Econ 512: Financial Econometrics
Final Exam

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Due: Monday 6/14/2010 at 9 am.

1 Instructions

This is a take-home open book final exam. It is due on Tuesday, June 9 at 10:30 am in my office or my mailbox (I have a final exam to proctor until 10:20). The exam is mostly a review of the main material covered during the term.

2 Empirical Properties of Returns

1. Throughout the course, we talked about some basic stylized facts of daily, monthly and intra-daily continuously compounded asset returns and transformations these returns (e.g., squared and absolute returns, daily realized variance from squared intra-day returns). Briefly describe these stylized facts (a bullet point list is fine). Distinguish between stylized facts for univariate series and stylized facts for multivariate series.

2. Consider the normal GARCH(1,1) model

$$
\begin{aligned}
r_t - \mu &= \varepsilon_t = u_t \sigma_t,\ u_t \sim N(0,1), \\
\sigma_t^2 &= a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2, \\
a_0 &> 0,\ a_1 > 0,\ b_1 \geq 0 \text{ and } a_1 + b_1 < 1,
\end{aligned}
$$

and the log-normal stochastic volatility (SV) model

$$
\begin{gathered}
r_t = \mu + \sigma_t u_t, \\
\ln(\sigma_t) - \alpha = \phi(\ln(\sigma_{t-1} - \alpha) + \eta_t, |\phi| < 1, \\
\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \sim iid\ N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \right).
\end{gathered}
$$

Which stylized facts of asset returns can be described by the GARCH(1,1) model and which cannot? Briefly justify your answers using the properties of the GARCH(1,1) model. (Note: you do not have to derive these properties of the GARCH(1,1), just state what they are.)

3. Consider the multivariate EWMA and DVEC(1,1) models based on the return decomposition

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t, \ t = 1, 2, \cdots, T, \\ & \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_t). \end{aligned}$$

The EWMA has the form

$$\begin{aligned} \boldsymbol{\Sigma}_t &= (1-\lambda)\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \lambda \boldsymbol{\Sigma}_{t-1} \\ 0 &< \lambda \leq 1 \end{aligned}$$

and the DVEC(1,1) has the form

$$\boldsymbol{\Sigma}_t = \mathbf{A}_0 + \mathbf{A}_1 \odot \boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \mathbf{B}_1 \odot \boldsymbol{\Sigma}_{t-1}$$

Which stylized facts of asset returns can be described by the DVEC(1,1) model and which cannot? Briefly justify your answers using the properties of the EWMA and DVEC(1,1) models. (Note: you do not have to derive these properties of the GARCH(1,1), just state what they are.)

3 Using Volatility Models

1. We spent a good deal of time in class studying univariate and multivariate volatility models for asset returns. Briefly explain why we care about modeling and forecasting asset return volatility, covariance and correlation.

2. GARCH models are commonly used to forecast future conditional volatility. In principle, we can evaluate the adequacy of a fitted GARCH model by examining the quality of its forecasts. However, conditional volatility is unobservable which makes a direct comparison between forecasted volatility and actual volatility impossible. Briefly explain how GARCH forecasts can be evaluated using observable proxies for conditional volatility. What problems, if any, are there associated with using proxies for volatility in evaluating volatility forecasts?

3. Value-at-risk (VaR) and expected shortfall (ES) are two commonly used risk measures for a portfolio of financial assets. Let R_t denote the daily return on a given portfolio of assets. VaR and ES can be estimated unconditionally from the distribution of R_t, and they can be estimated conditionally from a GARCH model. Typical unconditional models include (i) historical simulation; (2) normal distribution; (3) extreme value theory (EVT). Common conditional models include (4) GARCH; (5) GARCH + EVT. Briefly explain how VaR and ES are computed using each of these 5 methods.

4. Given a set of competing VaR models for returns, describe how these models can be evaluated. That is, describe how you can decide if one VaR model is better than other one.

4 Stochastic Volatility Models with Leverage

In the paper "On Leverage in a Stochastic Volatility Model," (*Journal of Econometrics*, 127, 2005, 165-178), Yu discusses leverage in the basic log-normal stochastic volatility model. Read this paper (it is available for download on the class syllabus page) as well as section 11.9 in APDVP and answer the following questions.

1. Yu discusses two forms of the SV model with leverage. Describe these two forms.

2. Which parameterization is preferred to capture the leverage effect? Why?

3. Briefly summarize Yu's empirical results comparing the two models (a few paragraphs is sufficient). Focus on the comparison and don't worry about the details of the Bayesian estimation.

5 Multivariate GARCH Models

In the paper "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models," (*Journal of Business and Economic Statistics*, 20, 2002, 339-350) Engle introduced the dynamic conditional correlation (DCC) multivariate GARCH model. Read this paper (it is available for download on the class syllabus page) and answer the following questions.

1. Describe the DCC model in the class of multivariate GARCH models

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu} + \boldsymbol{\epsilon}_t, \; t = 1, 2, \cdots, T, \\ & \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_t). \end{aligned}$$

   In particular, give the expression for the conditional covariance matrix $\boldsymbol{\Sigma}_t$. What is the main advantage of DCC over the other multivariate GARCH models?

2. Briefly describe Engle's two-step method for estimating the DCC model.

3. Briefly summarize Engle's empirical findings (a few paragraphs is sufficient).

6 High Frequency Methods

1. Given intra-day high frequency price data, describe how to compute the (naive) realized variance (RV) for the day.

   (a) Describe how to align irregularly spaced intra-day price data to a regularly spaced time clock. What is the preferred alignment method?

   (b) What does RV converge to as the sampling frequency goes to zero? Be specific about your assumptions regarding the underlying continuous time price process and the presence or absence of market microstructure noise.

   (c) In practice, you cannot let the sampling frequency go to zero so that there is always some measurement error in RV. How can you evaluate the magnitude of this measurement error? That is, how can you compute a 95% confidence interval for RV?

2. The convergence of RV assumes the absence of market microstructure noise.

   (a) Briefly describe the common sources of market microstructure noise.

   (b) Under the assumption that the microstructure noise is independent of the true price process, what happens to RV as the sampling interval goes to zero?

   (c) Briefly describe the logic behind Bandi and Russell's optimal sampling frequency and how that deals with the problems created by microstructure noise?