Econ 512 Lec 10

Note Title 4/28/2010

$$\ln(\sigma_t) - \alpha = \phi(\ln(\sigma_{t-1}) - \alpha) + \eta_t$$

$$\text{var}(\ln(\sigma_t)) = E[(\ln(\sigma_t) - \alpha)^2]$$

$$\Rightarrow \text{var}(\ln(\sigma_t)) = \phi^2 \text{var}(\ln(\sigma_t)) + \sigma_\eta^2$$

$$\Rightarrow \text{var}(\ln(\sigma_t)) = \frac{\sigma_\eta^2}{1-\phi^2}$$

Aside: Mixture-Normal distn

$r_t = \mu + \sigma_t \cdot u_t$, $u_t \sim$ iid $N(0,1)$

$\sigma_t \sim LN(\alpha, \beta^2)$

indep of u_t

$\Rightarrow r_t \sim$ continuous mixture normal distributions with mixing variable σ_t

$$r_t \mid \sigma_t \sim N(\mu, \sigma_t^2)$$

Discrete mixture Normals

$$r_t - \mu = \sigma_1 \cdot u_t \quad \text{with prob } p$$
$$= \sigma_2 \cdot u_t \quad \text{with prob } 1-p$$

$$S_t = 1 \quad \text{with prob } p$$
$$= 0 \quad \text{with prob } 1-p$$

$$r_t - \mu = \sigma_1 S_t u_t + (1 - S_t)\sigma_2 \cdot u_t$$

Aside: Defn of Student's t RV.

$$X \sim S_t(\nu)$$

$$X = \frac{N(0,1)}{\sqrt{\chi^2(\nu)/\nu}} = Z \cdot W$$

$$Z \sim N(0,1), \quad W = \frac{1}{\sqrt{\chi^2(\nu)/\nu}}$$