Econ 512: Financial Econometrics
HW 2

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Due: Monday 4/20/2009

1 Reading

1. Zivot, E. (2008). "Practical Issues in the Analysis of Univariate GARCH Models," Handbook of Financial Time Series (available on class webpage).
2. Tsay, R. (2005) *Analysis of Financial Time Series, Second Edition.* Chapters 3 and 7.
3. Taylor, S.J. (2005), *Asset Price Dynamics, Volatility, and Prediction.* Chapters 8-10.

2 Problems

1. Consider the GARCH(1,1) process

$$\begin{aligned} r_t &= \sigma_t z_t,\ z_t \sim iid\ N(0,1) \\ \sigma_t^2 &= a_0 + a_1 r_{t-1}^2 + b_1 \sigma_{t-1}^2 \end{aligned}$$

Derive the following results:

(a) $E[r_t] = 0, E[r_t^2] = E[\sigma_t^2] = a_0/(1 - a_1 - b_1)$

(b) $E[r_t|I_{t-1}] = 0,\ E[r_t^2|I_{t-1}] = \sigma_t^2$

(c) r_t^2 has an ARMA(1,1) representation of the form

$$r_t^2 = a_0 + (a_1 + b_1) r_{t-1}^2 + v_t - b_1 v_{t-1}$$

where $v_t = r_t^2 - \sigma_t^2$ is a MDS.

(d) σ_t^2 has an ARCH(∞) representation with $a_i = a_1 b_1^{i-1}$

(e) Write out the GARCH(1,1) log-likelihood function based on a sample of size T.

(f) σ^2_{T+k} has the forecasting equation

$$E_T[\sigma^2_{T+k}] - \bar{\sigma}^2 = (a_1 + b_1)^{k-1}(E[\sigma^2_{T+1}] - \bar{\sigma}^2).$$

2. Consider the simple stochastic volatility model

$$r_t = \mu + \sigma_t z_t$$

where $\{\sigma_t\}$ is a sequence of positive stationary random variables with $E[\sigma_t^4]$ finite and positive autocorrelations at all lags, z_t is iid $N(0,1)$ and is independent of σ_t^2 for all leads and lags. This is different from the GARCH(1,1) model above because there are two distinct sources of randomness: σ_t and z_t. In the GARCH(1,1) model above, the only source of randomness is z_t. Derive the following results (Hint: See Taylor pgs 268 - 271):

(a) $var(r_t) = E[\sigma_t^2]$

(b) $kurt(r_t) > 3$

(c) $cov(r_t, r_{t-j}) = 0$ for all $j > 0$

(d) $cov(s_t, s_{t-j}) > 0$ for all j where $s_t = (r_t - \mu)^2$

3. Tsay, R. (2005). *Analysis of Financial Time Series, Second Edition.* Chapter 3, Exercises 3.1 - 3.4,