Econ 512 Financial Econometrics
Lab #2 Part I

Eric Zivot

Due: Tuesday, May 7

1 Analytic Questions

Consider the GARCH(1,1) model

$$\begin{aligned} y_t &= c + \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t, \; z_t \sim iid \; N(0,1) \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

1. Determine the unconditional moments

$$E[\varepsilon_t], E[\varepsilon_t^2], E[\varepsilon_t^3] \text{ and } E[\varepsilon_t \varepsilon_{t-k}] \text{ for } k > 0$$

2. Determine the necessary conditions for

$$E[\varepsilon_t^2] = \sigma_\varepsilon^2 < \infty$$

3. Write the model as an infinite order ARCH process

$$\begin{aligned} \sigma_t^2 &= \omega^* + \delta(L)\varepsilon_t^2 \\ \delta(L) &= \sum_{k=1}^{\infty} \delta_k L^k \end{aligned}$$

   and express the coefficients ω^* and δ_k as functions of ω, α and β.

4. Determine the conditional moments

$$E[\varepsilon_t | I_{t-1}], E[\varepsilon_t^2 | I_{t-1}]$$

5. Express ε_t^2 as an ARMA(1,1) model in ε_t^2 and v_t where $v_t = \varepsilon_t^2 - \sigma_t^2$. Show that v_t is a MDS.

6. For a sample of size T, write down the log-likelihood function and discuss how a recursive algorithm could be used to compute it. Discuss issues related to the treatment of initial values.

Consider the stationary AR(1) process

$$\begin{aligned} y_t &= \phi y_{t-1} + \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t, \ z_t \,\tilde{}\, iid \ N(0,1) \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

where ε_t follows a GARCH(1,1) process.

1. Determine the best linear predictor of y_{T+s} based on information available at time T. Give a recursive formula for computing the forecast. What is the forecast error $y_{T+s} - y_{T+s|T}$?

2. Determine the variance of the forecast error for predicting y_{T+s} conditional on I_T and give a recursion for computing it.

3. Determine the best linear predictor of σ_{T+s}^2 based on information available at time T and give a recursive formula for computing it. What is the forecast for σ_{T+s}?

4. Briefly explain how you can compute standard errors for the forecasts of σ_{T+s}?