Econ 589: Financial Econometrics and Quantitative Risk Management
Final Exam

Eric Zivot

Due: Friday 6/14/2013 at 5 pm (or earlier).

1 Instructions

This is a take-home open book final exam. It is due no later than Friday, June 14th at 5 p.m. in my office, by email or my mailbox. The exam is mostly a review of the main material covered during the term. Please give short concise answers and do not just regurgitate my lecture notes. You will also read a few papers and comment on the results.

2 Empirical Properties of Returns

1. Throughout the course, we talked about some basic stylized facts of daily and monthly continuously compounded asset returns and transformations these returns (e.g., squared and absolute returns). Briefly describe these stylized facts (a bullet point list is fine). Distinguish between stylized facts for univariate series and stylized facts for multivariate series.

2. Consider the normal GARCH(1,1) model

$$\begin{aligned} r_t - \mu &= \varepsilon_t = u_t\sigma_t,\ u_t \sim N(0,1), \\ \sigma_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2, \\ a_0 &> 0,\ a_1 > 0,\ b_1 \geq 0 \text{ and } a_1 + b_1 < 1, \end{aligned}$$

   Which stylized facts of asset returns can be described by the simple GARCH(1,1) and which cannot? Briefly justify your answers using the analytical properties of the GARCH(1,1) . (Note: you do not have to derive these properties, just state what they are and give references.)

3. Consider the multivariate EWMA and DCC(1,1) models based on the return decomposition

$$\begin{aligned}\mathbf{r}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t,\ t = 1, 2, \cdots, T, \\ & \boldsymbol{\epsilon}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_t).\end{aligned}$$

The EWMA has the form

$$\begin{aligned}\boldsymbol{\Sigma}_t &= (1-\lambda)\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \lambda\boldsymbol{\Sigma}_{t-1} \\ 0 &< \lambda \leq 1\end{aligned}$$

and the DCC(1,1) has the form

$$var(\boldsymbol{\varepsilon}_t | I_{t-1}) = \underset{k\times k}{\boldsymbol{\Sigma}_t} = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t$$

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1k,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2k,t} \\ \vdots & & \ddots & \vdots \\ \rho_{1k,t} & \rho_{2k,t} & \cdots & 1 \end{bmatrix},\ \mathbf{D}_t = \begin{bmatrix} \sigma_{1t} & 0 & \cdots & 0 \\ 0 & \sigma_{2t} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{kt} \end{bmatrix}$$

where σ_{it} follows univariate GARCH(1,1) models and $\rho_{ij,t}$ is derived from a GARCH(1,1) model for $\hat{q}_{ij,t} = cov(\hat{z}_{it}, \hat{z}_{jt} | I_{t-1})$ where $\hat{z}_{it} = \frac{\hat{\varepsilon}_{it}}{\hat{\sigma}_{it}}$ is the standardized residual from the univariate GARCH(1,1) models. What are the advantages and disadvantages of each model? Which stylized facts of the multivariate distributions of asset returns can be described by the EWMA and DCC(1,1) models and which cannot? Briefly justify your answers using the properties of the EWMA and DCC(1,1) models. (Note: you do not have to derive these properties, just state what they are and give references.)

3 Using Volatility Models

1. We spent a good deal of time in class studying univariate volatility models for asset returns. Briefly explain why we care about modeling and forecasting asset return volatility.

2. GARCH models are commonly used to forecast future conditional volatility. In principle, we can evaluate the adequacy of a fitted GARCH model by examining the quality of its forecasts. However, conditional volatility is unobservable which makes a direct comparison between forecasted volatility and actual volatility impossible. Briefly explain how GARCH forecasts can be evaluated using observable proxies for conditional volatility. What problems, if any, are there associated with using proxies for volatility in evaluating volatility forecasts?

3. Value-at-risk (VaR) and expected shortfall (ES) are two commonly used downside risk measures for an asset. Let R_t denote the daily return on a given portfolio of assets. VaR and ES can be estimated unconditionally from the distribution of R_t, and they can be estimated conditionally from a GARCH model. Typical unconditional models include (1) historical simulation (non-parametric); (2) normal distribution; (3) non-normal distribution (e.g., Student's t, skew-t, Cornish-Fisher). Common conditional models include (4) normal GARCH; (5) non-normal GARCH; (6) filtered historical simulation. Briefly explain how VaR and ES are computed using each of these 6 methods.

4. Given a set of competing VaR models for returns, describe how these models can be evaluated. That is, describe how you can decide if one VaR model is better than other one.

4 Risk Budgeting Under the Multivariate Normal Distribution

Suppose the $n \times 1$ vector of asset returns $\mathbf{R}$ has a multivariate normal distribution: $\mathbf{R} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. For an $n \times 1$ vector of portfolio weights $\mathbf{w}$, the portfolio return is normally distributed: $R_p = \mathbf{w}'\mathbf{R} \sim N(\mathbf{w}'\boldsymbol{\mu}, (\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w})^{1/2})$.

1. For a confidence level α, give analytic expressions for portfolio VaR, $VaR_\alpha(\mathbf{w})$, and portfolio ES, $ES_\alpha(\mathbf{w})$, as explicit functions of the portfolio weights w.

2. Show that the functions $VaR_\alpha(\mathbf{w})$ and $ES_\alpha(\mathbf{w})$ are homogeneous functions of degree 1 in $\mathbf{w}$.

3. Using your analytic expressions for $VaR_\alpha(\mathbf{w})$ and $ES_\alpha(\mathbf{w})$, compute the vector of asset marginal contributions

$$\frac{\partial VaR_\alpha(\mathbf{w})}{\partial \mathbf{w}} \text{ and } \frac{\partial ES_\alpha(\mathbf{w})}{\partial \mathbf{w}}.$$

5 Multivariate Distributions and Risk Measures

1. We studied four ways of modeling the multivariate distribution of asset returns: (1) multivariate normal distribution; (2) multivariate GARCH with normally distributed errors; (3) multivariate distribution derived from univariate marginal distributions and a copula; (4) multivariate distributions derived from linear factor models. Explain how each approach is used to estimate the distribution of asset returns (i.e., explain the basic structure of each model and describe how the model is estimated. For the factor models, pick one of the three types of models - macro, fundamental or statistical - for your explanation)

2. Briefly explain the pros and cons of each approach for modeling the distribution of asset returns.

3. Risk budgeting is used to decompose a portfolio risk measure into contributions from individual assets or factors. When the portfolio risk measure is VaR or ES, these contributions generally do not have closed form analytical solutions unless the joint distribution of asset returns is multivariate normal and must be computed using simulation methods. Brief explain how the contributions to ES can be computed by simulating data from one of the four ways of modeling the joint distributions.

6 Modeling Systemic Risk

Read the paper "Volatility, correlation and tails for systemic risk measurement" (available on the class syllabus page) by Brownless and Engle. The questions below mainly concern the econometric methodology in section 3 of the paper

1. What is systemic risk and how is it different from market risk?

2. Brownlees and Engle measure the systemic risk of a firm using marginal expected shortfall (MES). In words, explain the intuition behind MES.

3. Brownlees and Engle propose a particular way of measuring MES based on Engle's DCC model. Briefly explain how they define this measure.

4. Brownlees and Engle discuss a short-term and long-term estimate of MES. Briefly describe how they propose to estimate these measures.

5. Do you think MES is a good measure of systemic risk? Why or why not?