Estimating Factor Models in R

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Eric Zivot

Estimation of Factor Models in R

- Data for examples
  - Berndt equity data from fEcofin package
- Estimation of fundamental factor model
  - BARRA-type industry model
- Estimation of statistical factor model
  - Principal components

Set Options and Load Packages

```
# set output options
> options(width = 70, digits=4)

# load required packages
> library(ellipse)                    # functions plotting
                                      # correlation matrices
> library(fEcofin)                    # various economic and
                                      # financial data sets
                                      # not available for R 3.0
> library(PerformanceAnalytics)       # performance and risk
                                      # analysis functions
> library(zoo)                        # time series objects
                                      # and utility functions
```

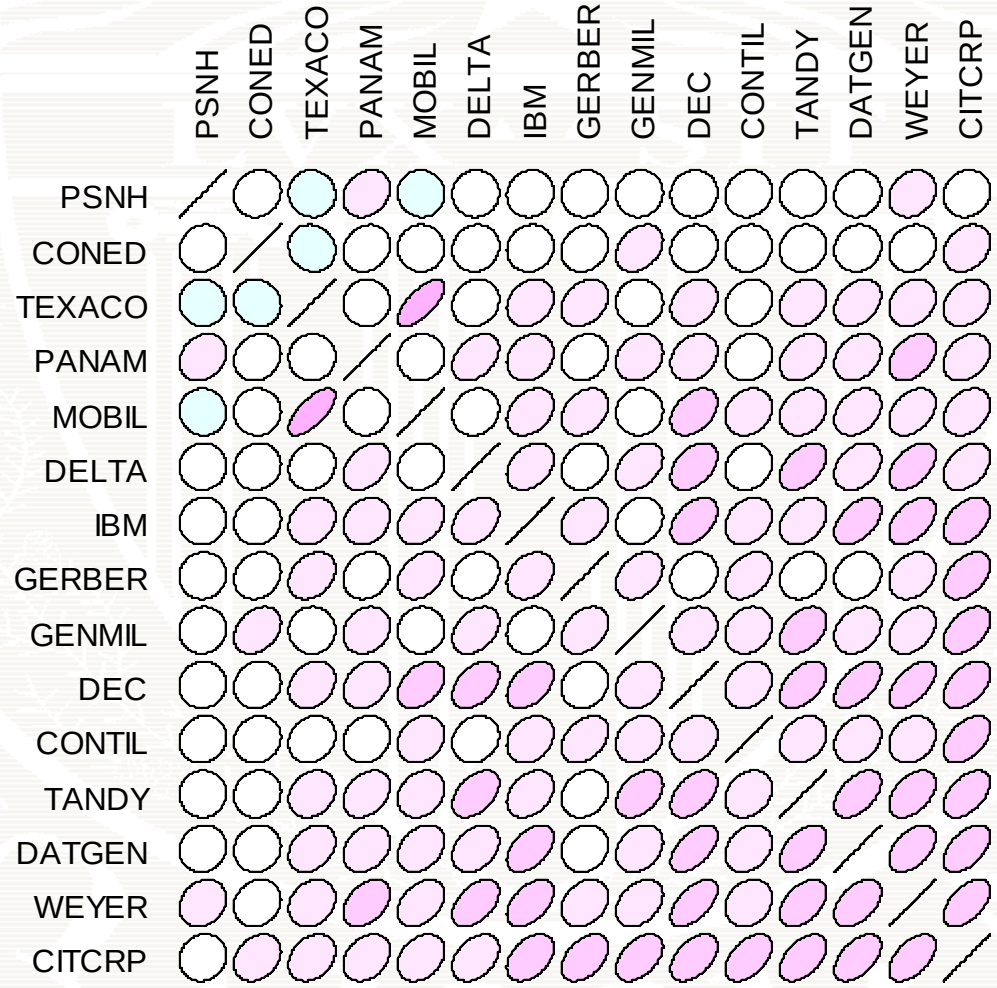
Berndt Data

```
# load Berndt investment data from fEcofin package
> data(berndtInvest)
> class(berndtInvest)
[1] "data.frame"

>  colnames(berndtInvest)
 [1] "X.Y..m..d" "CITCRP"    "CONED"     "CONTIL"
 [5] "DATGEN"    "DEC"       "DELTA"     "GENMIL"
 [9] "GERBER"    "IBM"       "MARKET"    "MOBIL"
[13] "PANAM"     "PSNH"      "TANDY"     "TEXACO"
[17] "WEYER"     "RKFREE"

 # create data frame with dates as rownames
 > berndt.df = berndtInvest[, -1]
 > rownames(berndt.df) = as.character(berndtInvest[, 1])
```

Berndt Data

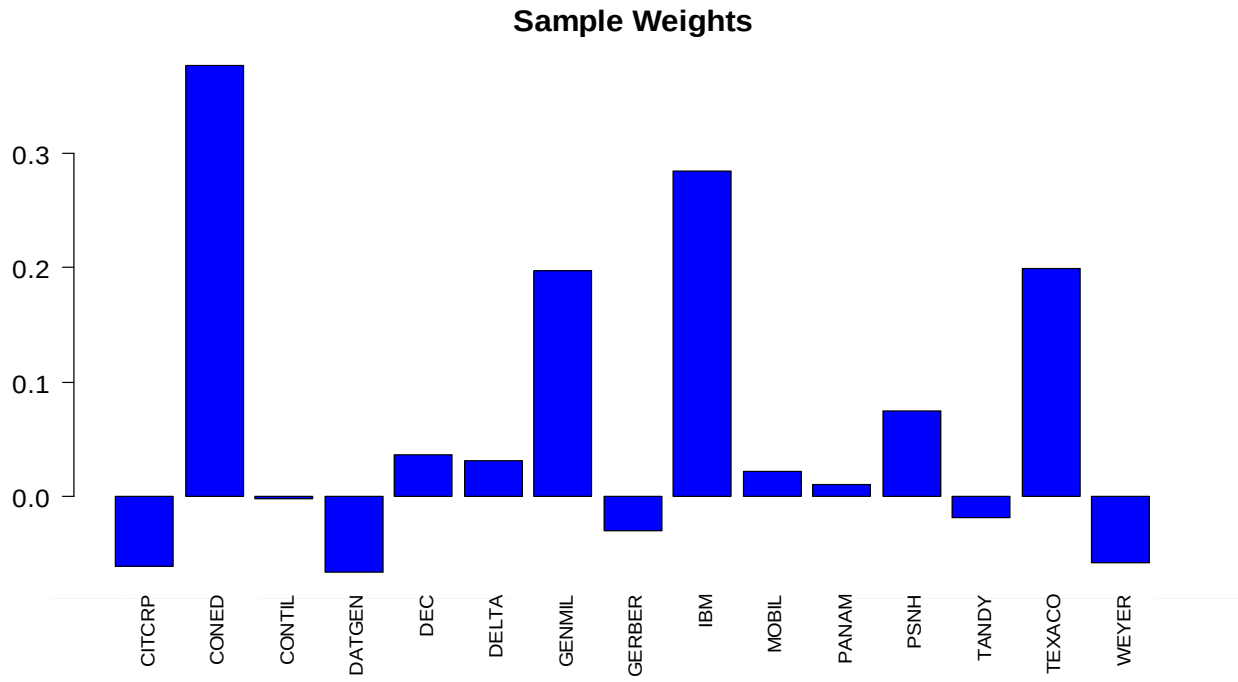
```
# monthly returns from January 1978 - December 1987
> head(berndt.df, n=3)
            CITCRP  CONED CONTIL DATGEN    DEC
1978-01-01 -0.115 -0.079 -0.129 -0.084 -0.100
1978-02-01 -0.019 -0.003  0.037 -0.097 -0.063
1978-03-01  0.059  0.022  0.003  0.063  0.010

> tail(berndt.df, n=3)
            CITCRP  CONED CONTIL DATGEN    DEC
1987-10-01 -0.282 -0.017 -0.372 -0.342 -0.281
1987-11-01 -0.136 -0.012 -0.148 -0.075 -0.127
1987-12-01  0.064 -0.006  0.050  0.181  0.134

> returns.mat = as.matrix(berndt.df[, c(-10, -17)])
> market.mat = as.matrix(berndt.df[,10, drop=F])
> n.obs = nrow(returns.mat)
> asset.names = colnames(returns.mat)
```

Sample Correlation Matrix



Minimum Variance Portfolio

```
> w.gmin.sample =
+     solve(var(returns.mat))%*%rep(1,nrow(cor.sample))
> w.gmin.sample = w.gmin.sample/sum(w.gmin.sample)
> colnames(w.gmin.sample) = "sample"
```



Industry Factor Model

```
# create loading matrix B for industry factor model
> n.stocks = ncol(returns.mat)
> tech.dum = oil.dum = other.dum =
+            matrix(0,n.stocks,1)
> rownames(tech.dum) = rownames(oil.dum) =
+                      rownames(other.dum) = asset.names
> tech.dum[c(4,5,9,13),] = 1
> oil.dum[c(3,6,10,11,14),] = 1
> other.dum = 1 - tech.dum - oil.dum
> B.mat = cbind(tech.dum,oil.dum,other.dum)
> colnames(B.mat) = c("TECH","OIL","OTHER")
```

Factor Sensitivity Matrix

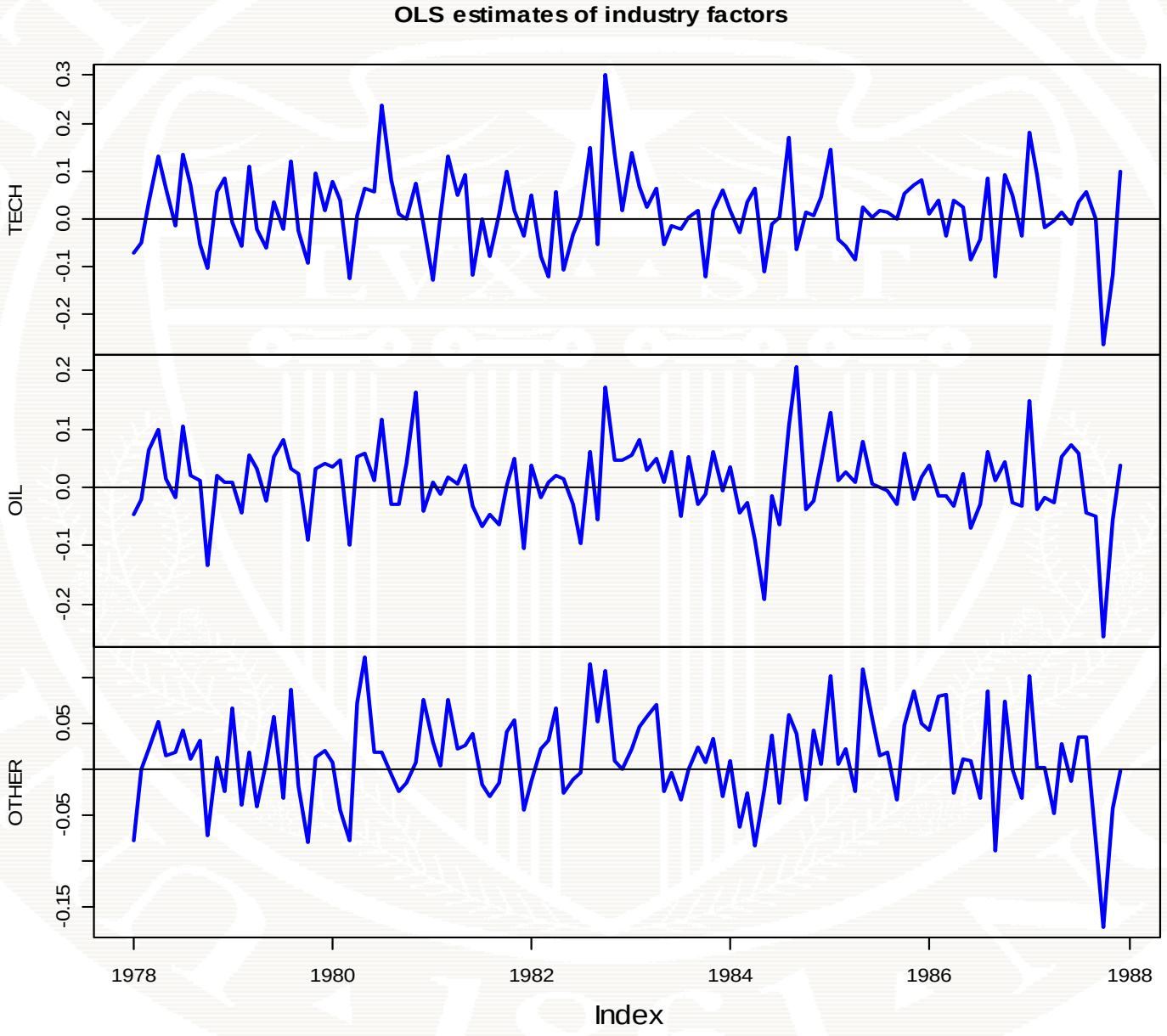
```
> B.mat
        TECH OIL OTHER
CITCRP     0   0     1
CONED      0   0     1
CONTIL     0   1     0
DATGEN     1   0     0
DEC        1   0     0
DELTA      0   1     0
GENMIL     0   0     1
GERBER     0   0     1
IBM        1   0     0
MOBIL      0   1     0
PANAM      0   1     0
PSNH       0   0     1
TANDY      1   0     0
TEXACO     0   1     0
WEYER      0   0     1
```

Multivariate Least Squares Estimation of Factor Returns

```
# returns.mat is T x N matrix, and fundamental factor
# model treats R as N x T.
> returns.mat = t(returns.mat)
# multivariate OLS regression to estimate K x T matrix
# of factor returns  (K=3)
> F.hat =
+ solve(crossprod(B.mat))%*%t(B.mat)%*%returns.mat

# rows of F.hat are time series of estimated industry
# factors
> F.hat
       1978-01-01 1978-02-01 1978-03-01 1978-04-01
TECH      -0.0720 -0.0517500     0.0335    0.13225
OIL       -0.0464 -0.0192000     0.0642    0.09920
OTHER     -0.0775 -0.0006667     0.0220    0.05133
```

Plot Industry Factors

```
# plot industry factors in separate panels - convert
# to zoo time series object for plotting with dates
> F.hat.zoo = zoo(t(F.hat), as.Date(colnames(F.hat)))
> head(F.hat.zoo, n=3)
TECH      OIL       OTHER
1978-01-01 -0.07200 -0.0464 -0.0775000
1978-02-01 -0.05175 -0.0192 -0.0006667
1978-03-01  0.03350  0.0642  0.0220000

# panel function to put horizontal lines at zero in each
panel
> my.panel <- function(...) {
+  lines(...)
+  abline(h=0)
+}
> plot(F.hat.zoo, main="OLS estimates of industry
+      factors“, panel=my.panel, lwd=2, col="blue")
```

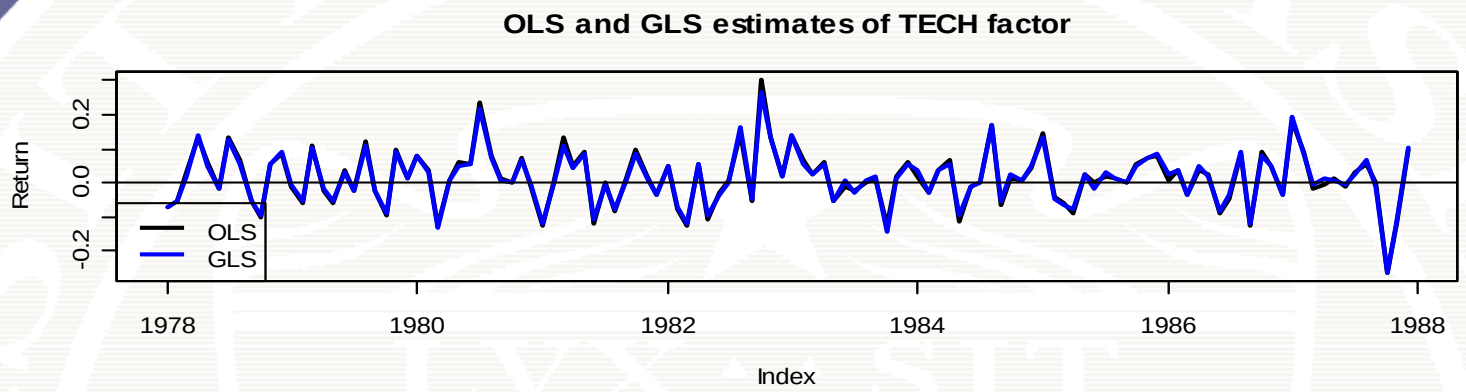


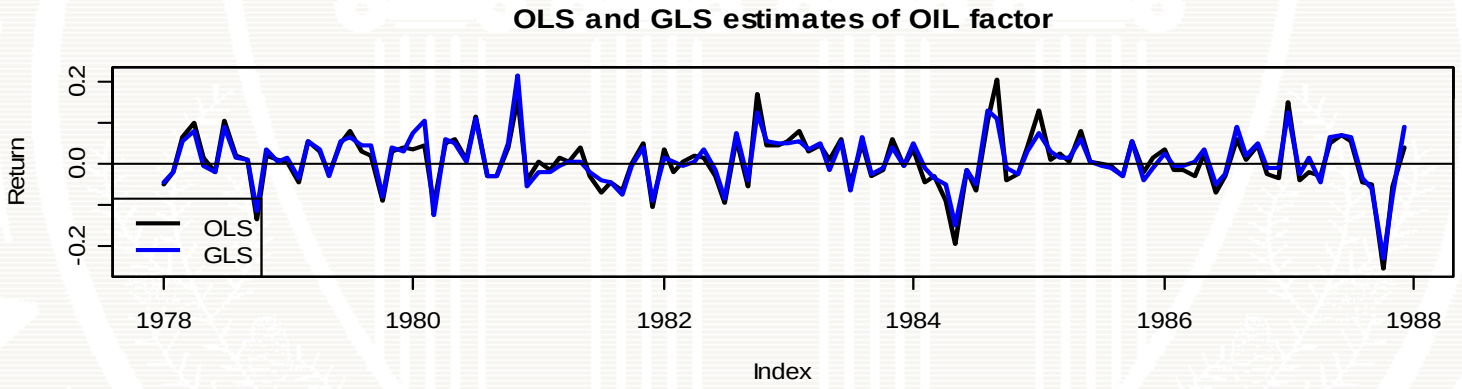
GLS Estimation of Factor Returns

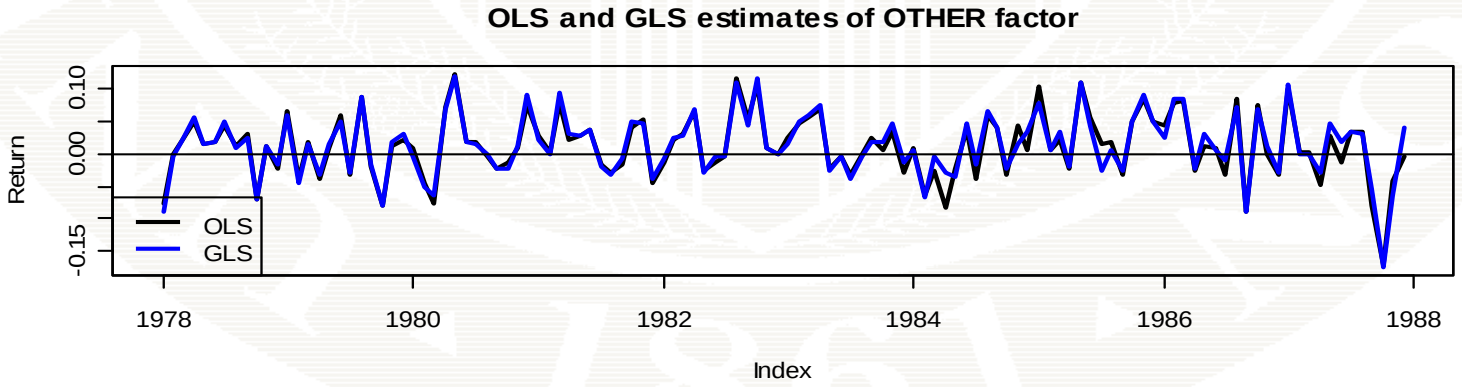
```
# compute N x T matrix of industry factor model residuals
> E.hat = returns.mat - B.mat%*%F.hat
# compute residual variances from time series of errors
> diagD.hat = apply(E.hat, 1, var)
> Dinv.hat = diag(diagD.hat^(-1))

# multivariate FGLS regression to estimate K x T matrix
# of factor returns
> H.hat = solve(t(B.mat)%*%Dinv.hat%*%B.mat)
+         %*%t(B.mat)%*%Dinv.hat
> colnames(H.hat) = asset.names
# note: rows of H sum to one so are weights in factor
# mimicking portfolios
> F.hat.gls = H.hat%*%returns.mat
```

GLS Factor Weights

```
> t(H.hat)
          TECH    OIL   OTHER
CITCRP 0.0000 0.0000 0.19918
CONED  0.0000 0.0000 0.22024
CONTIL 0.0000 0.0961 0.00000
DATGEN 0.2197 0.0000 0.00000
DEC    0.3188 0.0000 0.00000
DELTA  0.0000 0.2233 0.00000
GENMIL 0.0000 0.0000 0.22967
GERBER 0.0000 0.0000 0.12697
IBM    0.2810 0.0000 0.00000
MOBIL  0.0000 0.2865 0.00000
PANAM  0.0000 0.1186 0.00000
PSNH   0.0000 0.0000 0.06683
TANDY  0.1806 0.0000 0.00000
TEXACO 0.0000 0.2756 0.00000
WEYER  0.0000 0.0000 0.15711
```



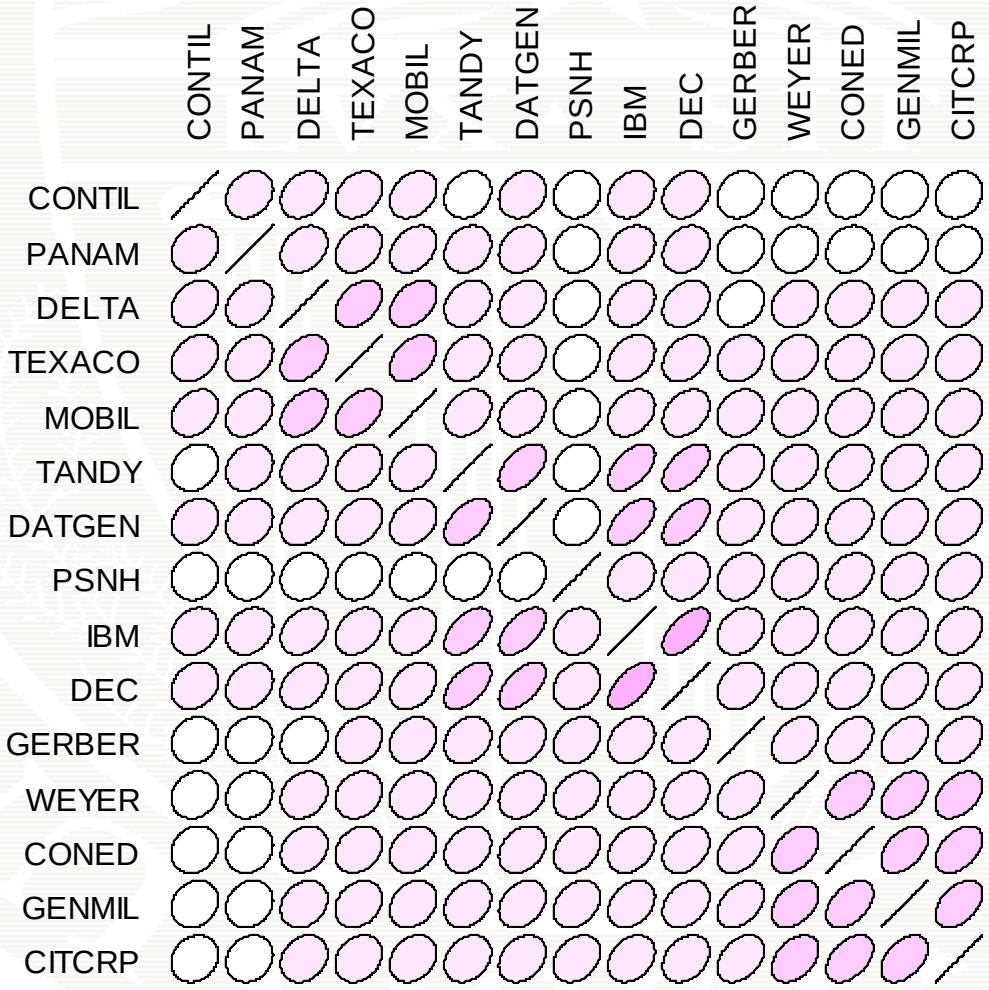




Industry Factor Model Covariance

```
# compute covariance and correlation matrices
> cov.ind = B.mat%*%var(t(F.hat.gls))%*%t(B.mat) +
+           diag(diagD.hat)
> cor.ind = cov2cor(cov.ind)
# plot correlations using plotcorr() from ellipse
# package
> rownames(cor.ind) = colnames(cor.ind)
> ord <- order(cor.ind[1,])
> ordered.cor.ind <- cor.ind[ord, ord]
> plotcorr(ordered.cor.ind,
+          col=cm.colors(11)[5*ordered.cor.ind + 6])
```

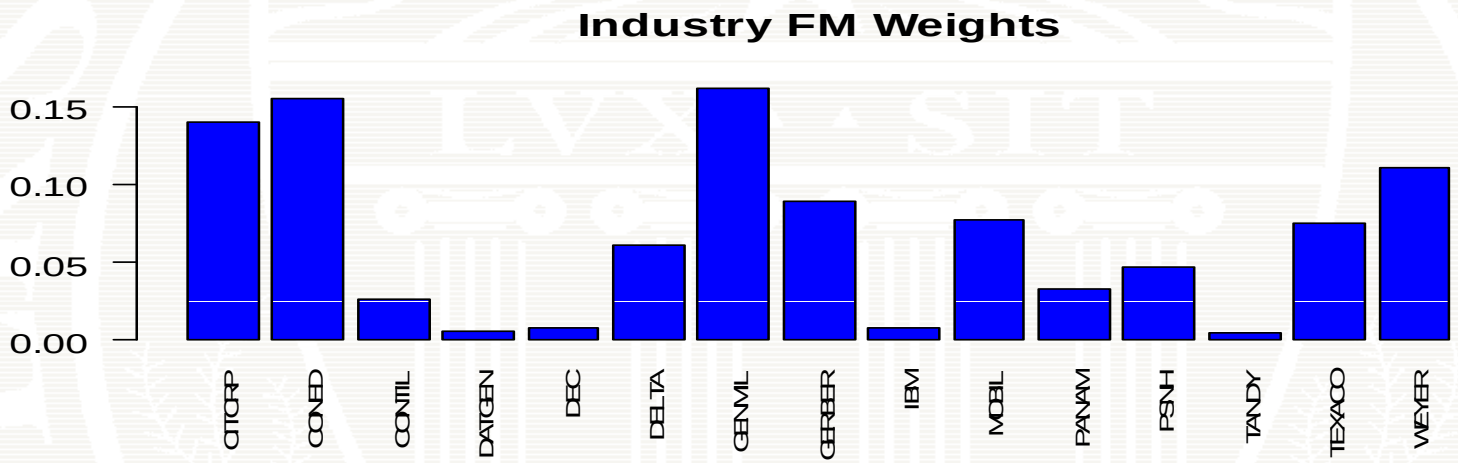
Industry Factor Model Correlations

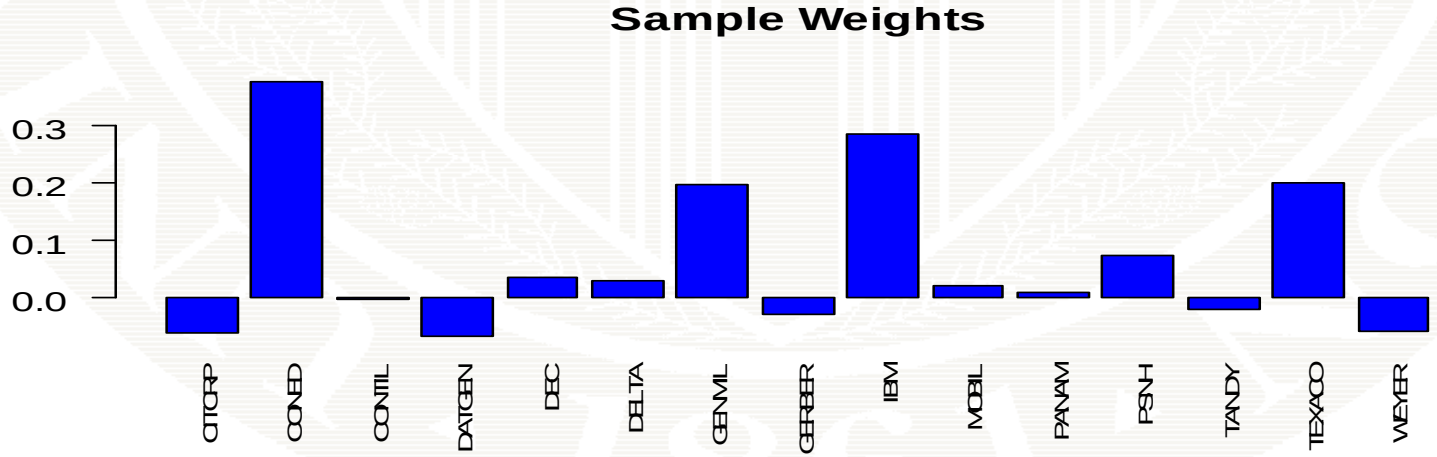


Industry Factor Model Summary

```
> ind.fm.vals
        TECH OIL OTHER   fm.sd residual.sd r.square
CITCRP     0   0     1 0.07291     0.05468   0.4375
CONED      0   0     1 0.07092     0.05200   0.4624
CONTIL     0   1     0 0.13258     0.11807   0.2069
DATGEN     1   0     0 0.10646     0.07189   0.5439
DEC        1   0     0 0.09862     0.05968   0.6338
DELTA      0   1     0 0.09817     0.07747   0.3773
GENMIL     0   0     1 0.07013     0.05092   0.4728
GERBER     0   0     1 0.08376     0.06849   0.3315
IBM        1   0     0 0.10102     0.06356   0.6041
MOBIL      0   1     0 0.09118     0.06839   0.4374
PANAM      0   1     0 0.12222     0.10630   0.2435
PSNH       0   0     1 0.10601     0.09440   0.2069
TANDY      1   0     0 0.11159     0.07930   0.4950
TEXACO     0   1     0 0.09218     0.06972   0.4279
WEYER      0   0     1 0.07821     0.06157   0.3802
```

Global Minimum Variance Portfolios





Statistical Factor Model: Principal Components Method

```
# continue to use Berndt data
> returns.mat = as.matrix(berndt.df[, c(-10, -17)])
# use R princomp() function for principal component
# analysis
> pc.fit = princomp(returns.mat)

> class(pc.fit)
[1] "princomp"
> names(pc.fit)
[1] "sdev"     "loadings" "center"   "scale"    "n.obs"
[6] "scores"   "call"
```

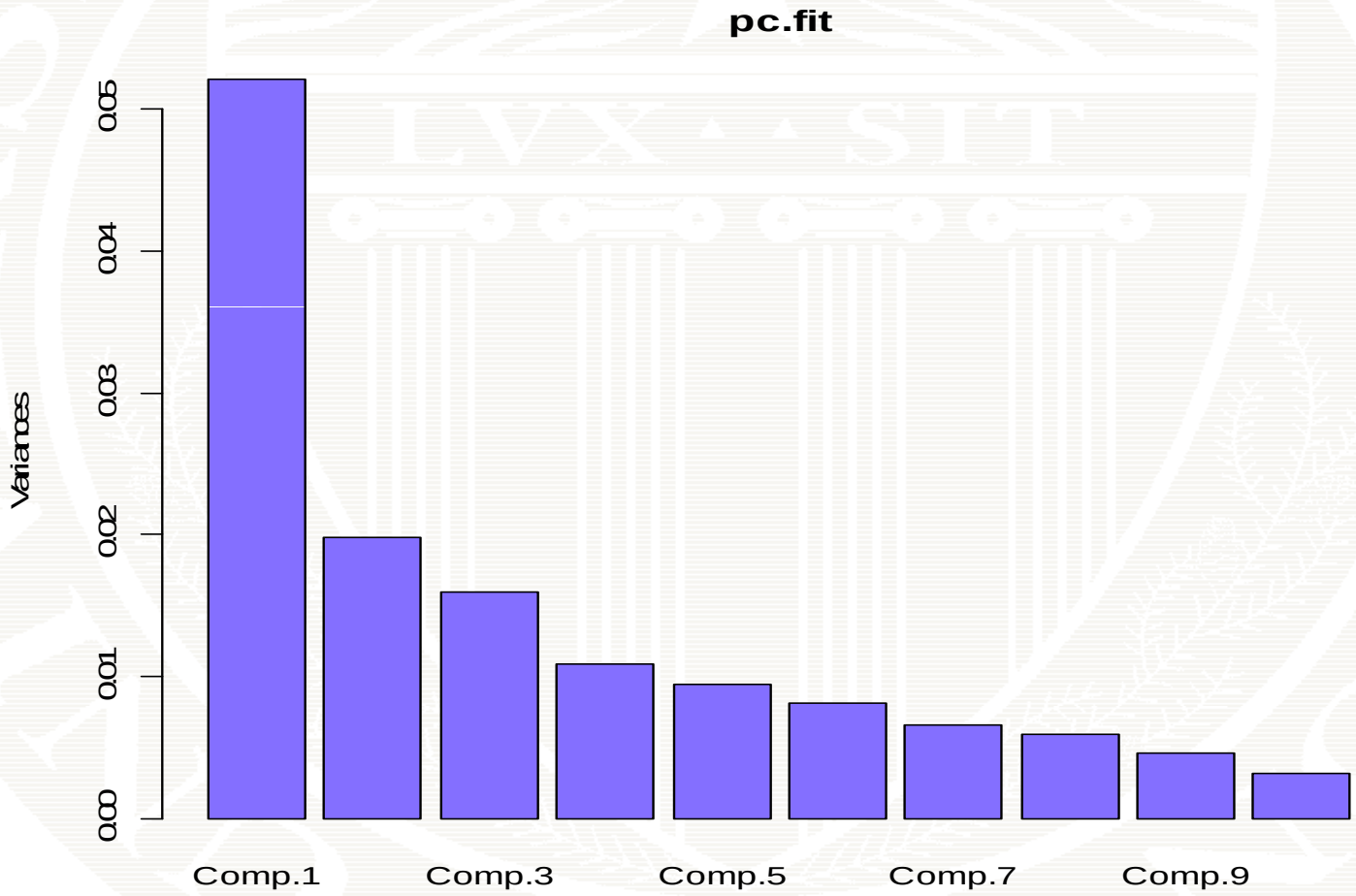
principal component factors

eigenvectors

Total Variance Contributions

```
> summary(pc.fit)
Importance of components:
                          Comp.1 Comp.2 Comp.3  Comp.4  Comp.5
Standard deviation     0.2282 0.1408 0.1264 0.10444 0.09741
Proportion of Variance 0.3543 0.1349 0.1087 0.07423 0.06458
Cumulative Proportion  0.3543 0.4892 0.5979 0.67218 0.73676
                           Comp.6  Comp.7  Comp.8  Comp.9
Standard deviation     0.09043 0.08123 0.07731 0.06791
Proportion of Variance 0.05565 0.04491 0.04068 0.03138
Cumulative Proportion  0.79241 0.83732 0.87800 0.90938
                          Comp.10 Comp.11 Comp.12 Comp.13
Standard deviation     0.05634 0.05353 0.04703 0.04529
Proportion of Variance 0.02160 0.01950 0.01505 0.01396
Cumulative Proportion  0.93098 0.95048 0.96553 0.97950
                          Comp.14  Comp.15
Standard deviation     0.04033 0.037227
Proportion of Variance 0.01107 0.009432
Cumulative Proportion  0.99057 1.000000
```

Eigenvalue Scree Plot




```
> plot(pc.fit)
```

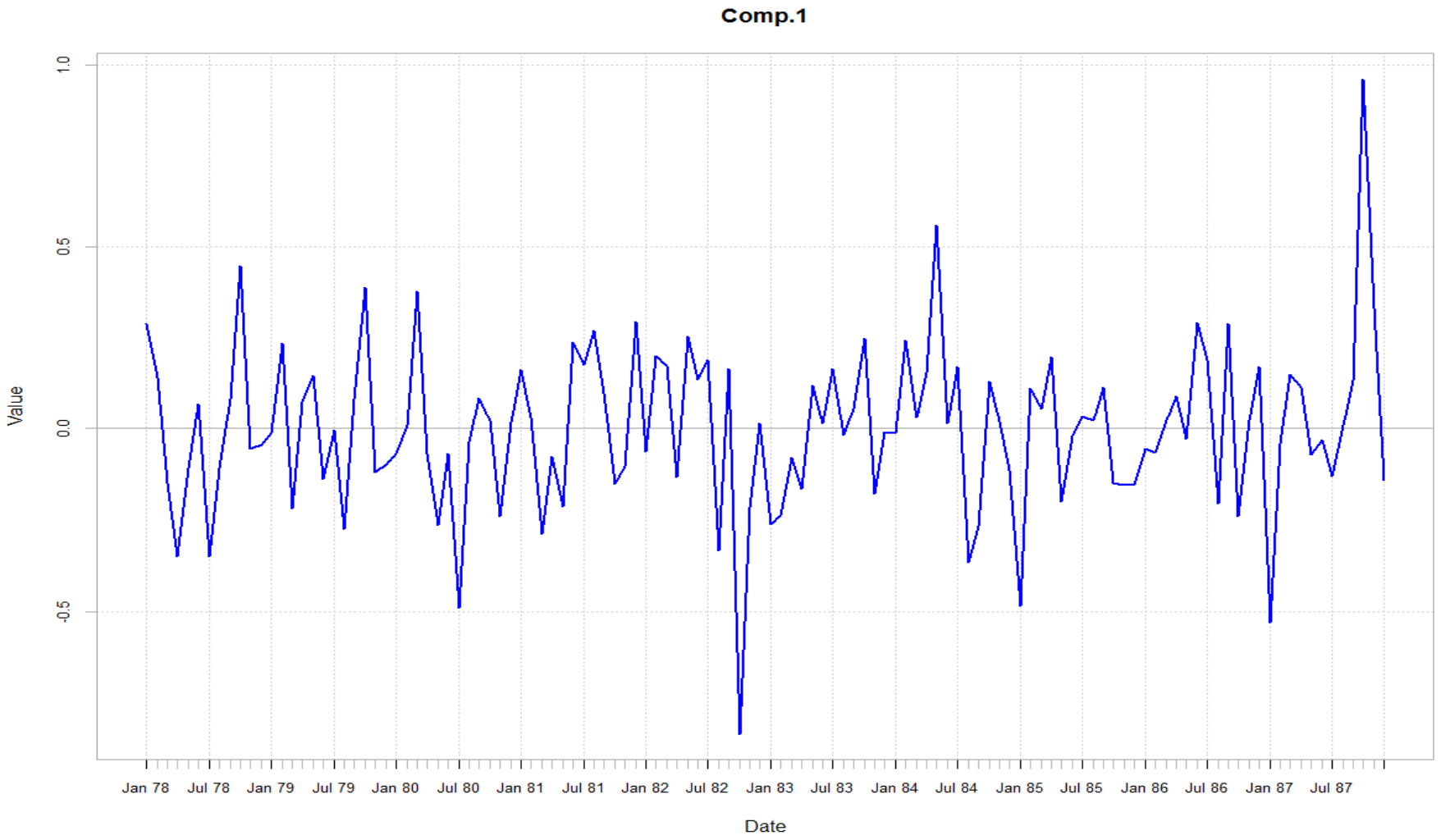
Loadings (eigenvectors)

```
> loadings(pc.fit) # pc.fit$loadings

Loadings:
       Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
CITCRP  0.273                                           
CONED                                                   
CONTIL  0.377 -0.824 -0.199  0.157  0.144 -0.191        
DATGEN  0.417  0.152  0.277 -0.329  0.287 -0.497        
DEC     0.305  0.129  0.202 -0.141                0.368 
DELTA   0.250  0.179         0.258         0.242  0.481 
GENMIL  0.133                0.128         0.249  0.117 
GERBER  0.167 -0.199                -0.418  0.349        
IBM     0.146                                       0.142 
MOBIL   0.155         0.248 -0.241 -0.459       -0.155 
PANAM   0.311  0.365 -0.630  0.227 -0.343 -0.390 -0.197 
PSNH                 -0.527 -0.692  0.249  0.360        
TANDY   0.412  0.207  0.188  0.323  0.356  0.385 -0.564 
TEXACO  0.132         0.245 -0.219 -0.430       -0.325 
WEYER   0.265  0.131        -0.128 -0.111  0.152  0.291 
```

Principal Component Factors

```
> head(pc.fit$scores[, 1:4])
              Comp.1    Comp.2   Comp.3     Comp.4
1978-01-01  0.28998  0.069162 -0.07621 -0.0217151
1978-02-01  0.14236 -0.141967 -0.01794 -0.0676476
1978-03-01 -0.14927  0.113295 -0.09307 -0.0326150
1978-04-01 -0.35056 -0.032904  0.01128  0.0168986
1978-05-01 -0.10874  0.004943 -0.04640 -0.0612666
1978-06-01  0.06948  0.041330 -0.06757  0.0009816
```

Note: Scores are based on centered (demeaned) returns



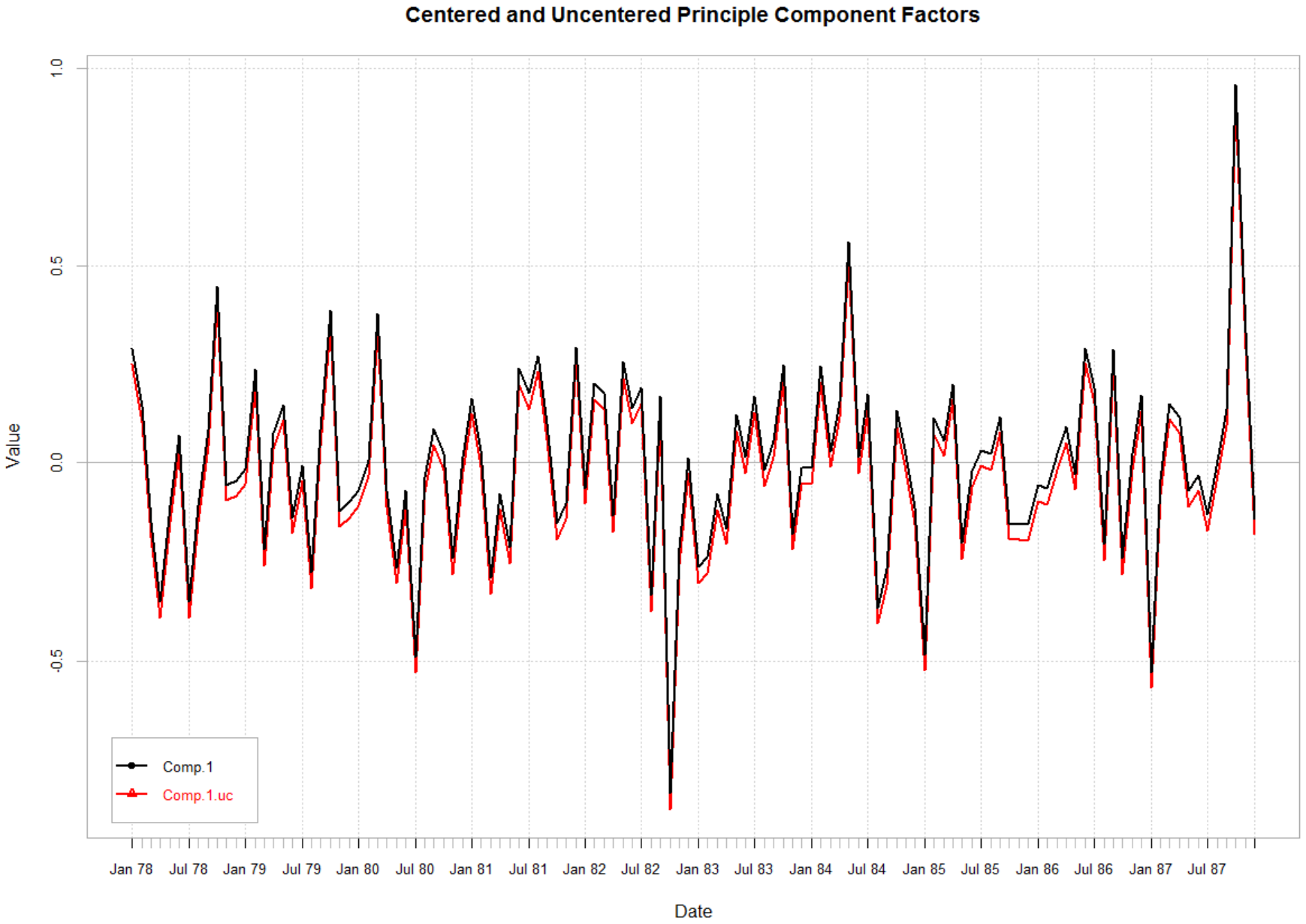

```
> chart.TimeSeries(pc.fit$scores[, 1, drop=FALSE],
+                  colorset="blue")
```

Direct Eigenvalue Computation

```
> eigen.fit = eigen(var(returns.mat))
> names(eigen.fit)
[1] "values"  "vectors"
> names(eigen.fit$values) =
+       rownames(eigen.fit$vectors) = asset.names

# compare princomp output with direct eigenvalue output
> cbind(pc.fit$loadings[,1:2], eigen.fit$vectors[, 1:2])
         Comp.1    Comp.2
CITCRP -0.27271 -0.085495 -0.27271 -0.085495
CONED  -0.04441  0.001193 -0.04441  0.001193
CONTIL -0.37694 -0.823575 -0.37694 -0.823575
DATGEN -0.41719  0.151818 -0.41719  0.151818
DEC    -0.30493  0.129067 -0.30493  0.129067…
```

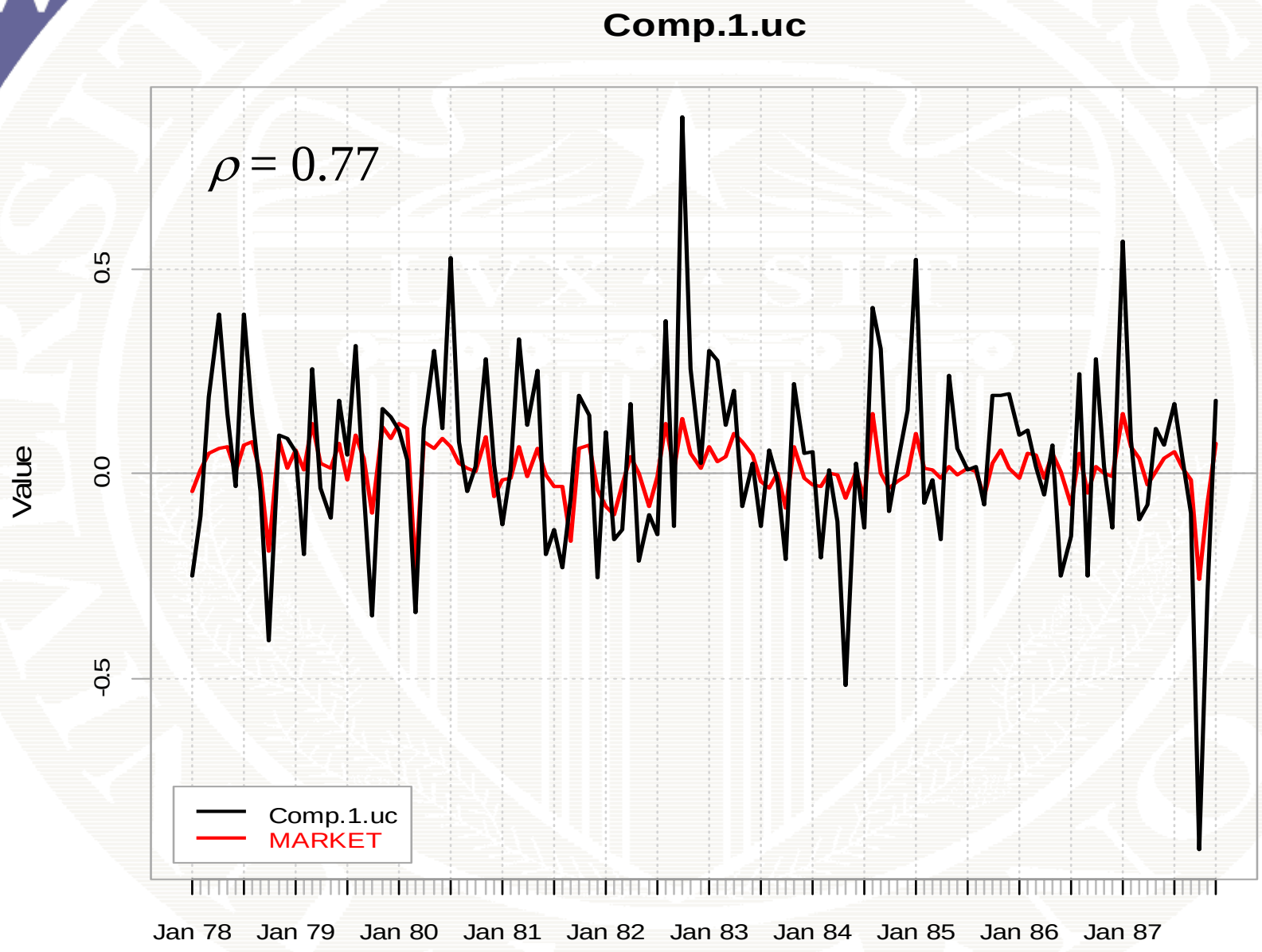
Compare Centered and Uncentered Principal Component Factors

```
# compute uncentered pc factors from eigenvectors
# and return data
> pc.factors.uc = returns.mat %*% eigen.fit$vectors
> colnames(pc.factors.uc) =
+          paste(colnames(pc.fit$scores),".uc",sep="")

# compare centered and uncentered scores. Note sign
# change on first factor
> cbind(pc.fit$scores[,1,drop=F],
+       -pc.factors.uc[,1,drop=F])
 Comp.1 Comp.1.uc
1978-01-01  0.289978  0.250237
1978-02-01  0.142355  0.102614
1978-03-01 -0.149273 -0.189015
1978-04-01 -0.350563 -0.390304
1978-05-01 -0.108743 -0.148484
```


Centered and Uncentered Principle Component Factors
Value
1.0
0.5
0.0
-0.5
Comp.1
Comp.1.uc
Jan 78
Jul 78
Jan 79
Jul 79
Jan 80
Jul 80
Jan 81
Jul 81
Jan 82
Jul 82
Jan 83
Jul 83
Jan 84
Jul 84
Jan 85
Jul 85
Jan 86
Jul 86
Jan 87
Jul 87
Date

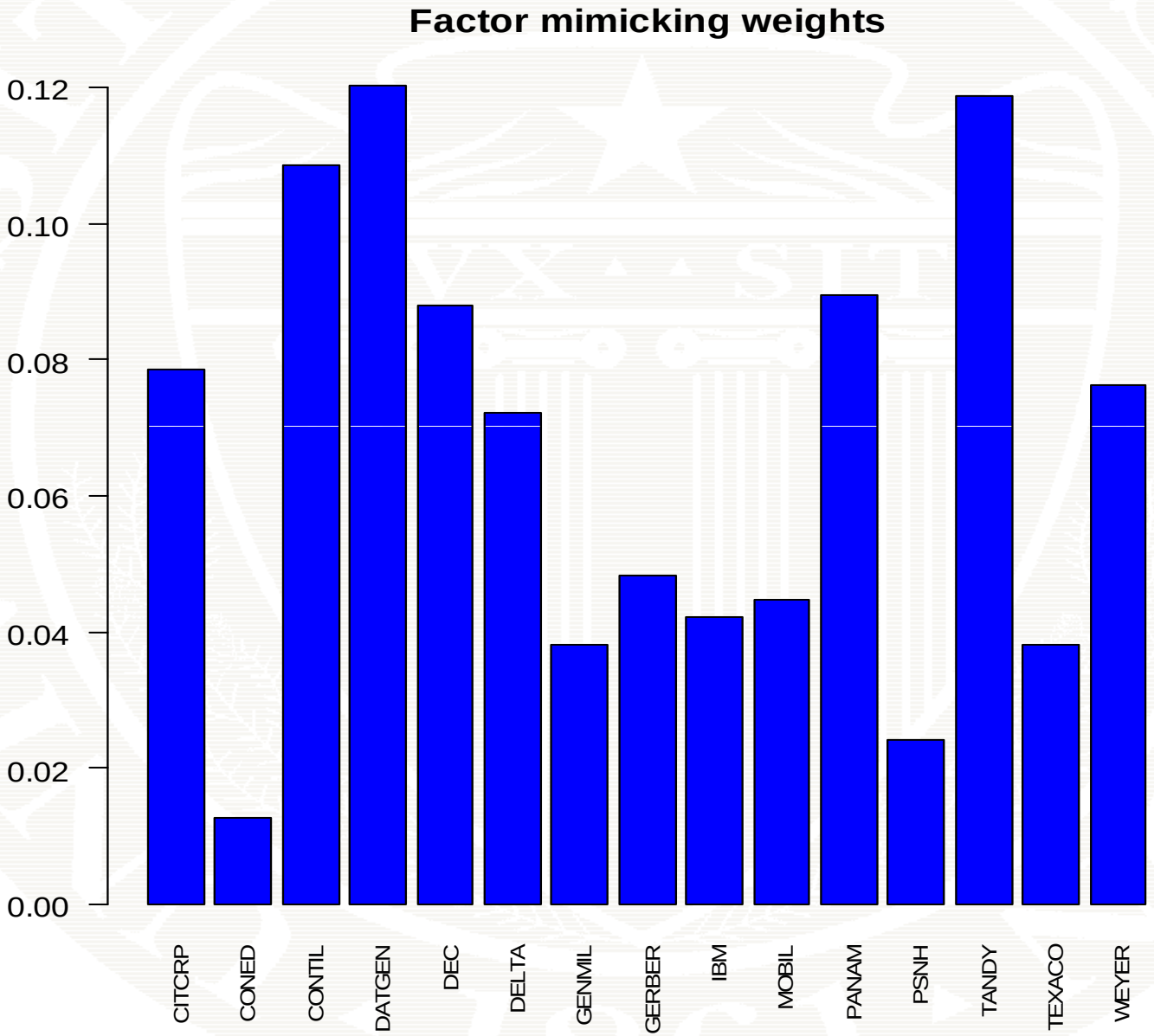
Interpreting Principal Component Factor

```
# Compute correlation with market return
> cor(cbind(pc.factors.uc[,1,drop=F],
+           berndt.df[, "MARKET",drop=F]))
           Comp.1.uc  MARKET
Comp.1.uc     1.0000 -0.7657
MARKET       -0.7657  1.0000

# Correlation with sign change
> cor(cbind(-pc.factors.uc[,1,drop=F],
+           berndt.df[, "MARKET",drop=F]))
           Comp.1.uc MARKET
Comp.1.uc     1.0000 0.7657
MARKET        0.7657 1.0000
```



Factor Mimicking Portfolio

```
# extract first eigenvector with sign change
> p1 = -pc.fit$loadings[, 1]
> p1
 CITCRP   CONED  CONTIL  DATGEN     DEC   DELTA  GENMIL
0.27271 0.04441 0.37694 0.41719 0.30493 0.25017 0.13256
 GERBER     IBM   MOBIL   PANAM    PSNH   TANDY  TEXACO
0.16716 0.14644 0.15517 0.31067 0.08407 0.41193 0.13225
  WEYER
0.26488
> sum(p1)
[1] 3.471
# create factor mimicking portfolio by normalizing
# weights to unity
> p1 = p1/sum(p1)
# normalized principle component factor
> f1 = returns.mat %*% p1
```



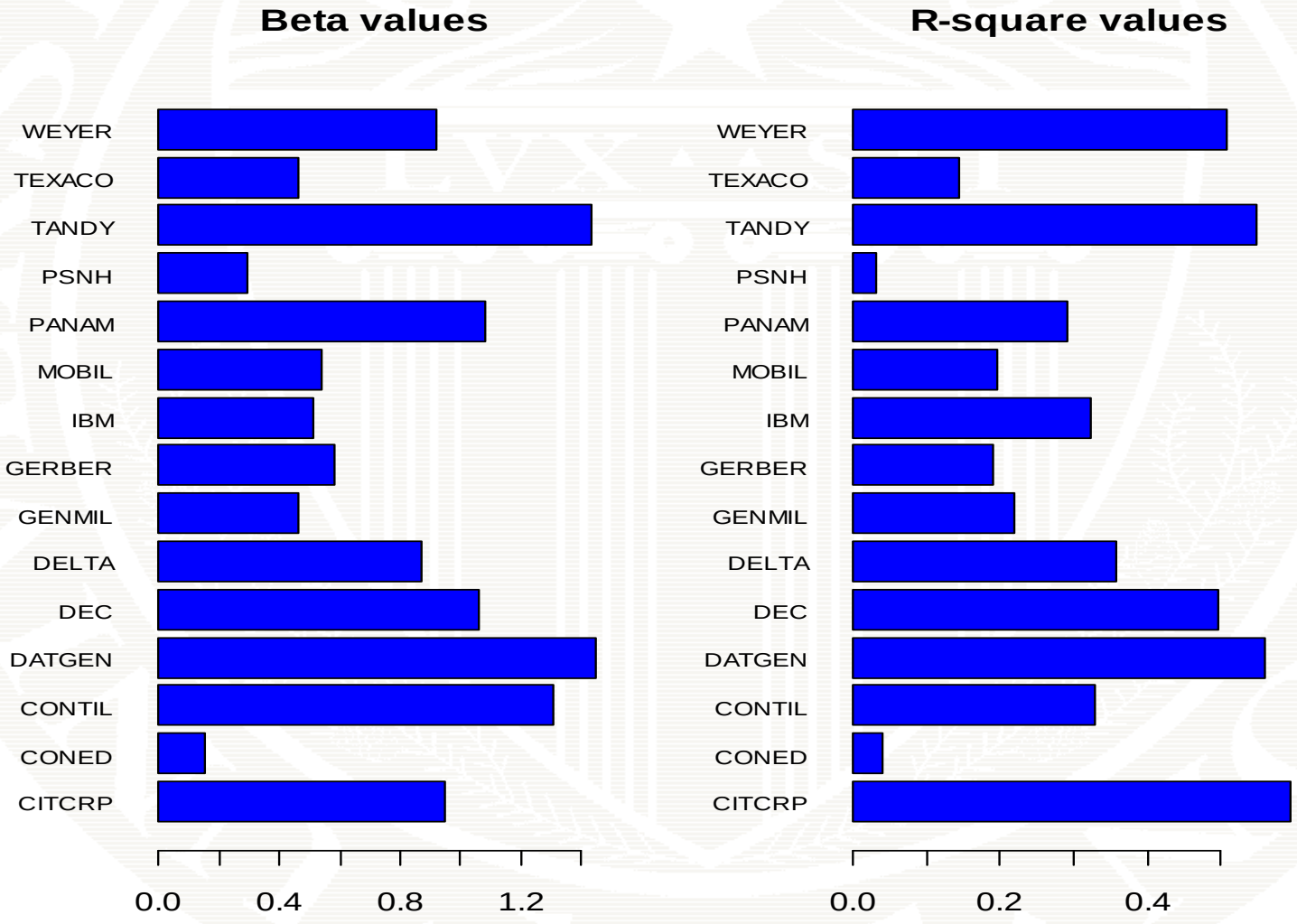
Estimate Factor Betas

```
# estimate factor betas by multivariate regression
> X.mat = cbind(rep(1,n.obs), f1)
> colnames(X.mat) = c("intercept", "Factor 1")
> XX.mat = crossprod(X.mat)
# multivariate least squares
> G.hat = solve(XX.mat)%*%crossprod(X.mat,returns.mat)
> beta.hat = G.hat[2,]
> E.hat = returns.mat - X.mat%*%G.hat
> diagD.hat = diag(crossprod(E.hat)/(n.obs-2))
# compute R2 values from multivariate regression
> sumSquares = apply(returns.mat, 2, function(x)
+                    {sum( (x - mean(x))^2 )})
> R.square = 1 - (n.obs-2)*diagD.hat/sumSquares
```

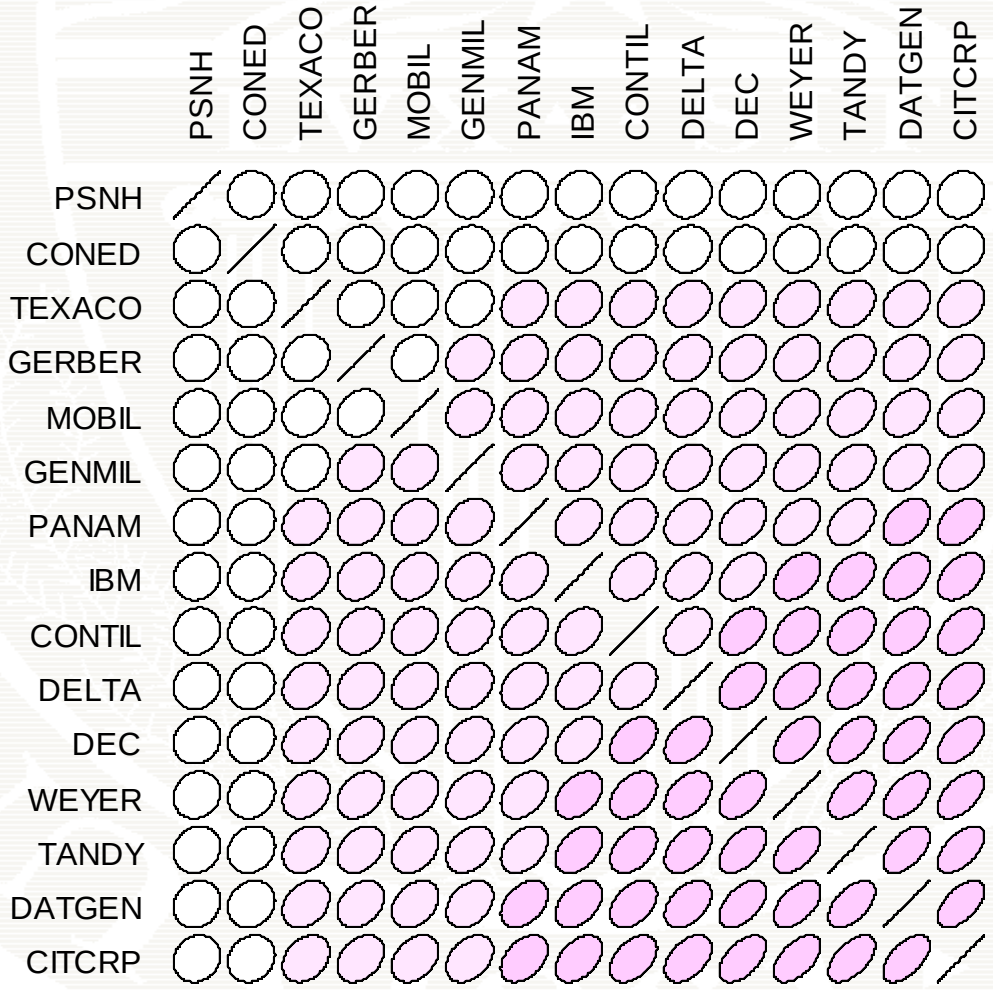
Regression Results

```
> cbind(beta.hat, diagD.hat, R.square)
        beta.hat diagD.hat R.square
CITCRP    0.9467  0.002674  0.59554
CONED     0.1542  0.002444  0.04097
CONTIL    1.3085  0.015380  0.32847
DATGEN    1.4483  0.007189  0.56176
DEC       1.0586  0.004990  0.49664
DELTA     0.8685  0.005967  0.35704
GENMIL    0.4602  0.003336  0.21808
GERBER    0.5803  0.006284  0.19058
IBM       0.5084  0.002378  0.32318
MOBIL     0.5387  0.005229  0.19600
PANAM     1.0785  0.012410  0.29168
PSNH      0.2918  0.011711  0.03096
TANDY     1.4300  0.007427  0.54746
TEXACO    0.4591  0.005480  0.14455
WEYER     0.9195  0.003583  0.50904
```

Regression Results



Principal Components Correlations



Global Minimum Variance Portfolios