

EWMA

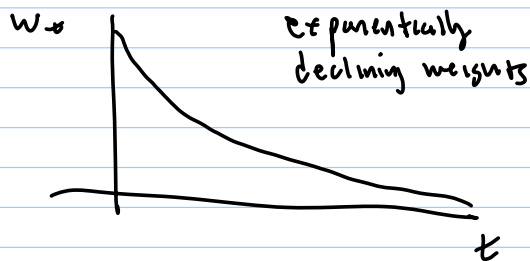
$$\{r_t\}_{t=1}^T$$

$$\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t$$

$$\bar{r}_w = \sum_{j=0}^T w_{t-j} r_{t-j}, \quad w_t = \frac{\lambda^t}{\sum_{j=1}^T \lambda^j}, \quad \lambda \in (0,1)$$

$$\Rightarrow \sum_{t=0}^T w_t = 1$$

typical $\lambda = 0.94$ = RiskMetrics value



Variance Ratio Statistics

$$\{r_t\}_{t=1}^T \quad \text{cc returns}$$

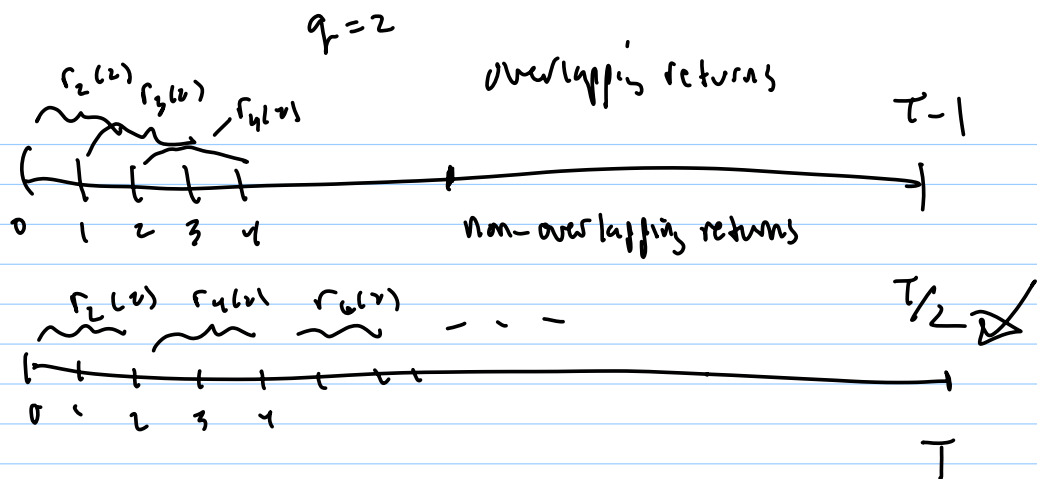
$$r_t(2) = r_t + r_{t-1} \quad \text{2 period return}$$

If RW is true then $\{r_t\}$ is iid ($0, \sigma^2$)

$$\begin{aligned} \text{var}(r_t(2)) &= \text{var}(r_t + r_{t-1}) = \text{var}(r_t) + \text{var}(r_{t-1}) \\ &\quad + 2 \text{cov}(r_t, r_{t-1}) \\ &= \text{var}(r_t) + \text{var}(r_{t-1}) \quad \text{|| under RW} \\ &= 2\sigma^2 \end{aligned}$$

$$VR(2) = \frac{\text{var}(r_t(2))}{2 \cdot \text{var}(r_t)} = \frac{2 \cdot \sigma^2}{2 \cdot \sigma^2} = 1$$

$$VR(10) = \frac{\text{var}(r_t + r_{t-1} + \dots + r_{t-9})}{10 \cdot \text{var}(r_t)} = \frac{10 \cdot \sigma^2}{10 \cdot \sigma^2} = 1$$



$$r_t(2) = r_t + r_{t-1}$$

$$\{r_2 + r_1, r_3 + r_2, r_4 + r_3, \dots\}$$

$$= \{r_2(t), r_3(t), r_4(t), \dots\} \sim MA(1)$$

$$\text{In general, } \{r_t(q)\} \sim MA(q-1)$$