data to estimate intensity models.

Credit Spread Decomposition

Intensity-based models utilize the following decomposition

$$
\begin{aligned}
CS(t) &= PD(t) \times LGD(t) \\
CS(t) &= \text{credit spread} = \text{risky debt yield - risk free rate} \\
PD(t) &= \text{probability of default} \\
LGD(t) &= \text{loss rate given default rate} = 1 \text{ - recovery rate}
\end{aligned}
$$

Here, $CS(t)$ is a measure of the expected cost of default.

How to Identify $PD(t)$ and $LGD(t)$ from $CS(t)$?

- Without additional assumptions, it is not possible to uniquely recover $PD(t)$ and $LGD(t)$ from $CS(t)$

- Different models use different identifying assumptions (e.g., assume $LGD(t)$ is a known fraction of Bond face value just prior to maturity and $PD(t)$ follows a Poisson process with intensity parameter that depends on economic state variables)

- $CS(t)$ also incorporates liquidity effects which further complicates analysis