AMATH 546/ECON 589

Factor Model Risk Analysis in R

29 May, 2013

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Outline

- Data for examples
- Fit Macroeconomic factor model
- Factor risk budgeting
- Portfolio risk budgeting
- Factor model Monte Carlo

Set Options and Load Packages

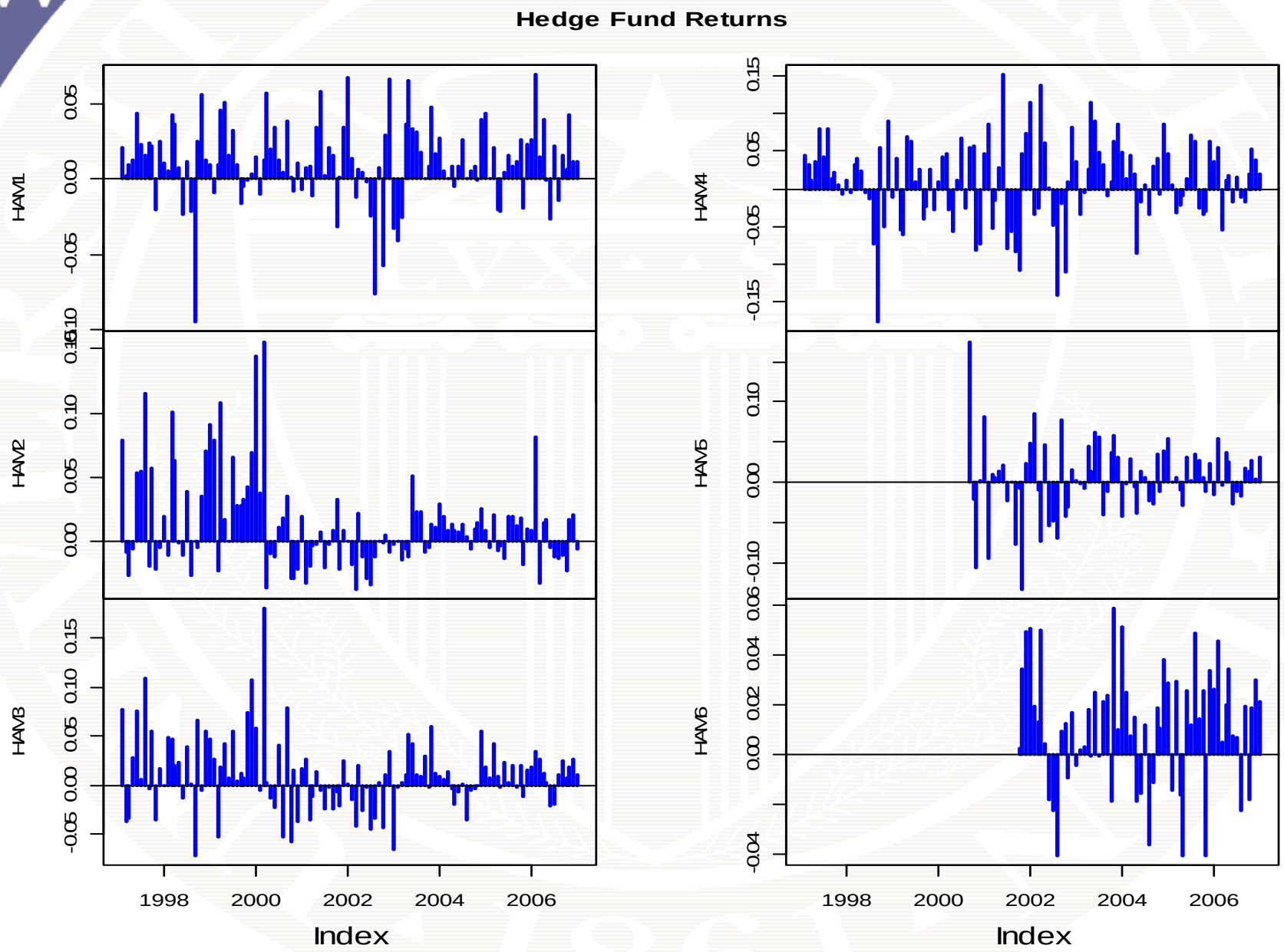
```
# set output options
> options(width = 70, digits=4)

# load required packages
> library(ellipse)                    # plot ellipses
> library(fEcofin)                    # various economic and
                                      # financial data sets
> library(PerformanceAnalytics)       # performance and risk
                                      # analysis functions
> library(tseries)                    # MISC time series funs
> library(xts)                        # time series objects
> library(zoo)                        # and utility functions
```

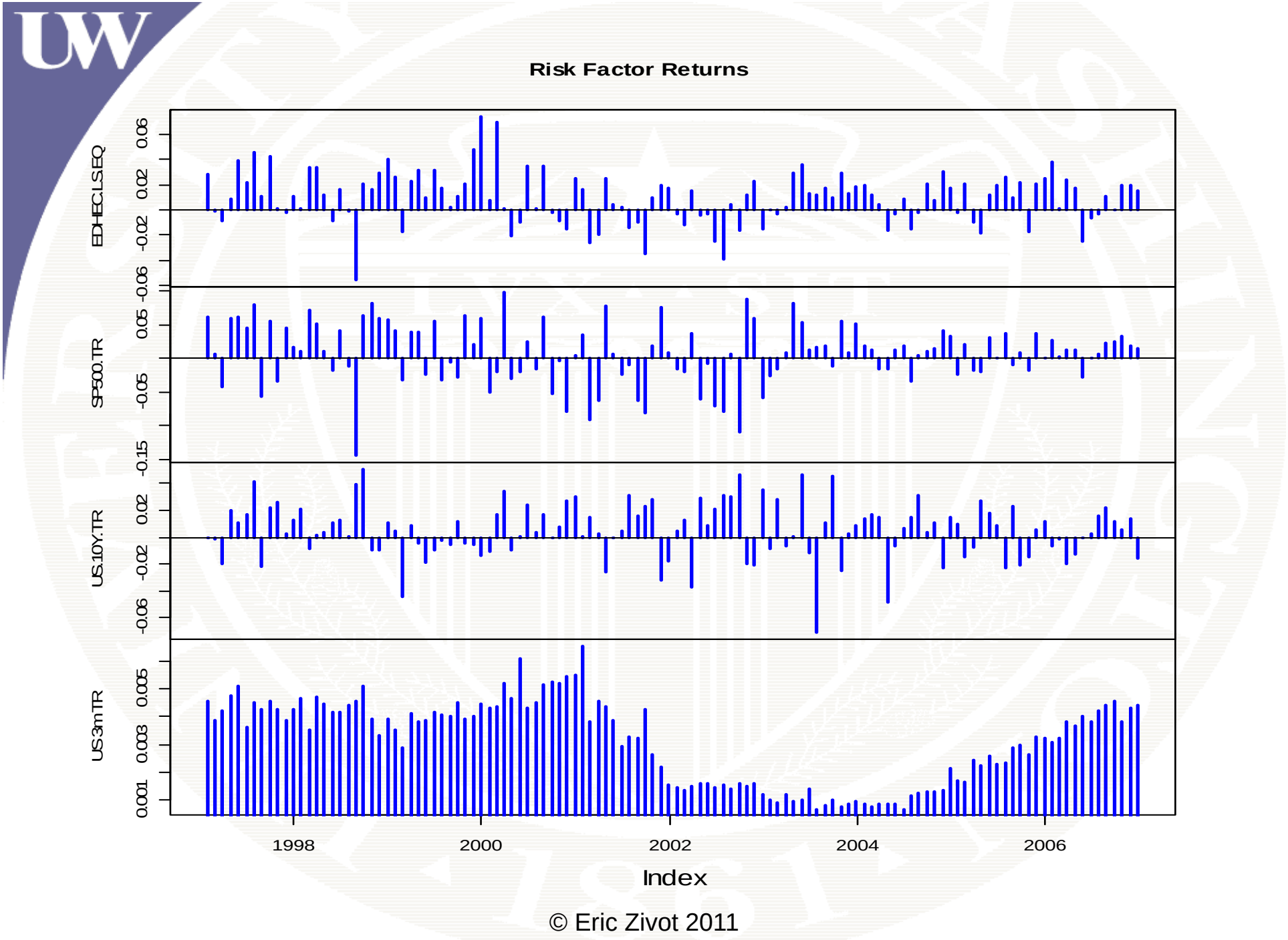
Hedge Fund Data

```
# load hypothetical long-short equity asset managers data
# from PerformanceAnalytics package
> data(managers)
> class(managers)
[1] "xts" "zoo"
> start(managers)
[1] "1996-01-30"
> end(managers)
[1] "2006-12-30"
> colnames(managers)
 [1] "HAM1"        "HAM2"        "HAM3"          "HAM4"
 [5] "HAM5"        "HAM6"        "EDHEC LS EQ" "SP500 TR"
 [9] "US 10Y TR"   "US 3m TR"

# remove data prior to 1997-01-30 due to missing vals
> managers = managers["1997::2006"]
```

Plot Hedge Fund and Factor Returns

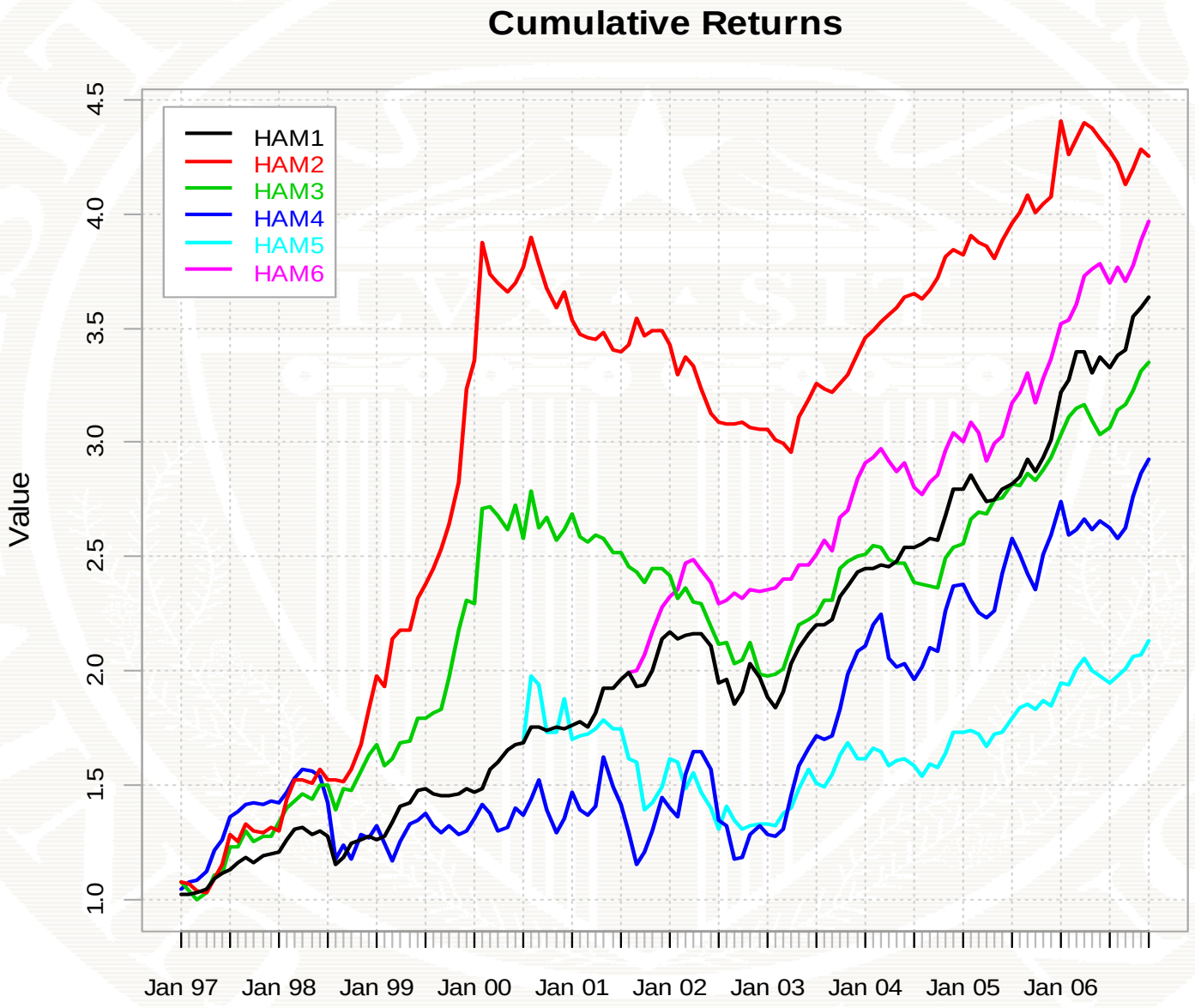
```
> my.panel <- function(...) {
+   lines(...)
+   abline(h=0)
+ }

# use plot.zoo() from zoo package for multi-panel plots
> plot.zoo(managers[, 1:6], main="Hedge Fund Returns",
+   plot.type="multiple", type="h", lwd=2, col="blue",
+   panel=my.panel)

> plot.zoo(managers[, 7:10], main="Risk Factor Returns",
+   plot.type="multiple", type="h", lwd=2, col="blue",
+   panel=my.panel)
```

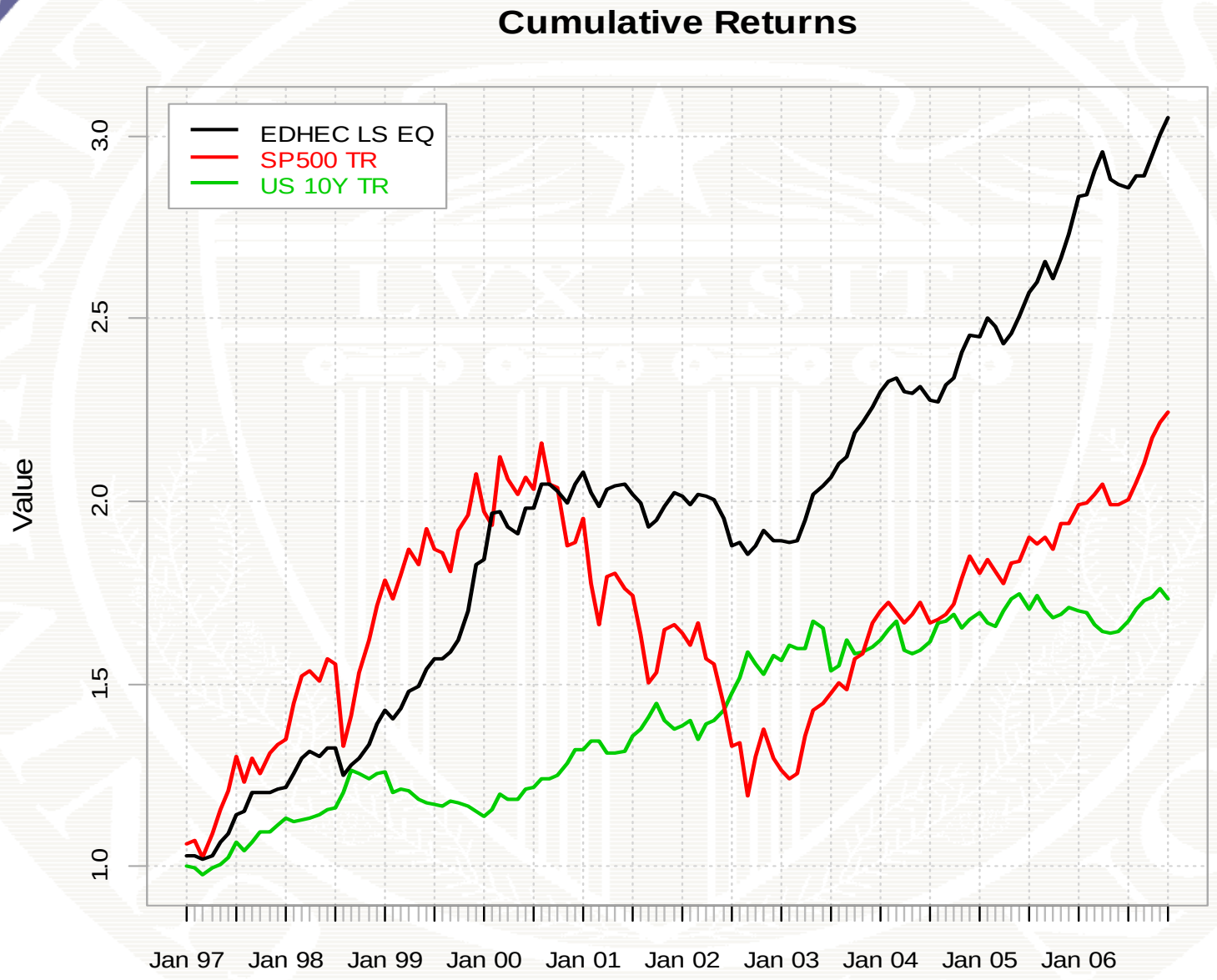




Plot Cumulative Returns

```
# plot cumulative returns using PerformanceAnalytics
# function chart.CumReturns()

# hedge funds
> chart.CumReturns(managers[,1:6],
+                  main="Cumulative Returns",
+                  wealth.index=TRUE,
+                  legend.loc="topleft")

# risk factors
> chart.CumReturns(managers[,7:9],
+                  main="Cumulative Returns",
+                  wealth.index=TRUE,
+                  legend.loc="topleft")
```





Descriptive Statistics: Funds

```
# Use table.Stats() function from PerformanceAnalytics package
> table.Stats(managers[, 1:6])
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6
Observations	120.0000	120.0000	120.0000	120.0000	77.0000	64.0000
NAs	0.0000	0.0000	0.0000	0.0000	43.0000	56.0000
Minimum	-0.0944	-0.0371	-0.0718	-0.1759	-0.1320	-0.0404
Quartile 1	0.0000	-0.0108	-0.0059	-0.0236	-0.0164	-0.0016
Median	0.0107	0.0075	0.0082	0.0128	0.0038	0.0128
Arithmetic Mean	0.0112	0.0128	0.0108	0.0105	0.0041	0.0111
Geometric Mean	0.0108	0.0121	0.0101	0.0090	0.0031	0.0108
Quartile 3	0.0252	0.0224	0.0263	0.0468	0.0309	0.0255
Maximum	0.0692	0.1556	0.1796	0.1508	0.1747	0.0583
SE Mean	0.0024	0.0033	0.0033	0.0050	0.0052	0.0030
LCL Mean (0.95)	0.0064	0.0062	0.0041	0.0006	-0.0063	0.0051
UCL Mean (0.95)	0.0159	0.0193	0.0174	0.0204	0.0145	0.0170
Variance	0.0007	0.0013	0.0013	0.0030	0.0021	0.0006
Stdev	0.0264	0.0361	0.0367	0.0549	0.0457	0.0238
Skewness	-0.6488	1.5406	0.9423	-0.4064	0.0724	-0.2735
Excess Kurtosis	2.1223	2.7923	3.0910	0.6453	2.1772	-0.4311

Descriptive Statistics: Factors

```
> table.Stats(managers[, 7:9])
                  EDHEC LS EQ SP500 TR US 10Y TR
Observations         120.0000 120.0000  120.0000
NAs                    0.0000   0.0000    0.0000
Minimum               -0.0552  -0.1446   -0.0709
Quartile 1            -0.0032  -0.0180   -0.0075
Median                 0.0110   0.0105    0.0051
Arithmetic Mean        0.0095   0.0078    0.0048
Geometric Mean         0.0093   0.0068    0.0046
Quartile 3             0.0214   0.0390    0.0167
Maximum                0.0745   0.0978    0.0506
SE Mean                0.0019   0.0040    0.0019
LCL Mean (0.95)        0.0058  -0.0003    0.0011
UCL Mean (0.95)        0.0132   0.0158    0.0085
Variance               0.0004   0.0020    0.0004
Stdev                  0.0205   0.0443    0.0204
Skewness               0.0175  -0.5254   -0.4389
Kurtosis               0.8456   0.3965    0.9054
```

Testing for Normality

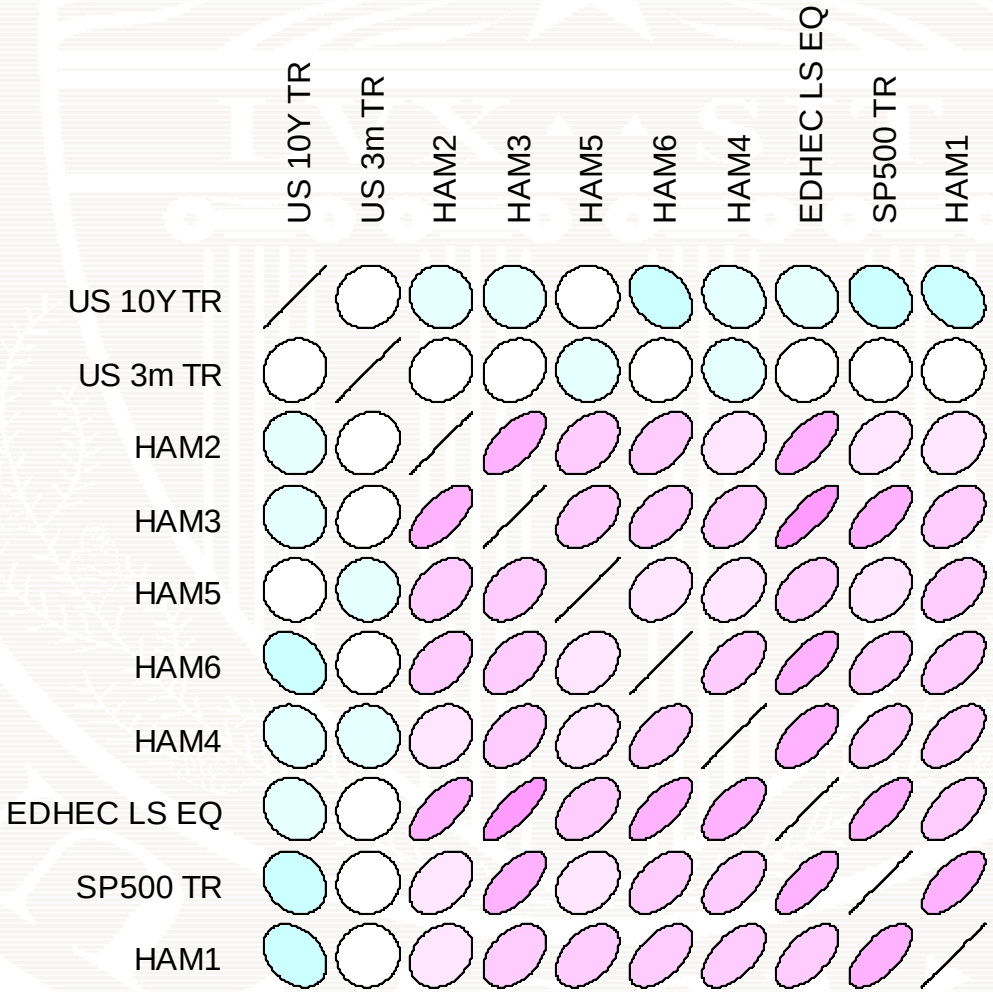
```
# use jarque.bera.test() function from tseries package
> jarque.bera.test(managers.df$HAM1)

        Jarque Bera Test

data:  managers.df$HAM1
X-squared = 33.7, df = 2, p-value = 4.787e-08
```

	HAM1	HAM2	HAM3	HAM4	HAM5	HAM6
statistic	33.7	85.4	67.2	5.88	15.2	1.02
P-value	0.000	0.000	0.000	0.053	0.000	0.602

Conclusion: All assets non-normal except HAM6

Sample Correlations

(pairwise complete obs)



Macroeconomic Factor Model (FM)

$$R_{it} = \alpha_i + \beta_{1i} EDHEC.LS.EQ_t + \beta_{2i} SP500.TR_t + \beta_{3i} US.10Y.TR_t + \varepsilon_{it}$$

- R_{it} = return in excess of T-Bill rate on hedge fund *i* in month *t*.
- $EDEC.LS.EQ_t$ = excess total return on EDHEC long-short equity index ("exotic risk factor")
- $SP500.TR_t$ = excess total return on S&P 500 index (traditional equity risk factor)
- $US.10.YR_t$ = excess total return on US 10 year T-Note (traditional rates risk factor)

Prepare Data for Regression

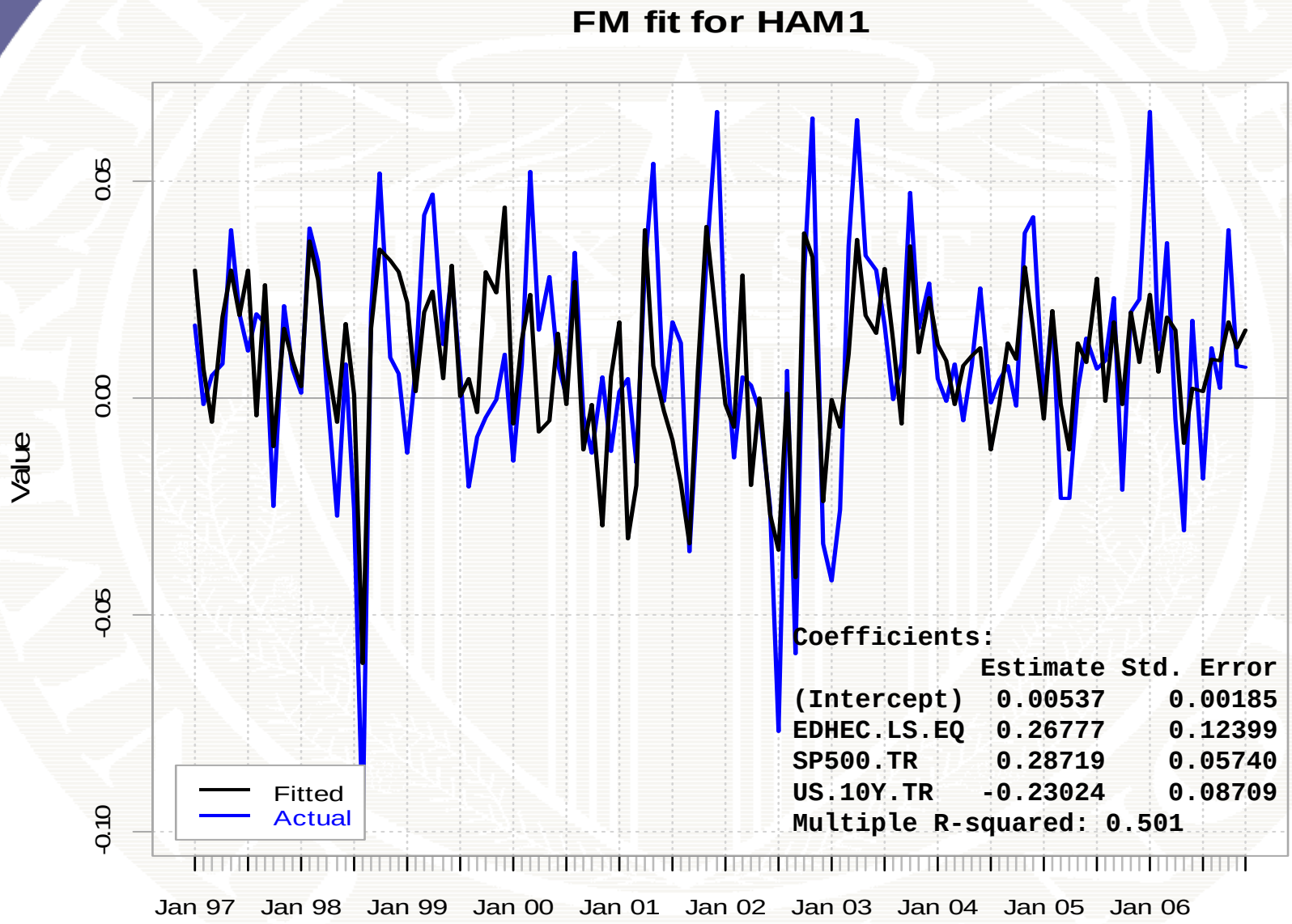
```
# subtract "US 3m TR" (risk free rate) from all
# returns. note: apply() changes managers.df to class
# "matrix"
> managers.df = apply(managers.df, 2,
+         function(x) {x - managers.df[,"US 3m TR"]})
> managers.df = as.data.frame(managers.df)
# remove US 3m TR from data.frame
> managers.df = managers.df[, -10]
# extract variable names for later use
> manager.names = colnames(managers.df)[1:6]
# eliminate spaces in factor names
> factor.names = c("EDHEC.LS.EQ", "SP500.TR",
+                  "US.10Y.TR")
> colnames(managers.df)[7:9] = colnames(managers)[7:9]
+                              = factor.names
>managers.zoo = as.zoo(na.omit(managers[,
+                               manager.names]))
```

Fit FM by Least Squares

```
# initialize list object to hold regression objects
> reg.list = list()

# initialize matrices and vectors to hold regression
# alphas, betas, residual variances and r-squared values
> Betas = matrix(0, length(manager.names),
+                   length(factor.names))
> colnames(Betas) = factor.names
> rownames(Betas) = manager.names
> Alphas = ResidVars = R2values =
+          rep(0,length(manager.names))
> names(Alphas) = names(ResidVars) = names(R2values) =
+                   manager.names
```

Fit FM by Least Squares

```
# loop over all assets and estimate time series
# regression
> for (i in manager.names) {
+  reg.df = na.omit(managers.df[, c(i, factor.names)])
+  fm.formula = as.formula(paste(i,"~", ".", sep=" "))
+  fm.fit = lm(fm.formula, data=reg.df)
+  fm.summary = summary(fm.fit)
+  reg.list[[i]] = fm.fit
+  Alphas[i] = coef(fm.fit)[1]
+  Betas[i, ] = coef(fm.fit)[-1]
+  ResidVars[i] = fm.summary$sigma^2
+  R2values[i] =  fm.summary$r.squared
+ }

> names(reg.list)
[1] "HAM1" "HAM2" "HAM3" "HAM4" "HAM5" "HAM6"
```



Regression Results

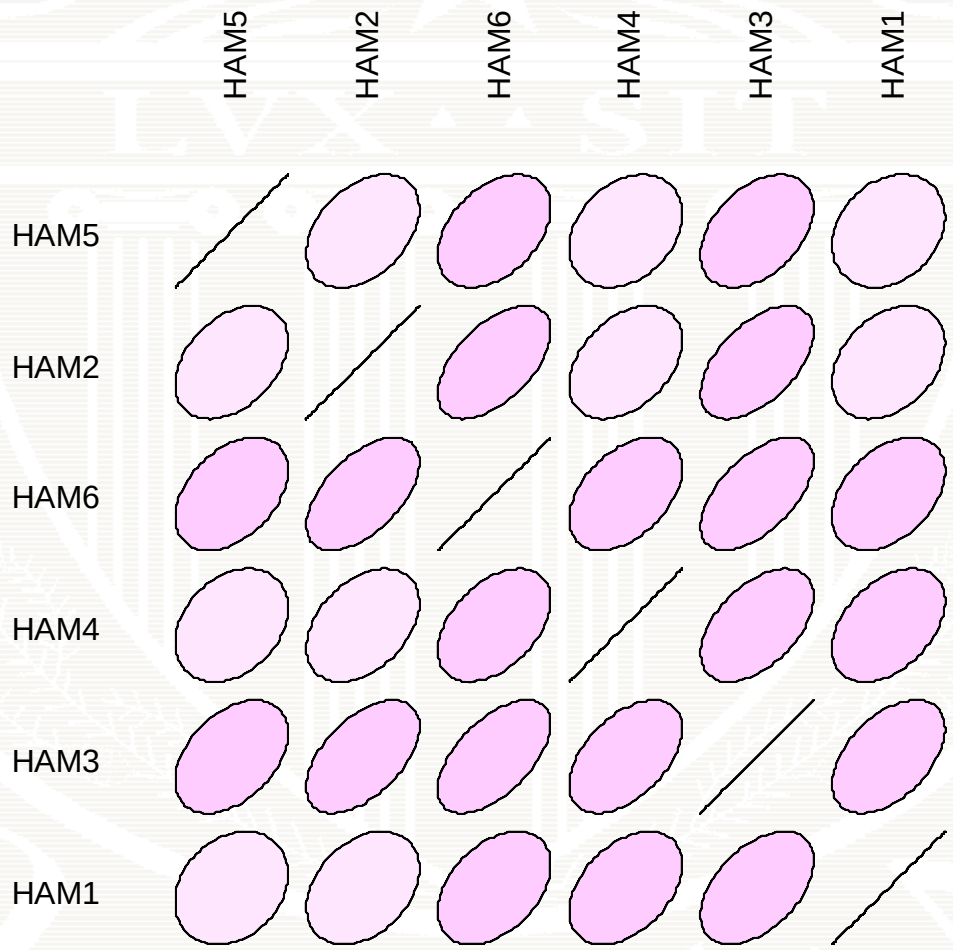
Fund	Intercept	LS.EQ	SP500	US.10YR	σ	R^2
HAM1	0.005***	0.268**	0.287***	-0.230***	0.019	0.501
HAM2	0.001	1.547***	-0.195**	0.050	0.025	0.514
HAM3	-0.001	1.251***	0.131**	0.144	0.022	0.657
HAM4	-0.002	1.222***	0.273**	-0.139	0.043	0.413
HAM5	-0.005	1.621***	-0.184	0.271	0.040	0.232
HAM6	0.004***	1.250***	-0.175*	-0.174*	0.016	0.564

***, **, * denote significance at the 1%, 5% and 10% level, respectively

FM Covariance Matrix

```
# risk factor sample covariance matrix
> cov.factors = var(managers.df[, factor.names])

# FM covariance matrix
> cov.fm = Betas%*%cov.factors%*%t(Betas) +
+          diag(ResidVars)

# FM correlation matrix
> cor.fm = cov2cor(cov.fm)
```

FM Correlations



Fund of Hedge Funds (FoHF)

Equally weighted portfolio (fund of hedge funds):

$$w_i = \frac{1}{6},\ i = \text{HAM1},\ldots,\text{HAM6}$$

```
> w.vec = rep(1,6)/6
> names(w.vec) = manager.names
> w.vec
 HAM1  HAM2  HAM3  HAM4  HAM5  HAM6
0.167 0.167 0.167 0.167 0.167 0.167
```

```
# portfolio returns. Note: need to eliminate NA values
# from HAM5 and HAM6
> r.p = as.matrix(na.omit(managers.df[,
                          manager.names]))%*%w.vec
> r.p.zoo = zoo(r.p, as.Date(rownames(r.p)))
```

FoHF (Portfolio) FM

```
# portfolio factor model
> alpha.p = as.numeric(crossprod(Alphas,w.vec))
> beta.p = t(Betas)%*%w.vec

> var.systematic = t(beta.p)%*%cov.factors%*%beta.p
> var.specific = t(w.vec)%*%diag(ResidVars)%*%w.vec
> var.fm.p = var.systematic + var.specific
> var.fm.p = as.numeric(var.fm.p)
> r.square.p = as.numeric(var.systematic/var.fm.p)

> fm.p = c(alpha.p, beta.p, sqrt(var.fm.p), r.square.p)
> names(fm.p) = c("intercept", factor.names, "sd", “r2")
> fm.p
  intercept EDHEC.LS.EQ    SP500.TR   US.10Y.TR          sd
   0.000455    1.193067    0.022973   -0.012990    0.027817
  r-squared
   0.812435
```

Factor Risk Budgeting

```
# use factorModelFactorSdDecomposition() function
# from factorAnalytics package
> args(factorModelFactorSdDecomposition)
function (beta.vec, factor.cov, sig2.e)

# Compute factor SD decomposition for HAM1
> factor.sd.decomp.HAM1 =
factorModelFactorSdDecomposition(Betas["HAM1",],
+                  cov.factors, ResidVars["HAM1"])

> names(factor.sd.decomp.HAM1)
[1] "sd.fm"  "mcr.fm" "cr.fm"  "pcr.fm"
```

Factor Contributions to SD

```
> factor.sd.decomp.HAM1
$sd.fm
[1] 0.0265

$mcr.fm
    EDHEC.LS.EQ SP500.TR US.10Y.TR residual
MCR      0.0119   0.0295  -0.00638    0.711

$cr.fm
   EDHEC.LS.EQ SP500.TR US.10Y.TR residual
CR     0.00318  0.00847   0.00147   0.0134

$pcr.fm
    EDHEC.LS.EQ SP500.TR US.10Y.TR residual
PCR        0.12    0.319    0.0553    0.506
```

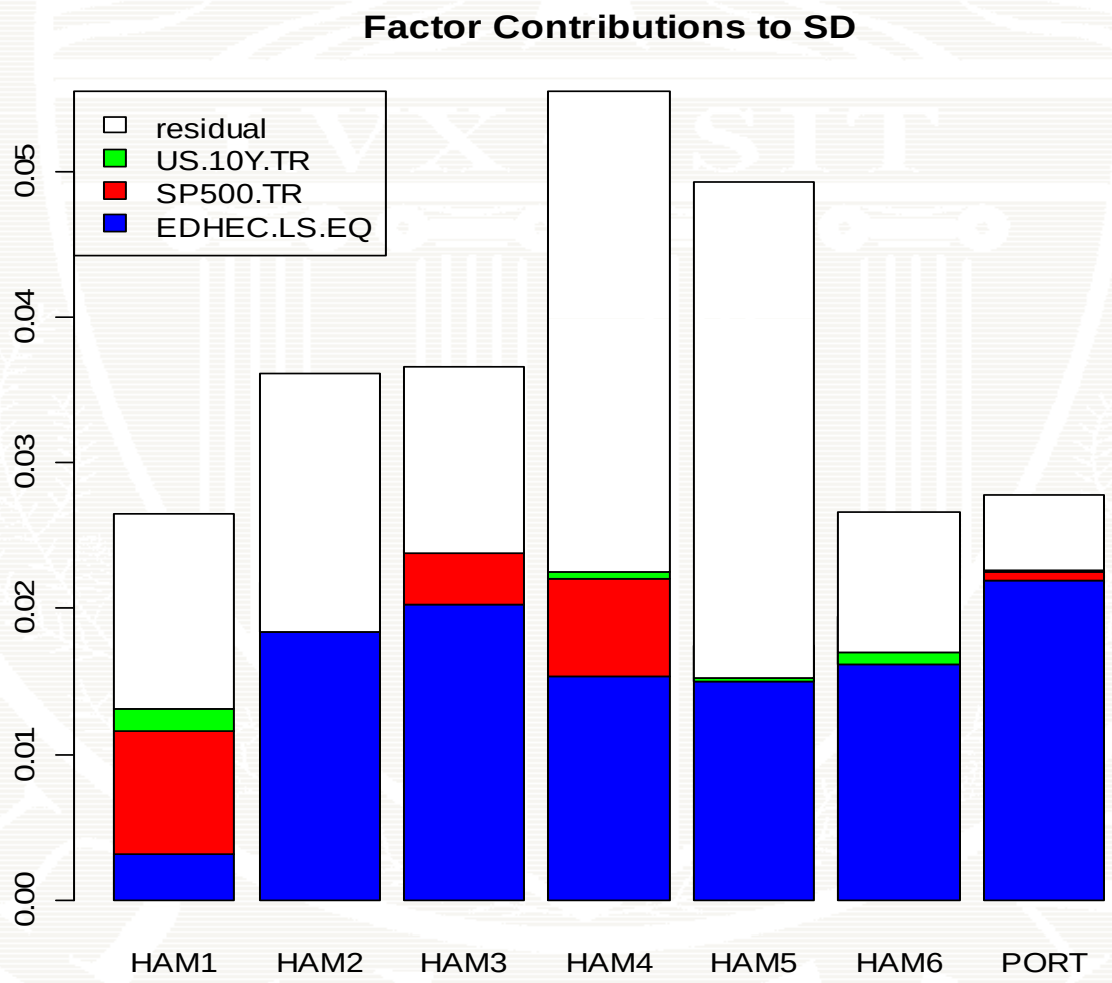
Factor Contributions to SD

```
# loop over all assets and store results in list
> factor.sd.decomp.list = list()
> for (i in manager.names) {
+  factor.sd.decomp.list[[i]] =
factorModelFactorSdDecomposition(Betas[i,],
+                 cov.factors, ResidVars[i])
+ }

# add portfolio factor SD decomposition to list
> factor.sd.decomp.list[["PORT"]] =
factorModelFactorSdDecomposition(beta.p,
+                cov.factors, var.p.resid)

> names(factor.sd.decomp.list)
[1] "HAM1" "HAM2" "HAM3" "HAM4" "HAM5" "HAM6" "PORT"
```

Factor Contributions to SD

```
# function to extract contribution to sd from list
> getCSD = function(x) {
+   x$cr.fm
+}

# extract contributions to SD from list
> cr.sd = sapply(factor.sd.decomp.list, getCSD)
> rownames(cr.sd) = c(factor.names, "residual")


# create stacked barchart
> barplot(cr.sd, main="Factor Contributions to SD",
+         legend.text=T, args.legend=list(x="topleft"),
+         col=c("blue","red","green","white"))
```

Factor Contributions to SD



Factor Contributions to ETL

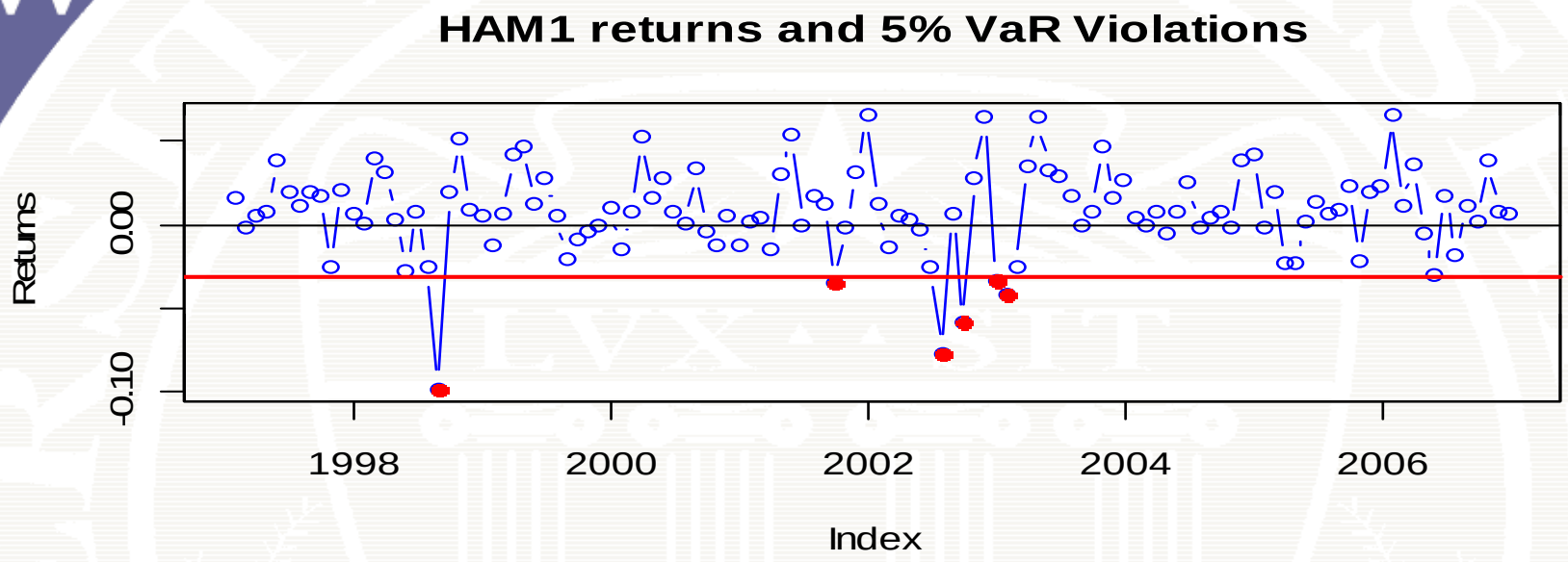
```
# first combine HAM1 returns, factors and std residuals
> tmpData = cbind(managers.df[,1],
+                 managers.df[,factor.names],
+           residuals(reg.list[[1]])/sqrt(ResidVars[1]))
> colnames(tmpData)[c(1,5)] = c(manager.names[1],
                                "residual")

> factor.es.decomp.HAM1 =
factorModelFactorEsDecomposition(tmpData, Betas[1,],
+                                ResidVars[1], tail.prob=0.05)
> names(factor.es.decomp.HAM1)
[1] "VaR.fm"     "n.exceed"   "idx.exceed" "ES.fm"
[5] "mcES.fm"    "cES.fm"     "pcES.fm"
```

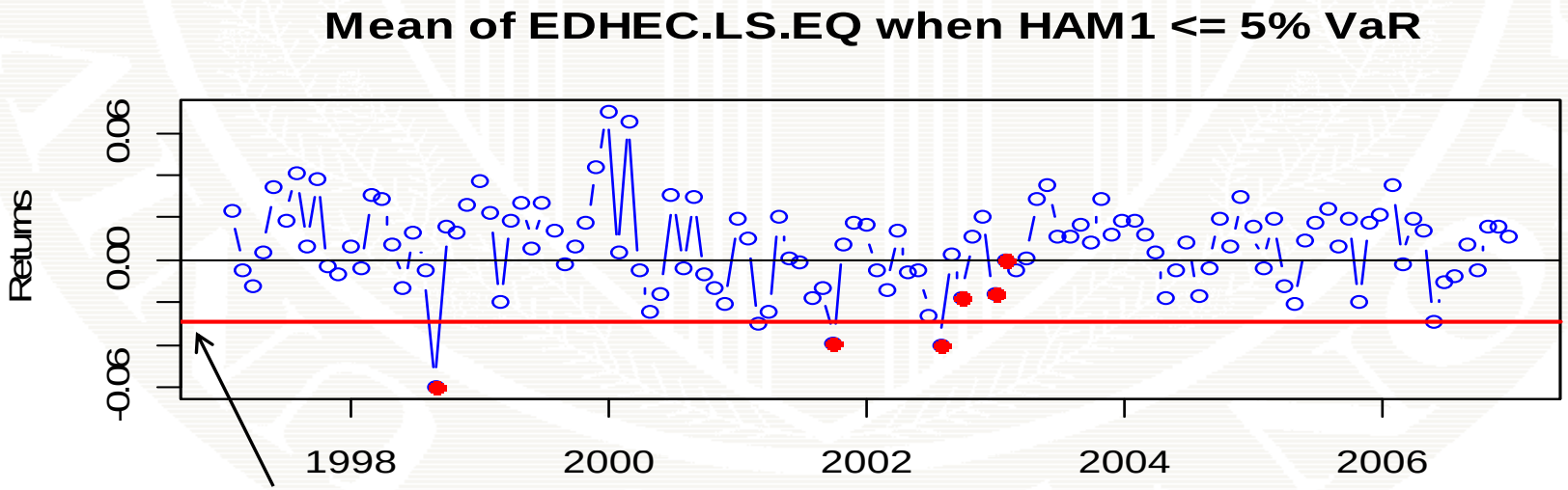
Factor Contributions to ETL: HAM1

```
> factor.es.decomp.HAM1
$VaR.fm
     5%
0.0309

$n.exceed
[1] 6

$idx.exceed
1998-08-30 2001-09-29 2002-07-30 2002-09-29 2002-12-30
        20         57         67         69         72
2003-01-30
        73
```

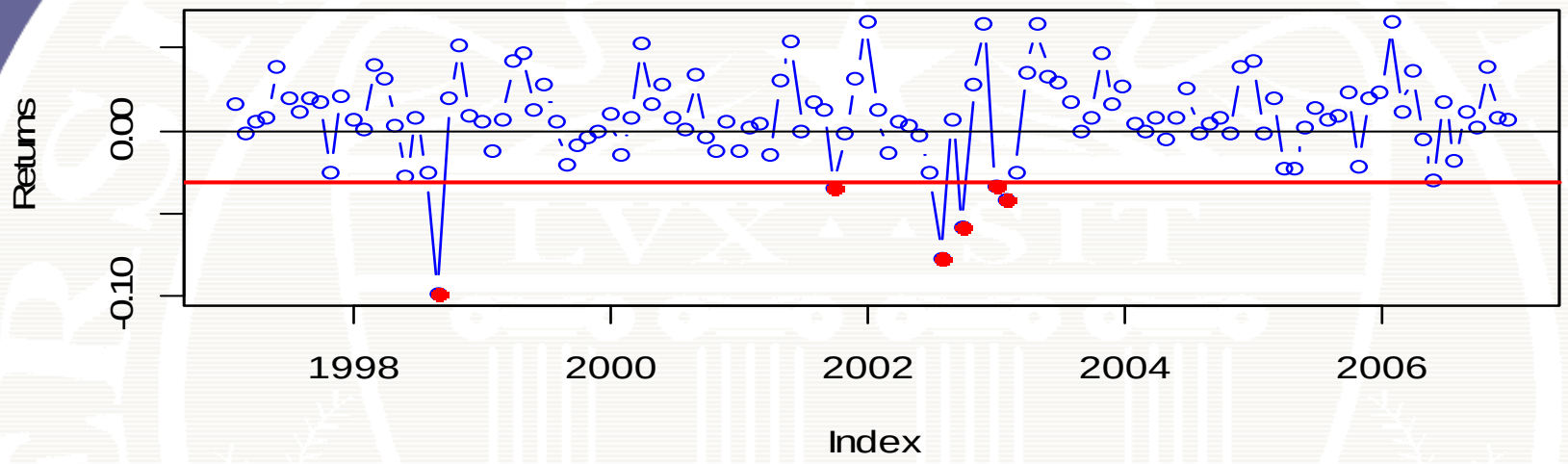
Factor Contributions to ETL: HAM1

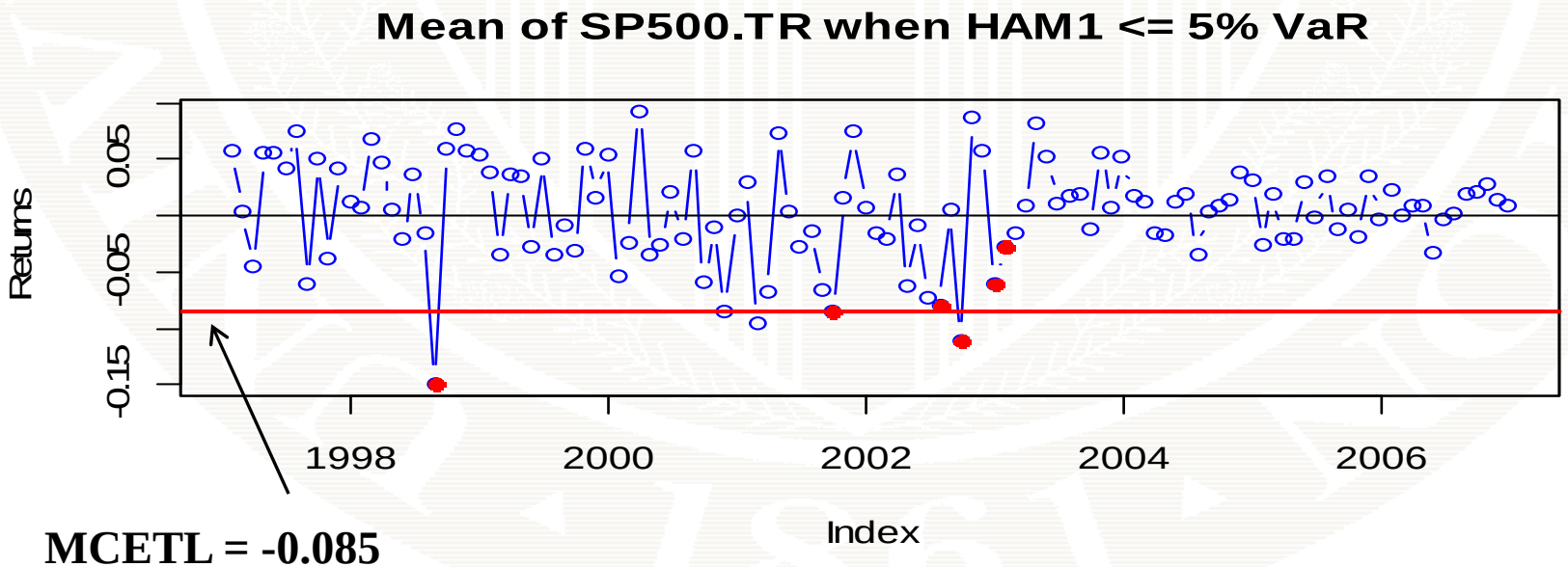
```
$ES.fm
[1] 0.0577

$mcES.fm
     EDHEC.LS.EQ SP500.TR US.10Y.TR residual
MCES      0.0289   0.0852   -0.0255     1.32

$cES.fm
    EDHEC.LS.EQ SP500.TR US.10Y.TR residual
CES     0.00774   0.0245   0.00587    0.025

$pcES.fm
     EDHEC.LS.EQ SP500.TR US.10Y.TR residual
PCES       0.134    0.424     0.102    0.433
```



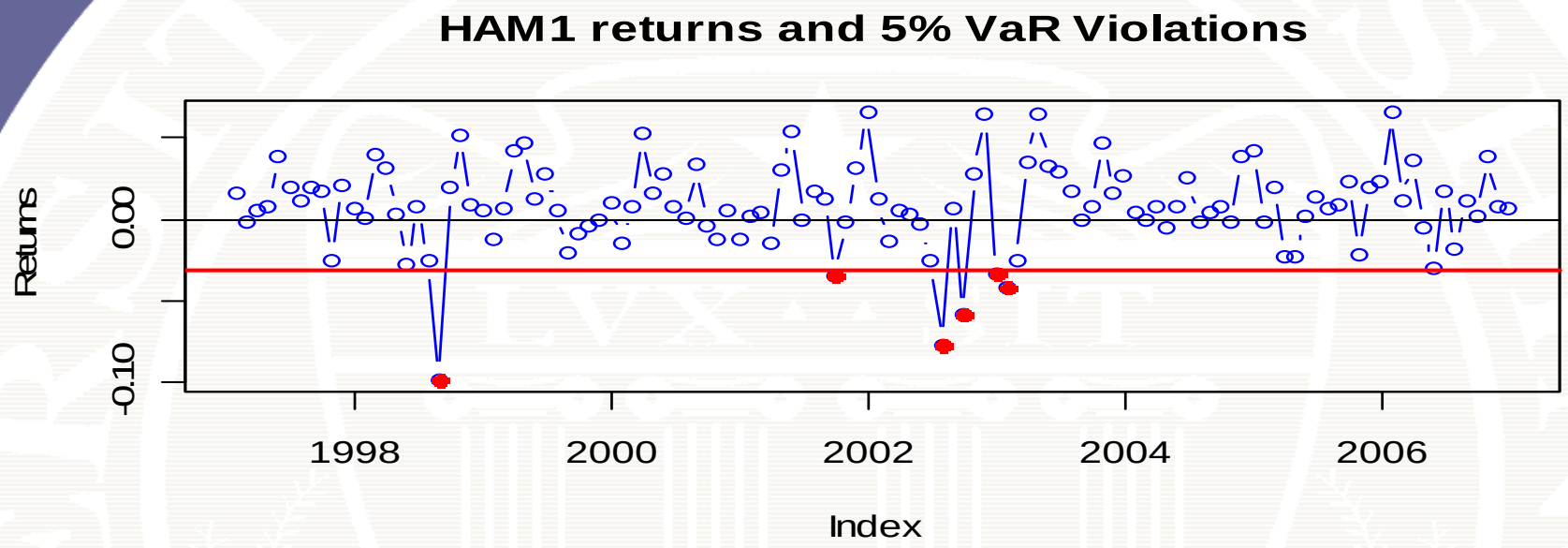


MCETL = -0.029

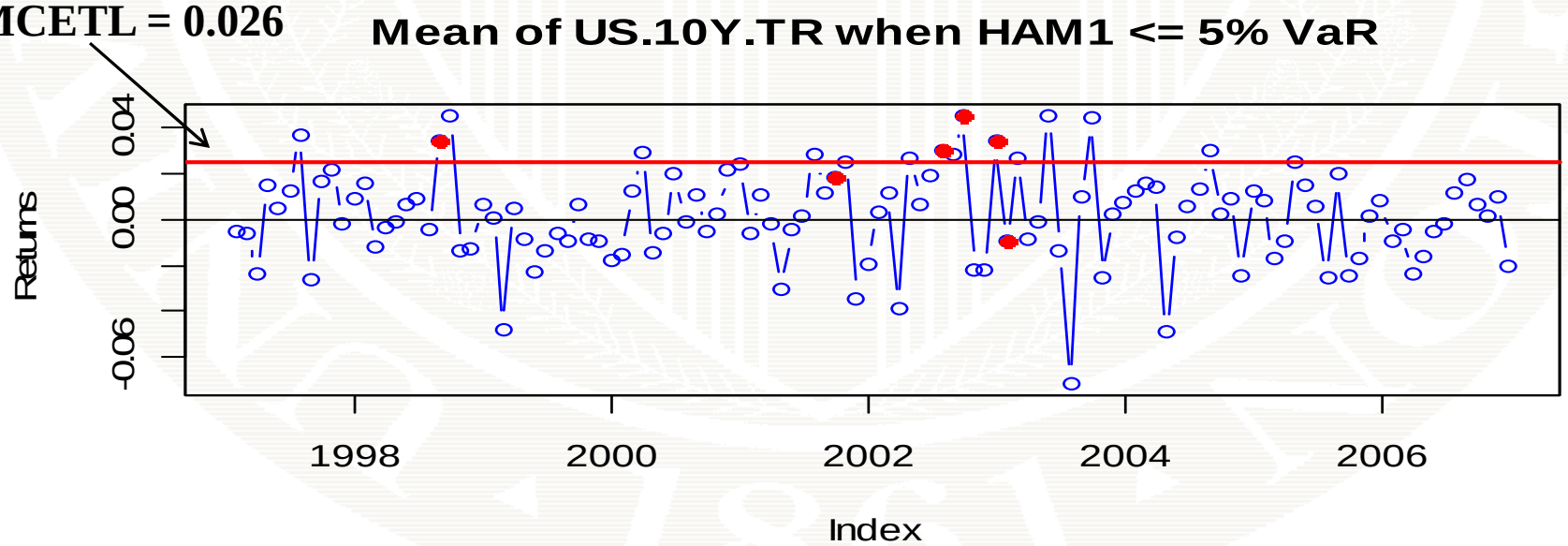


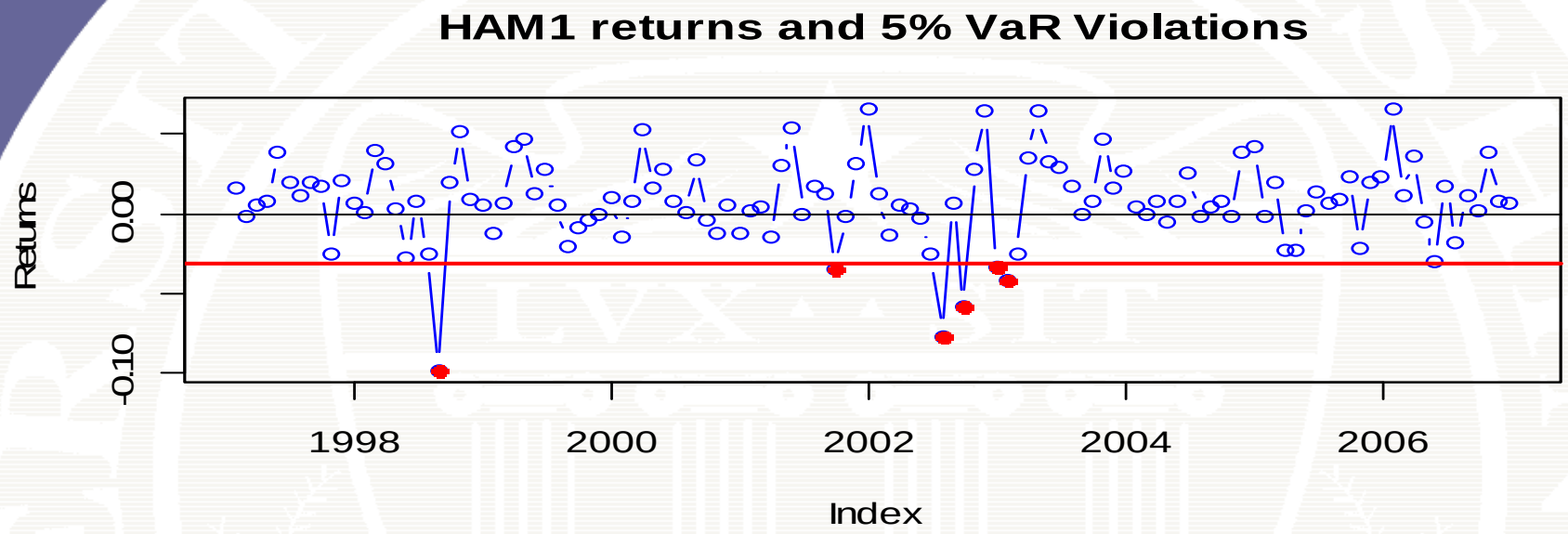


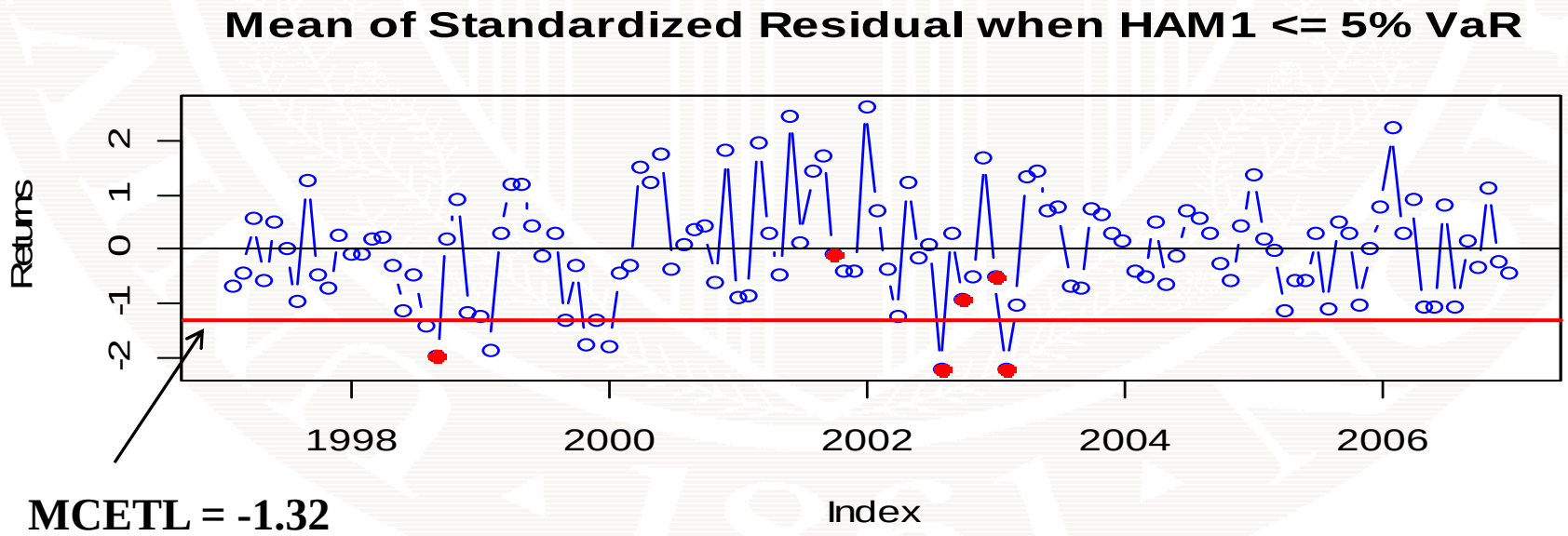
MCETL = -0.085



MCETL = 0.026

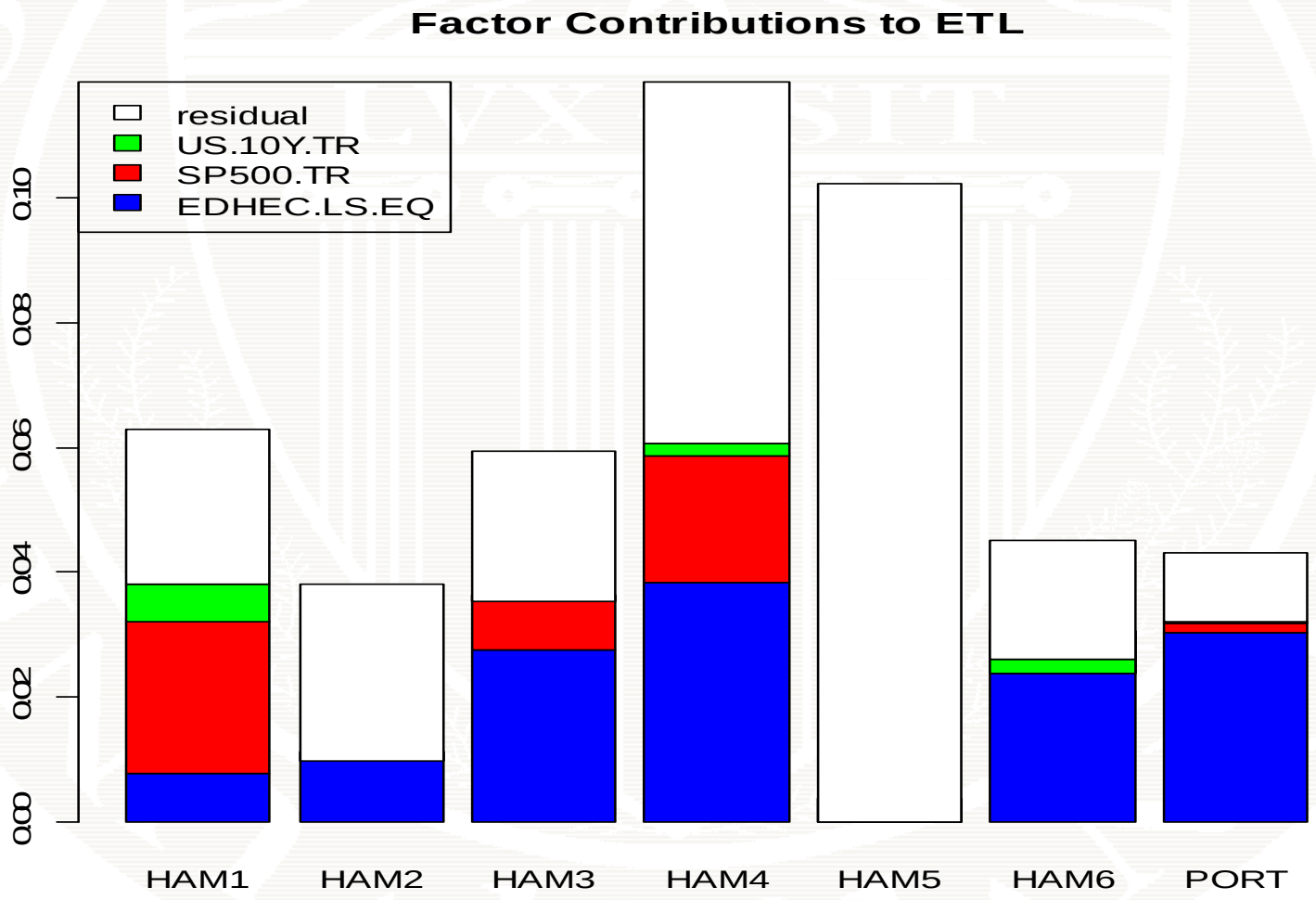






MCETL = -1.32

Factor Contributions to ETL



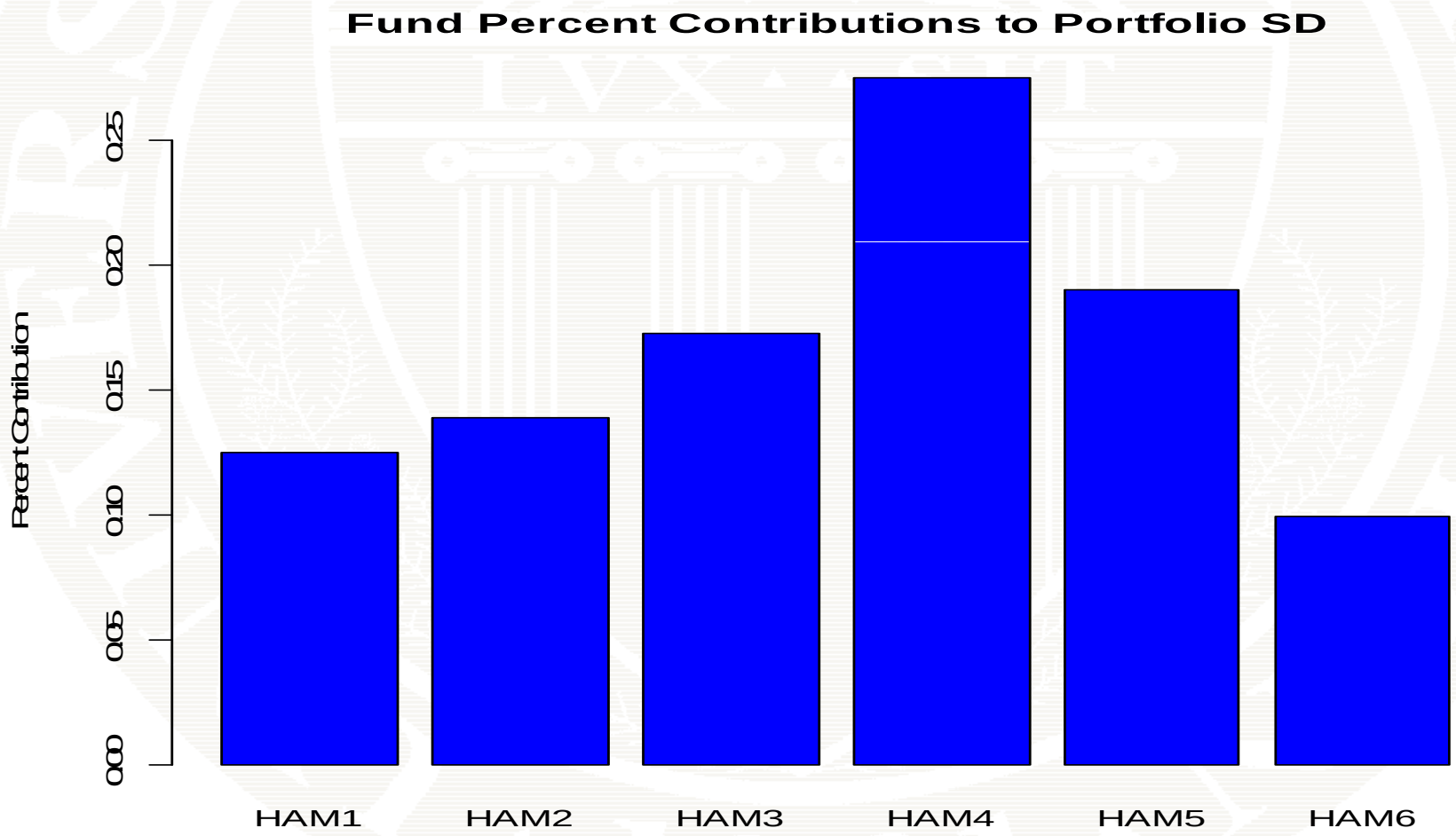
Portfolio Risk Budgeting

```
# use portfolioSdDecomposition() function from
# factorAnalytics package
> args(portfolioSdDecomposition)
function (w.vec, cov.assets)
# compute with sample covariance matrix (pairwise
# complete observations)
> cov.sample = cov(managers.df[,manager.names],
+                  use="pairwise.complete.obs")
> port.sd.decomp.sample =
+ portfolioSdDecomposition(w.vec, cov.sample)

> names(port.sd.decomp.sample)
[1] "sd.p"   "mcsd.p" "csd.p"  "pcsd.p"
```

Portfolio SD Decomposition

```
> port.sd.decomp.sample
$sd.p
[1] 0.0261

$mcsd.p
       HAM1   HAM2   HAM3   HAM4   HAM5   HAM6
MCSD 0.0196 0.0218 0.0270 0.0431 0.0298 0.0155

$csd.p
       HAM1    HAM2    HAM3    HAM4    HAM5    HAM6
CSD 0.00327 0.00363 0.00451 0.00718 0.00497 0.00259

$pcsd.p
      HAM1  HAM2  HAM3  HAM4 HAM5  HAM6
PCSD 0.125 0.139 0.172 0.275 0.19 0.099
```

Fund Contributions to Portfolio SD



Portfolio ETL Decomposition

```
# use ES() function in PerformanceAnalytics package
> port.ES.decomp =
ES(na.omit(managers.df[,manager.names]),
+                    p=0.95, method="historical",
+                    portfolio_method = "component",
+                    weights = w.vec)
> port.ES.decomp
$`-r_exceed/c_exceed`
[1] 0.0479

$c_exceed
[1] 3

$realizedcontrib
  HAM1   HAM2   HAM3   HAM4   HAM5   HAM6
0.1874 0.0608 0.1479 0.3518 0.1886 0.0635
```

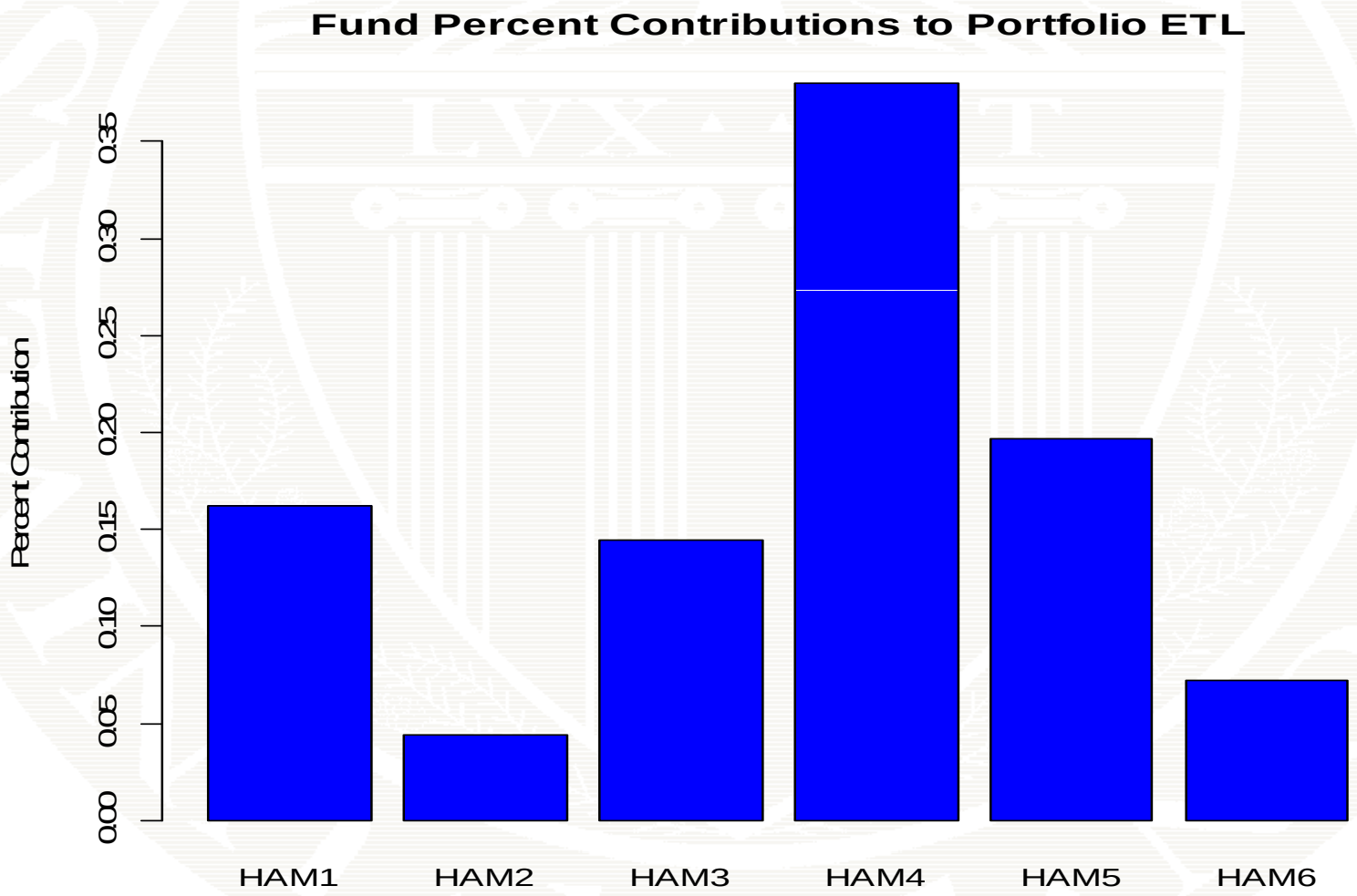
Portfolio ETL Decomposition

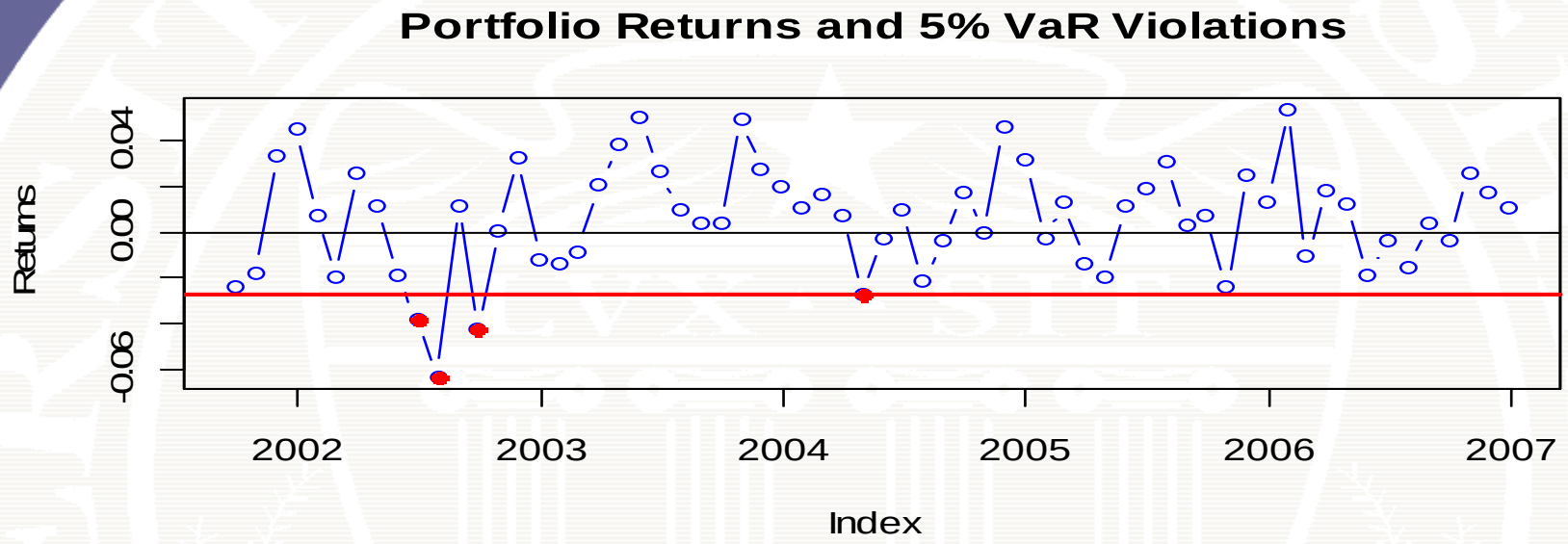
```
# use portfolioEsDecomposition from factorAnalytics
# package.
> args(portfolioEsDecomposition)
function (bootData, w, delta.w = 0.001, tail.prob =
     0.01, method = c("derivative",
    "average"), VaR.method = c("HS", "CornishFisher"))

> port.ES.decomp =
portfolioEsDecomposition(na.omit(managers.df[,manager.
                         names]),w.vec, tail.prob=0.05)

> names(port.ES.decomp)
[1] "VaR.fm"      "ES.fm"       "n.exceed"    "idx.exceed"
[5] "MCES"        "CES"         "PCES"
```

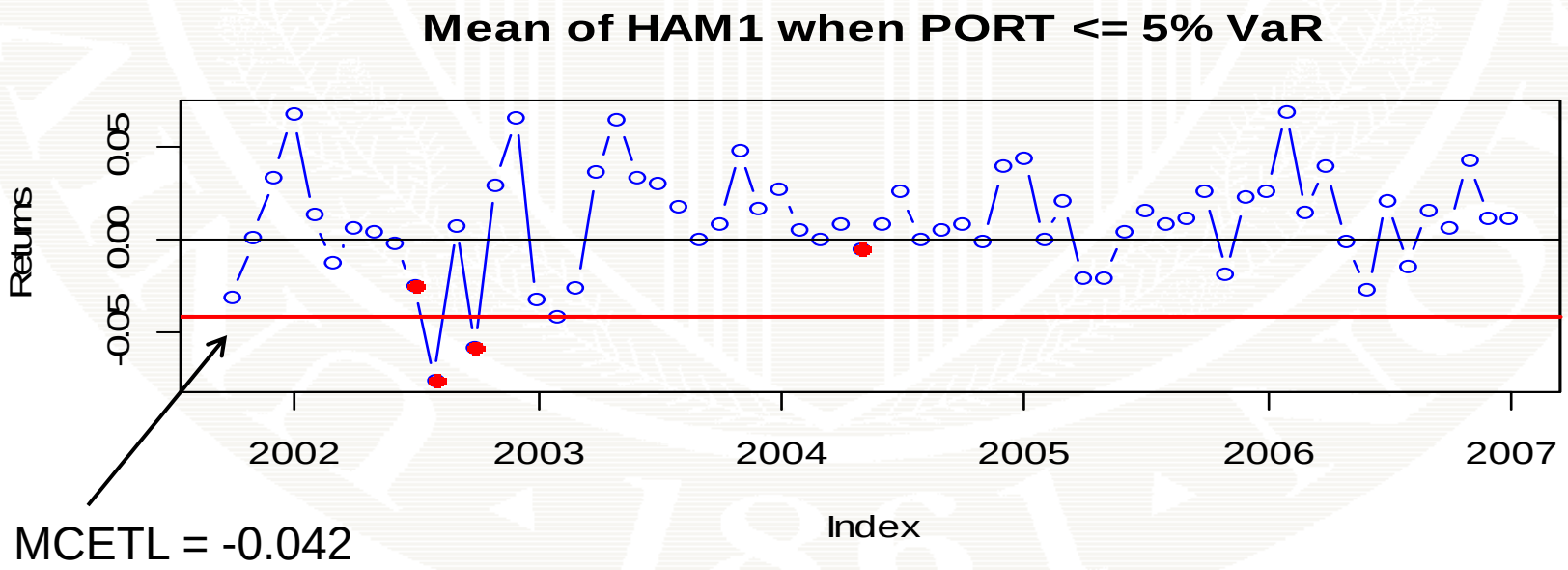
Portfolio ETL Decomposition

```
> port.ES.decomp
$VaR.fm      $ES.fm        $n.exceed    $idx.exceed
     5%      [1] 0.0428    [1] 4        [1] 10 11 13 32
 0.0269

$MCES
       HAM1   HAM2   HAM3   HAM4   HAM5   HAM6
MCES 0.0417 0.0113 0.0371 0.0976 0.0505 0.0186

$CES
       HAM1    HAM2    HAM3   HAM4    HAM5    HAM6
CES 0.00695 0.00188 0.00618 0.0163 0.00842 0.00310

$PCES
      HAM1   HAM2  HAM3 HAM4  HAM5   HAM6
PCES 0.162 0.0439 0.145 0.38 0.197 0.0724
```

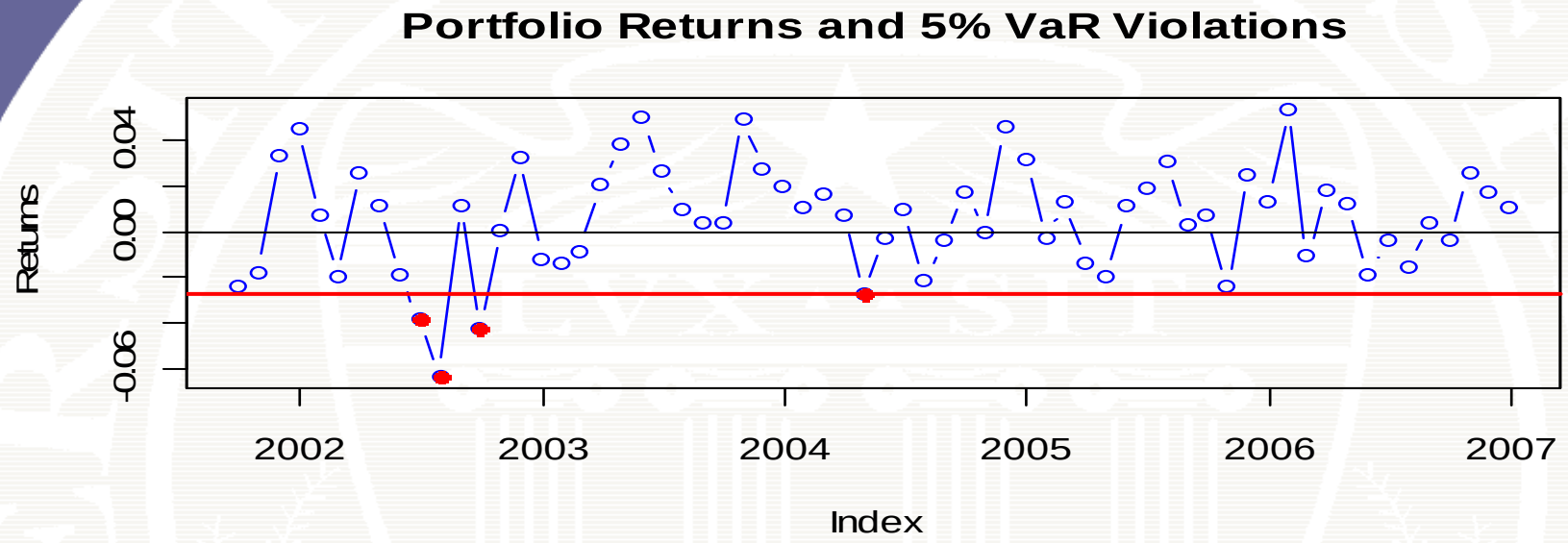
Fund Contributions to Portfolio ETL

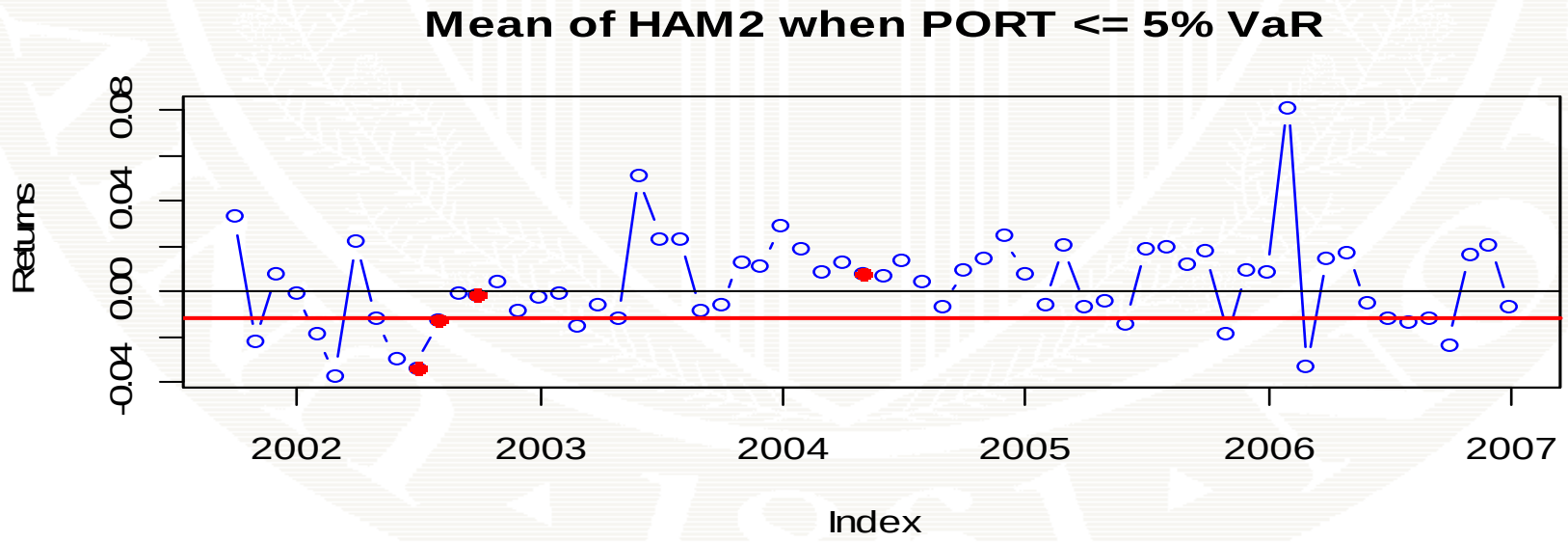


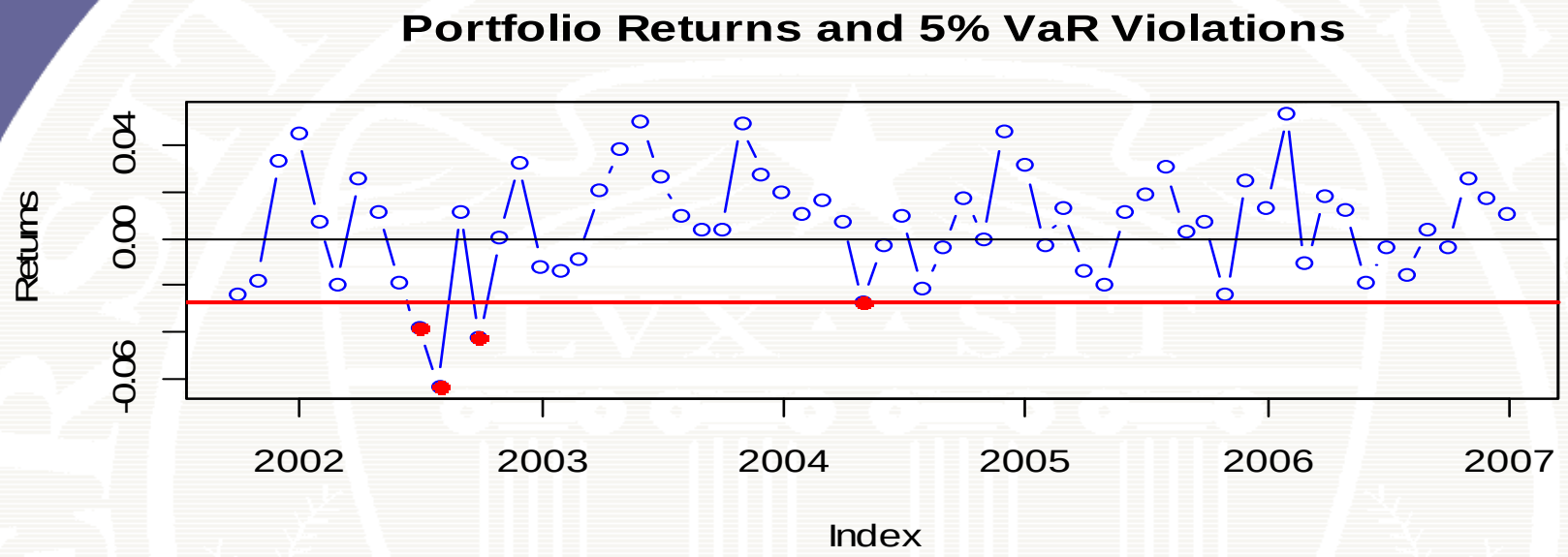


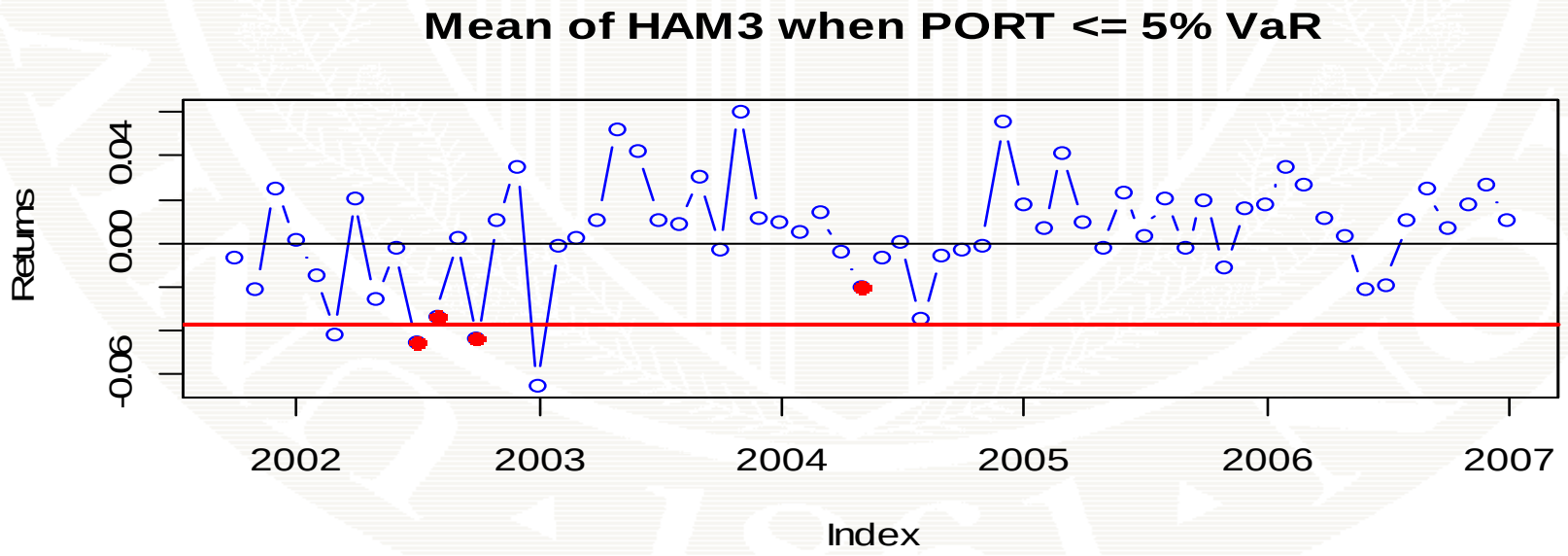


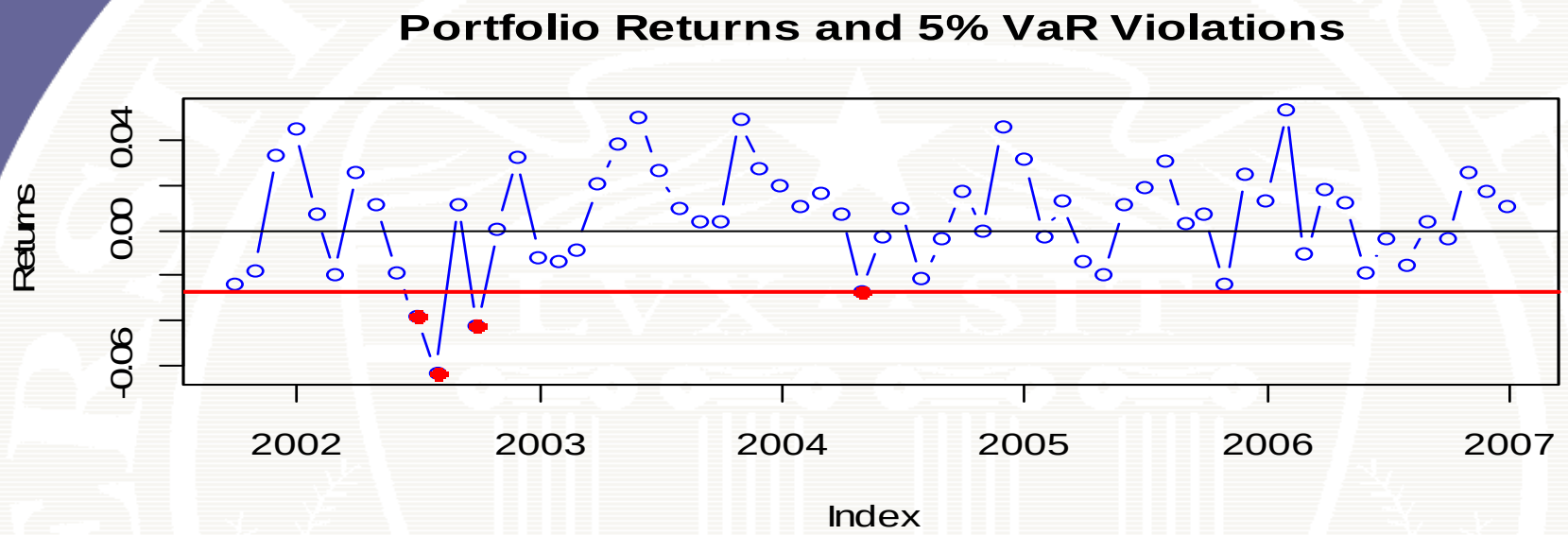
MCETL = -0.042

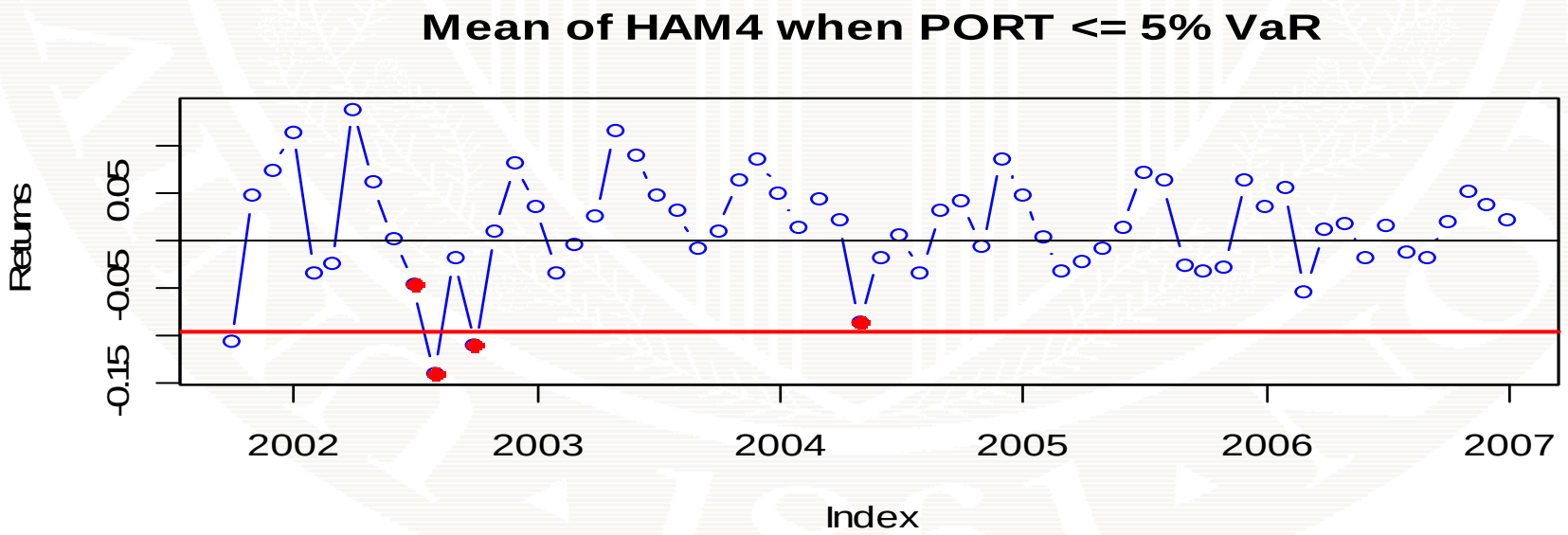


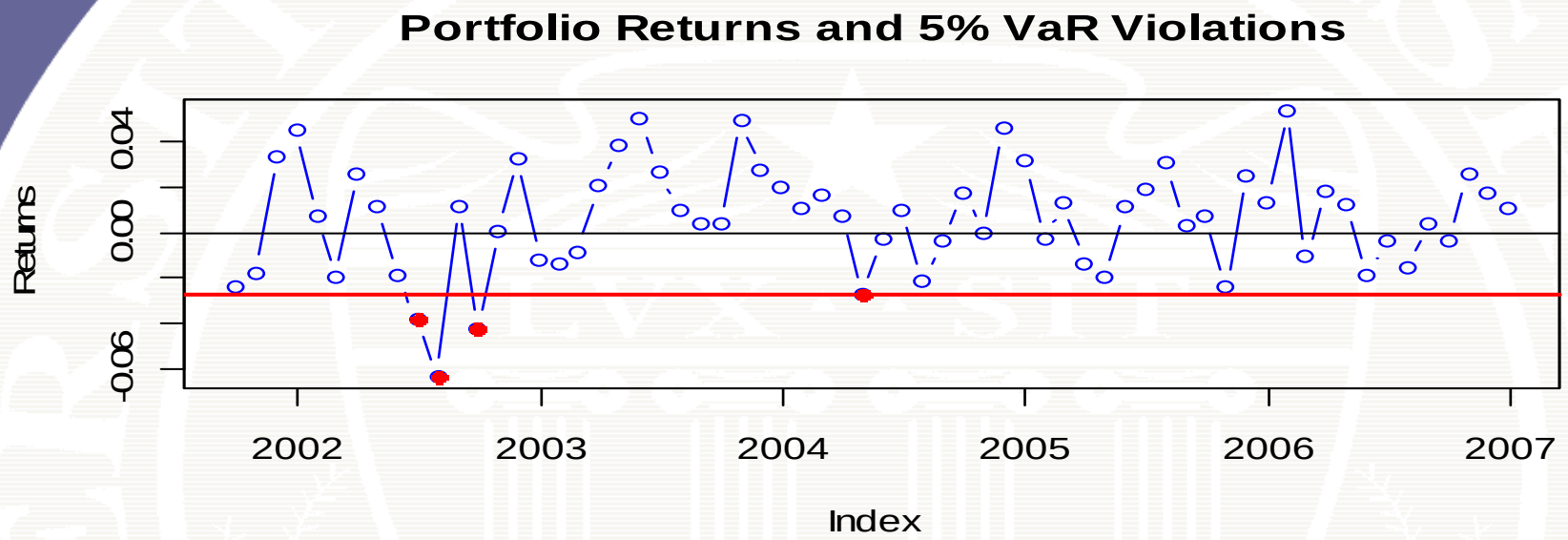


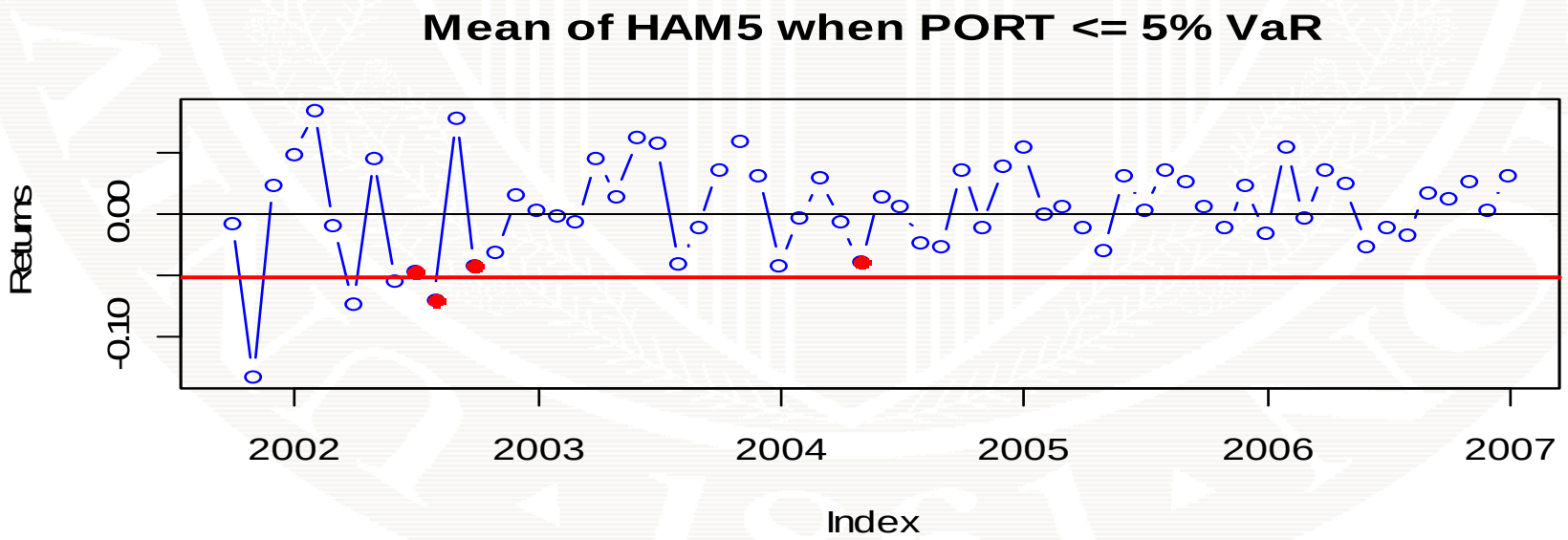


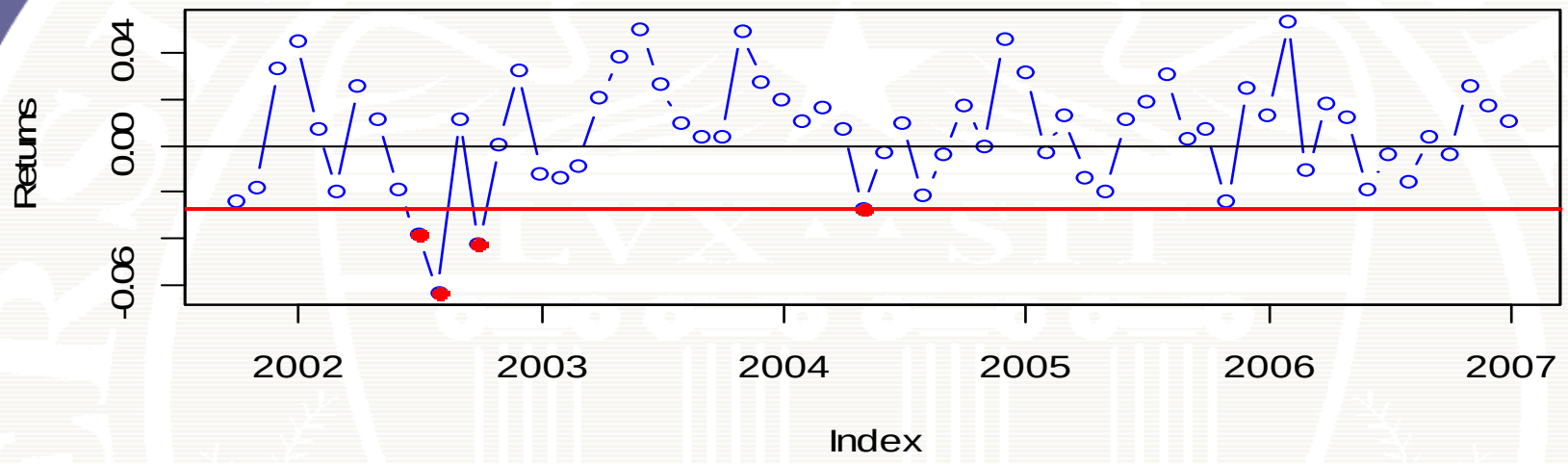


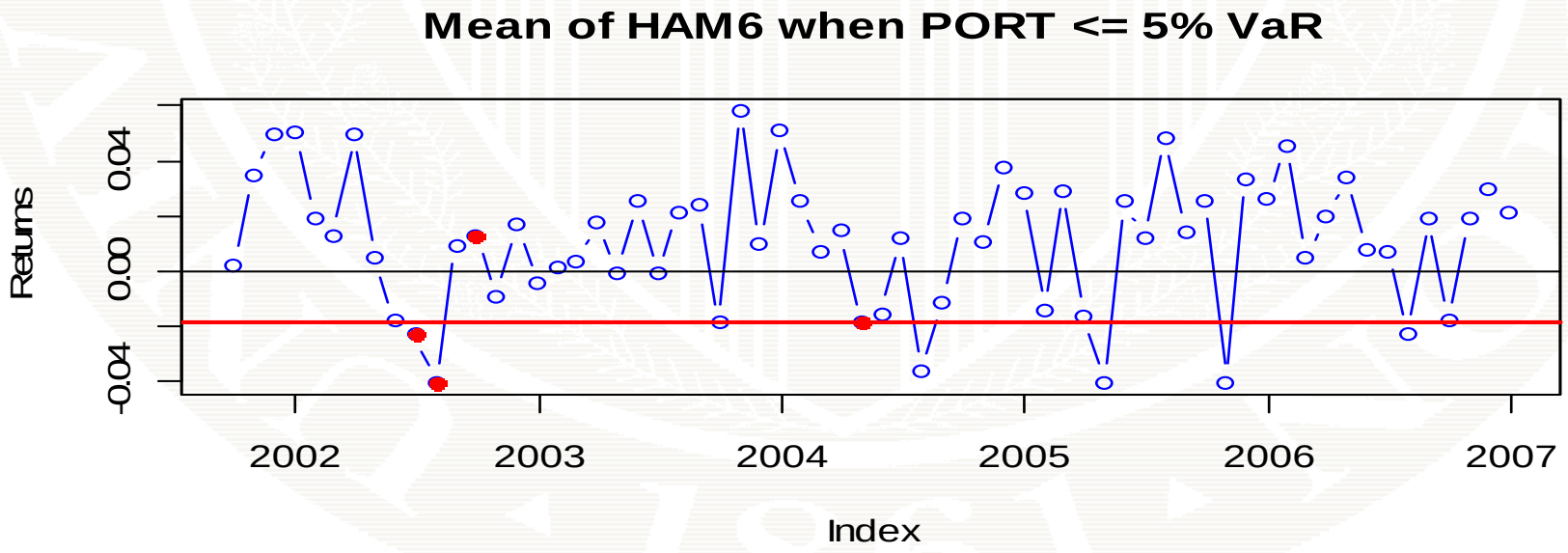












Factor Model Monte Carlo (FMMC)

```
# resample from historical factors
> n.boot = 5000

# set random number seed
> set.seed(123)

# n.boot reshuffled indices
> bootIdx = sample(nrow(managers.df),  n.boot,
+                  replace=TRUE)

# resampled factor data
> factorDataBoot.mat = as.matrix(managers.df[bootIdx,
+                                factor.names])
```

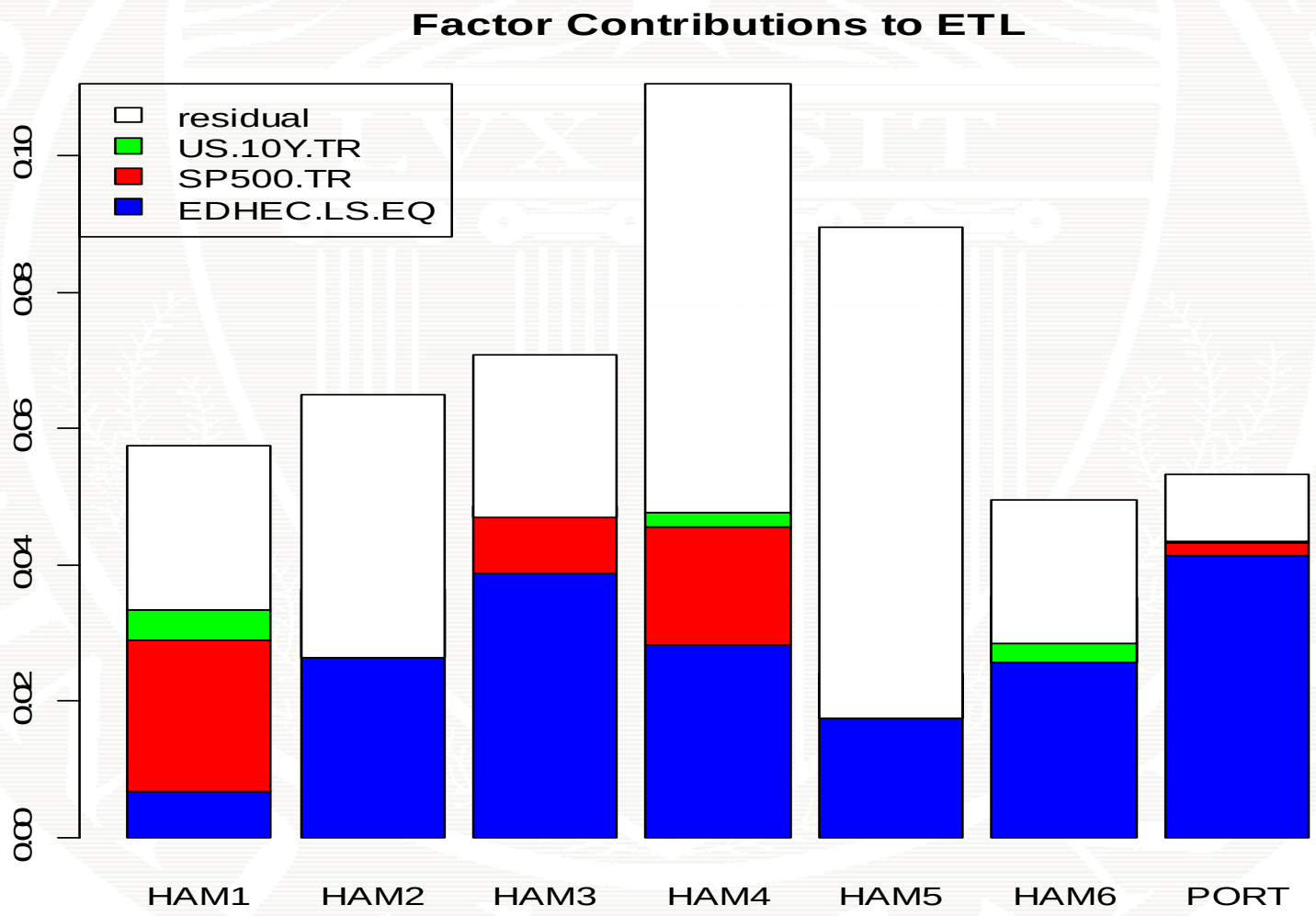
FMMC with Normal Residuals

```
# FMMC using normal distribution for residuals and
# alpha = 0
> returns.boot = matrix(0, n.boot, length(manager.names))
> resid.sim = matrix(0, n.boot, length(manager.names))
> colnames(returns.boot) = colnames(resid.sim) =
+                          manager.names
# FMMC loop
for (i in manager.names) {
   returns.fm = factorDataBoot.mat%*%Betas[i, ]
   resid.sim[, i] = rnorm(n.boot,sd=sqrt(ResidVars[i]))
   returns.boot[, i] = returns.fm + resid.sim[, i]
}

# compute portfolio return and fm residual
> return.p.boot = returns.boot%*%wvec
> resid.fm.p = resid.sim%*%w.vec
```

FMMC Factor Contribution to ETL

```
# compute decomposition in loop
> factor.es.decomp.list = list()
> for (i in manager.names) {
+ tmpData = cbind(returns.boot[, i], factorDataBoot.mat,
+                  resid.sim[, i]/sqrt(ResidVars[i]))
> colnames(tmpData)[c(1,5)] = c(manager.names[i], "residual")
> factor.es.decomp.list[[i]] =
  factorModelFactorEsDecomposition(tmpData, Betas[i,],
+                         ResidVars[i], tail.prob=0.05)
}
# add portfolo retsults - need factor model residuals
> tmpData = cbind(r.p.boot, factorDataBoot.mat,
+                  resid.fm.p/sqrt(as.numeric(var.p.resid)))
> colnames(tmpData)[c(1,5)] = c("PORT", "residual")
> factor.es.decomp.list[["PORT"]] =
factorModelFactorEsDecomposition(tmpData, beta.p,
+                       var.p.resid, tail.prob=0.05)
```

FMMC Factor Contributions to ETL



FMMC Fund Contribution to Portfolio ETL

```
> port.ES.decomp.fmmc =
portfolioEsDecomposition(returns.boot,
+                         w.vec, tail.prob=0.05)
```

FMMC Fund Contributions to Portfolio ETL