Econ 512 Lec 5

Note Title 4/12/2010

ARCH(1) $r_t = \mu + \epsilon_t$, $\hat{\epsilon}_t = r_t - \hat{\mu}$

$\epsilon_t = z_t \sigma_t$

$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$

$\hat{\epsilon}_t = \beta_0 + \beta_1 \hat{\epsilon}_{t-1}^2 + \cdots + \beta_5 \hat{\epsilon}_{t-5}^2$

$LM = T \cdot R^2$ or $F_{\beta_1 = \cdots = \beta_5 = 0}$

GARCH-M

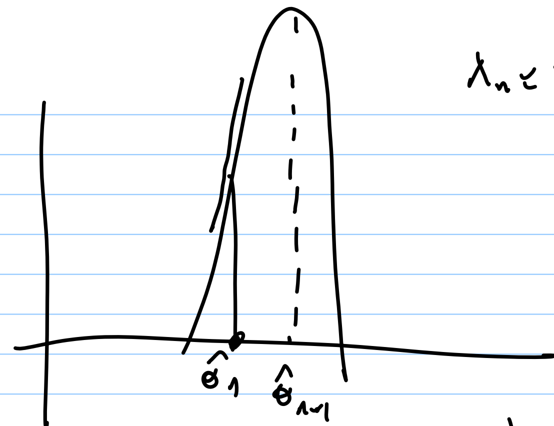
$r_t = \mu + \pi \cdot \sigma_t + \epsilon_t$

$\epsilon_t = z_t \cdot \sigma_t$

volatility or risk impacts expected return.

$E[r_t | I_{t-1}] = \mu + \pi \sigma_t$

expect $\pi > 0$: risk $\uparrow \Rightarrow E\{r_t | I_{t-1}\} \uparrow$



λ_n = step length

NR optimization: $\hat{\theta}_{n+1} = \hat{\theta}_n - H^{-1}(\hat{\theta}_n) \cdot S(\hat{\theta}_n)$

H = hessian, S = score

$$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$$

If $a_1 = 0$

$\Rightarrow \sigma_t^2 = a_0 + b_1 \sigma_{t-1}^2 \Rightarrow$ No randomness in volatility

$\Rightarrow b_1$ is not identified!

ARMA(1,1)

$$r_t = \phi r_{t-1} + \epsilon_t + \theta \epsilon_{t-1}$$

$$(1-\phi L) r_t = (1+\theta L)\epsilon_t$$

$$\theta = \phi$$

$$y_t = x_t'\beta + \epsilon_t$$

$\hat{\beta}_{OLS} = \hat{\beta}_{MLE}$ under normality.

GARCH(1,1)

$r_t = \mu + \epsilon_t$

$\epsilon_t = z_t \sigma_t$

$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$

2 Forecasts

(1) r_{t+j} , $j = 1, 2, \ldots$: $\hat{\mu}$

(2) σ_{t+j}^2 , $j = 1, 2, \ldots$

$\hat{\epsilon}_t = r_t - \hat{\mu}$

$\epsilon_t = z_t \sigma_t$

$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + \sigma_{t-1}^2$

$\hat{\sigma}_t^2 = \hat{a}_0 + \hat{a}_1 \hat{\epsilon}_{t-1}^2 + \hat{\sigma}_{t-1}^2$

$\hat{\sigma}_1^2 = \hat{a}_0 + \hat{a}_1 \hat{\epsilon}_0^2 + \hat{\sigma}_0^2$

$\frac{1}{T}\sum_{t=1}^{T} \hat{\epsilon}_t^2$, $\hat{\sigma}_0^2 = \frac{1}{T}\Sigma_1^T \epsilon_t^2$

$\hat{\sigma}_2^2 = a_0 + \hat{a}_1 \hat{\epsilon}_1^2 + \hat{b}_1 \hat{\sigma}_1^2$

$\hat{\epsilon}_1 = r_1 - \hat{\mu}$

$\hat{\sigma}_1^2$