

Econ 512: Financial Econometrics and Volatility Models

HW 1

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Due: Tuesday 4/12/2011

1 Reading

1. Zivot, E. (2008). “Practical Issues in the Analysis of Univariate GARCH Models,” *Handbook of Financial Time Series* (available on class webpage).
2. Jondeau, E., Poon, S.-H., and Rockinger, M. (2006), *Financial Modeling Under Non-Gaussian Distributions*, Chapter 3.
3. Taylor, S.J. (2005), *Asset Price Dynamics, Volatility, and Prediction*. Chapters 8-10.

2 Analytic Problems

1. Consider the GARCH(1,1) process

$$\begin{aligned}r_t - \mu &= \varepsilon_t = \sigma_t z_t, \quad z_t \sim iid N(0, 1) \\ \sigma_t^2 &= a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2\end{aligned}$$

Derive the following results:

- (a) $E[\varepsilon_t] = 0, E[\varepsilon_t^2] = E[\sigma_t^2] = a_0/(1 - a_1 - b_1) = \bar{\sigma}^2$
- (b) $E[\varepsilon_t | I_{t-1}] = 0, E[\varepsilon_t^2 | I_{t-1}] = \sigma_t^2$
- (c) ε_t^2 has an ARMA(1,1) representation of the form

$$\varepsilon_t^2 = a_0 + (a_1 + b_1)\varepsilon_{t-1}^2 + v_t - b_1 v_{t-1}$$

where $v_t = \varepsilon_t^2 - \sigma_t^2$ is a MDS.

- (d) σ_t^2 has an ARCH(∞) representation with $a_i = a_1 b_1^{i-1}$

(e) Write out the GARCH(1,1) log-likelihood function $\ln L(\mu, a_0, a_1, b_1)$ based on a sample $\{r_1, \dots, r_T\}$ of size T . Specify initial values for ε_0^2 and σ_0^2 and the parameters a_0, a_1 and b_1 .

(f) Compute the following derivatives of the log-likelihood function:

$$\frac{\partial \ln L(\mu, a_0, a_1, b_1)}{\partial \mu}, \frac{\partial \ln L(\mu, a_0, a_1, b_1)}{\partial a_0}, \frac{\partial \ln L(\mu, a_0, a_1, b_1)}{\partial a_1}, \frac{\partial \ln L(\mu, a_0, a_1, b_1)}{\partial b_1}$$

(g) σ_{T+k}^2 has the forecasting equation

$$E_T[\sigma_{T+k}^2] - \bar{\sigma}^2 = (a_1 + b_1)^{k-1}(E[\sigma_{T+1}^2] - \bar{\sigma}^2).$$

3 Empirical Problems

Consider the daily returns on Microsoft and the S&P 500 that we have been using in the class examples. These are available in the S+FinMetrics module and are posted in Excel files on the class webpage (for use in other software programs). You may use any software you like (e.g. Eviews, Matlab, Ox, R, Stata, S-PLUS) but I recommend using either Eviews, R or S-PLUS. The garch modeling tools in Eviews and S-PLUS are very extensive and quite similar. The tools available in R are less extensive and less tested. For those using S-PLUS, you will find the example script files on the homework page very useful.

1. Create time plots of the daily returns, absolute returns and squared returns on Microsoft and the S&P 500. Comment on any "stylized facts" of asset returns that we discussed in class.
2. In this exercise, you will assess the distributional properties of the daily returns on Microsoft and the S&P 500. Create histograms and normal qq-plots for the returns. Compute sample statistics (mean, sd, skewness, kurtosis) and test the null hypothesis of normality using the JB statistic.
3. In this exercise, you will assess the serial correlation properties of the daily returns on Microsoft and the S&P 500. Plot the sample ACF and PACF and comment. Compute the Ljung-Box Q statistic for lags 1-10 and use these statistics to test the hypothesis that the returns are uncorrelated. Is the Q statistic valid for testing RW3? Next, compute the heteroskedasticity robust variance ratio statistics for lags 1-10 and test the hypothesis that the returns follow RW3. What do you find?
4. In this exercise, you will assess the serial correlation properties of the daily squared and absolute returns on Microsoft and the S&P 500. Plot the sample ACF and PACF and comment. Compute the Ljung-Box Q statistic for lags 1-10 and use these statistics to test the hypothesis that the returns are uncorrelated.

5. In this exercise, you will test for ARCH effects in the daily returns on Microsoft and the S&P 500. For each series, compute Engle's LM test for ARCH effects using 5 and 10 lags.
6. In this exercise, you will estimate ARCH and GARCH models for the daily returns on Microsoft and the S&P 500. First, estimate an ARCH(5) model for the each series. What is the sum of the ARCH coefficients? Plot the in-sample estimates of the conditional volatility and compare with the absolute returns. What is the estimate of the unconditional volatility? Compare this with the sample standard deviation of returns. Next, estimate a GARCH(1,1) model for each series. What is the sum of the ARCH and GARCH coefficients? Plot the in-sample estimates of the conditional volatility and compare with the absolute returns. Do the conditional volatility estimates look similar to the ARCH(5) estimates? What is the estimate of the unconditional volatility? Compare this with the sample standard deviation of returns and the estimate from the ARCH(5) model.
7. Using the estimated ARCH(5) and GARCH(1,1) models, compute h-step-ahead volatility forecasts for $h = 1, \dots, 100$ days and plot these along with the estimate of unconditional volatility. Which forecasts converge faster to the unconditional volatility?