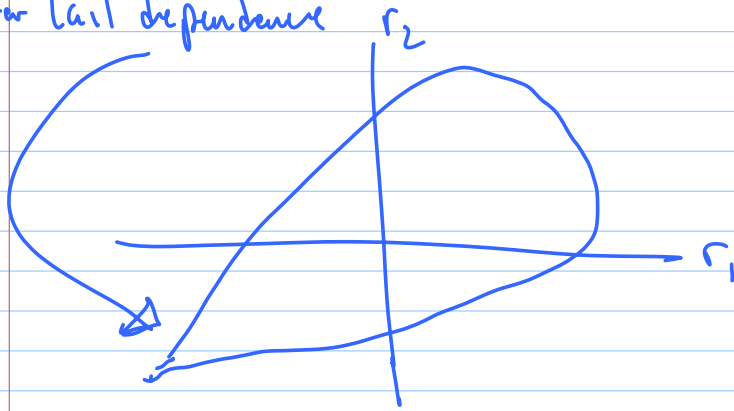


Econ 512 Lec 8

Note Title

4/21/2010

Lower Tail dependence



Upper and Lower
Tail Dependence



Multivariate GARCH

$$e_t = \Sigma_t^{1/2} z_t, \quad z_t \sim iid(0, I_K)$$

$$\begin{aligned} E(e_t e_t') &= E\left[\Sigma_t^{1/2} z_t z_t' \Sigma_t^{1/2'}\right] \\ &= E\left[E\left[\Sigma_t^{1/2} z_t z_t' \Sigma_t^{1/2'} \mid \mathcal{I}_{t-1}\right]\right] \\ &= E\left[\Sigma_t^{1/2} E(z_t z_t' \mid \mathcal{I}_{t-1}) \Sigma_t^{1/2'}\right] \end{aligned}$$

$$= E\left[\Sigma_t^{1/2} \cdot I_K \Sigma_t^{1/2'}\right]$$

$$= E\left[\Sigma_t^{1/2} \Sigma_t^{1/2'}\right]$$

$$= E[\Sigma_t]$$

$$\text{BEKK}(1,1) \quad \epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$$

$$\Sigma_t = A_0 A_0' + A_1 \epsilon_{t-1} \epsilon_{t-1}' A_1' + B_1 \Sigma_{t-1} B_1'$$

$$\begin{bmatrix} \sigma_t^{11} & \sigma_t^{12} \\ \sigma_t^{12} & \sigma_t^{22} \end{bmatrix} = \begin{bmatrix} a_0^{11} & 0 \\ a_0^{12} & a_0^{22} \end{bmatrix} \begin{bmatrix} a_0^{11} a_0^{12} \\ 0 \quad a_0^{22} \end{bmatrix}$$

$$+ \begin{bmatrix} a_1^{11} & 0 \\ a_1^{12} & a_1^{22} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1}^1 \epsilon_{t-1}^1 & \epsilon_{t-1}^1 \epsilon_{t-1}^2 \\ \epsilon_{t-1}^1 \epsilon_{t-1}^2 & \epsilon_{t-1}^2 \epsilon_{t-1}^2 \end{bmatrix} \begin{bmatrix} a_1^{11} a_1^{12} \\ 0 \quad a_1^{22} \end{bmatrix}$$

$$+ \begin{bmatrix} b_1^{11} & 0 \\ b_1^{12} & b_1^{22} \end{bmatrix} \begin{bmatrix} \sigma_{t-1}^{11} & \sigma_{t-1}^{12} \\ \sigma_{t-1}^{12} & \sigma_{t-1}^{22} \end{bmatrix} \begin{bmatrix} b_1^{11} b_1^{12} \\ 0 \quad b_1^{22} \end{bmatrix}$$

$$\Rightarrow \sigma_t^{ij} \text{ depends on } \epsilon_{t-1}^1, \epsilon_{t-1}^2, \sigma_{t-1}^{11}, \sigma_{t-1}^{12} \text{ and } \sigma_{t-1}^{22}$$