Ecm 512 lec 6

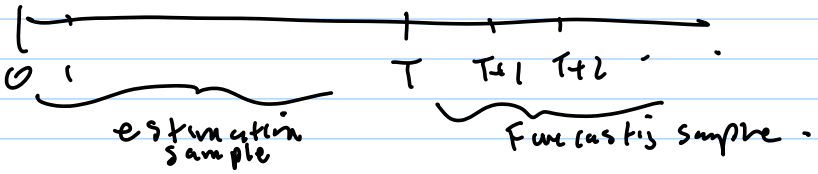
Note Title 4/14/2010

$r_t - \mu = \epsilon_t$

$\epsilon_t = \sigma_t z_t$, z_t iid $N(0,1)$

$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$

Note: $\epsilon_t^2 = \sigma_t^2 z_t^2$



$E[\epsilon_{T+1}^2 | I_T] = E[z_{T+1}^2 \sigma_{T+1}^2 | I_T]$

$= E[\sigma_{T+1}^2 | I_T]$

$I_T = \{r_1, \ldots, r_T\}$

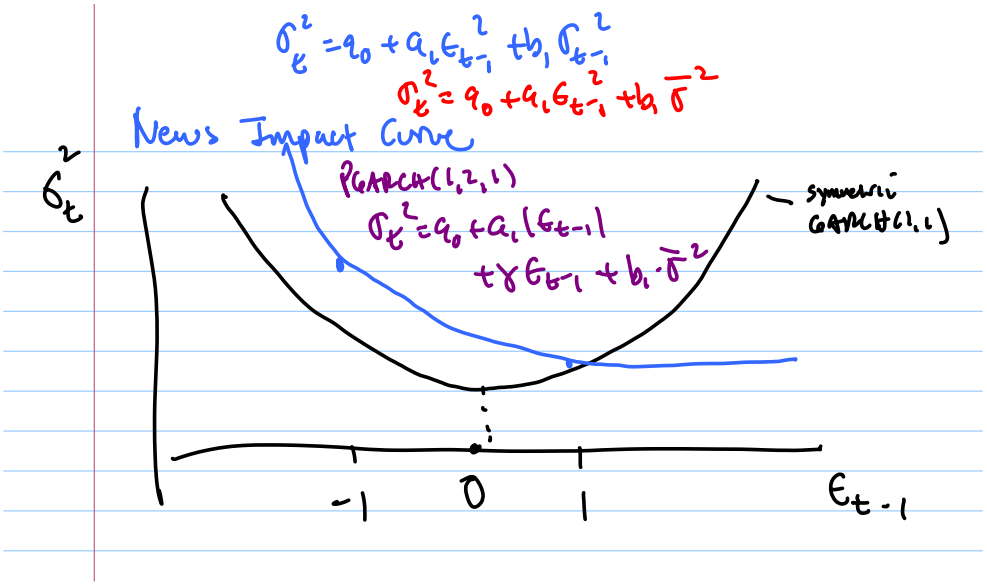
$E[\sigma_{T+1}^2 | I_T] = a_0 + a_1 E[\epsilon_T^2 | I_T] + b_1 E[\sigma_T^2 | I_T]$

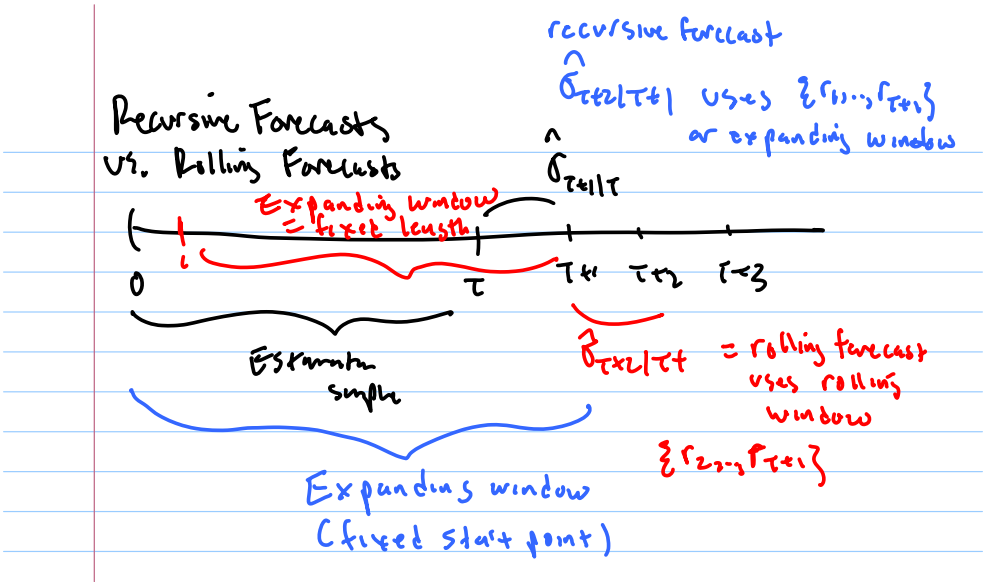
$= a_0 + a_1 \epsilon_T^2 + b_1 \sigma_T^2$ = known

$E[\sigma_{T+2}^2 | I_T] = a_0 + a_1 E[\epsilon_{T+1}^2 | I_T] + b_1 E[\sigma_{T+1}^2 | I_T]$

$= a_0 + a_1 E[r_{T+1}^2 | I_T] + b_1 E[\sigma_{T+1}^2 | I_T]$

$= a_0 + (a_1 + b_1) E[\sigma_{T+1}^2 | I_T]$


$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$
$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + b_1 \bar{\sigma}^2$
News Impact Curve
σ_t^2
PGARCH(1,2,1)
$\sigma_t^2 = \alpha_0 + \alpha_1 (\epsilon_{t-1}$
$+ \gamma \epsilon_{t-1} + b_1 \cdot \bar{\sigma}^2$
symmetric GARCH(1,1)
-1
0
1
ϵ_{t-1}


Recursive Forecasts
vs. Rolling Forecasts
recursive forecast
$\hat{\sigma}_{\tau+2|\tau+1}$ uses $\{r_1, \ldots, r_{\tau+1}\}$
or expanding window
$\hat{\sigma}_{\tau+1|\tau}$
Expanding window
= fixed length
1
0
τ
$\tau+1$
$\tau+2$
$\tau+3$
Estimation sample
$\hat{\sigma}_{\tau+2|\tau+1}$ = rolling forecast uses rolling window
$\{r_{2}, \ldots, r_{\tau+1}\}$
Expanding window
(fixed start point)