Econ 589: Financial Econometrics
Final Exam

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Due: Friday 6/10/2011 at 5 pm (or earlier).

1 Instructions

This is a take-home open book final exam. It is due no later than Friday, June 10 at 5 p.m. in my office or my mailbox. The exam is mostly a review of the main material covered during the term. Please give short concise answers and do not just regurgitate my lecture notes. You will also read a few papers and comment on the results.

2 Empirical Properties of Returns

1. Throughout the course, we talked about some basic stylized facts of daily, monthly and intra-daily continuously compounded asset returns and transformations these returns (e.g., squared and absolute returns, daily realized variance from squared intra-day returns). Briefly describe these stylized facts (a bullet point list is fine). Distinguish between stylized facts for univariate series and stylized facts for multivariate series.

2. Consider the normal GARCH(1,1) model

$$\begin{aligned} r_t - \mu &= \varepsilon_t = u_t\sigma_t,\ u_t \sim N(0,1), \\ \sigma_t^2 &= a_0 + a_1\varepsilon_{t-1}^2 + b_1\sigma_{t-1}^2, \\ a_0 &> 0,\ a_1 > 0,\ b_1 \geq 0 \text{ and } a_1 + b_1 < 1, \end{aligned}$$

and the log-normal stochastic volatility (SV) model

$$\begin{gathered} r_t = \mu + \sigma_t u_t, \\ \ln(\sigma_t) - \alpha = \phi(\ln(\sigma_{t-1} - \alpha) + \eta_t, |\phi| < 1, \\ \begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim iid\ N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \right). \end{gathered}$$

Which stylized facts of asset returns can be described by the GARCH(1,1) and log-normal stochastic volatility models and which cannot? Briefly justify your answers using the analytical properties of these models . (Note: you do not have to derive these properties, just state what they are and give references.)

3. Consider the multivariate EWMA and DVEC(1,1) models based on the return decomposition

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t, \ t = 1, 2, \cdots, T, \\ & \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_t). \end{aligned}$$

The EWMA has the form

$$\begin{aligned} \boldsymbol{\Sigma}_t &= (1 - \lambda)\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \lambda \boldsymbol{\Sigma}_{t-1} \\ 0 &< \lambda \leq 1 \end{aligned}$$

and the DVEC(1,1) has the form

$$\boldsymbol{\Sigma}_t = \mathbf{A}_0 + \mathbf{A}_1 \odot \boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \mathbf{B}_1 \odot \boldsymbol{\Sigma}_{t-1}$$

Which stylized facts of the multivariate distributions of asset returns can be described by the EWMA and DVEC(1,1) models and which cannot? Briefly justify your answers using the properties of the EWMA and DVEC(1,1) models. (Note: you do not have to derive these properties, just state what they are and give references.)

3 Using Volatility Models

1. We spent a good deal of time in class studying univariate and multivariate volatility models for asset returns. Briefly explain why we care about modeling and forecasting asset return volatility, covariance and correlation.

2. GARCH models are commonly used to forecast future conditional volatility. In principle, we can evaluate the adequacy of a fitted GARCH model by examining the quality of its forecasts. However, conditional volatility is unobservable which makes a direct comparison between forecasted volatility and actual volatility impossible. Briefly explain how GARCH forecasts can be evaluated using observable proxies for conditional volatility. What problems, if any, are there associated with using proxies for volatility in evaluating volatility forecasts?

3. Value-at-risk (VaR) and expected shortfall (ES) are two commonly used risk measures for a portfolio of financial assets. Let R_t denote the daily return on a given portfolio of assets. VaR and ES can be estimated unconditionally from

the distribution of R_t, and they can be estimated conditionally from a GARCH model. Typical unconditional models include (i) historical simulation (non-parametric); (2) normal distribution; (3) non-normal distribution (e.g., Student's t or skew-t). Common conditional models include (4) normal GARCH; (5) non-normal GARCH. Briefly explain how VaR and ES are computed using each of these 5 methods.

4. Given a set of competing VaR models for returns, describe how these models can be evaluated. That is, describe how you can decide if one VaR model is better than other one.

4 Predicting Asset Return Volatility

In the paper "A Comprehensive Look at Financial Volatility Prediction by Economic Variables," (SSRN working paper no. 1737433 available on the class homework page), Christiansen, Schmeling and Schrimpf address the question of what drives volatility on financial markets. Read this paper and answer the following questions.

1. What financial and macroeconomic variables do the authors consider in their analysis? How do they measure monthly financial asset volatility?

2. What is the econometric methodology used by the authors to evaluate the predictability of asset volatility?

3. What are their main conclusions regrading the predictability of asset return volatilty by macroeconomic variables?

5 High Frequency Methods

1. Empirical analysis of high frequency equity data typically uses intra-day data taken from the TAQ database. The raw data, however, is typically not suitable for direct analysis and needs to be cleaned. Briefly describe some of the data problems associated with the TAQ data and recommended procedures for cleaning the TAQ data prior to analysis. You may find the discussion in Barndorff-Nielsen, O.E., P. Hansen, A. Lunde, and N. Shephard (2009), "Realized Kernels in Practice: Trades and Quotes", *Econometrics Journal*, helpful for answering this question.

2. Given intra-day high frequency price data, describe how to compute the (naive) realized variance (RV) for the day.

   (a) Describe how to align irregularly spaced intra-day price data to a regularly spaced time clock. What is the preferred alignment method?

(b) What does RV converge to as the sampling frequency goes to zero? Be specific about your assumptions regarding the underlying continuous time price process and the presence or absence of market microstructure noise.

(c) In practice, you cannot let the sampling frequency go to zero so that there is always some measurement error in RV. How can you evaluate the magnitude of this measurement error? That is, how can you compute a 95% confidence interval for RV?

3. The convergence of RV assumes the absence of market microstructure noise.

   (a) Briefly describe the common sources of market microstructure noise.

   (b) Under the assumption that the microstructure noise is independent of the true price process, what happens to RV as the sampling interval goes to zero?

4. Three types of estimators for integrated variance (IV) have been proposed to deal with microstructure noise: (1) Bandi and Russell's realized variance estimator based on an optimal sampling frequency; (2) Zhang, Mykland and Ait-Sahalia's two time scale estimator; (3) Hansen and Lunde's autocovariance corrected (kernel-type) estimator Briefly describe the logic behind these three estimators how they deal with the problems created by microstructure noise.