AMATH546/ECON589 HW6

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1 Reading

- My lectures on factor model risk analysis
- Meucci, A. (2007). "Risk Contributions from Generic User-Defined Factors", Risk.
- SDAFE chapter 17.

2 Data and Programs

For this assignment you will use the managers data set from the Performance-Analytics package. The analysis you will do is similar to that done in the class examples. Please refer to the lecture slides on factor model risk analysis and the R code for factor model risk analysis.

3 Factor Model Analytics

Consider the factor model for asset returns

$$\begin{aligned} R_{it} &= \alpha_i + \beta_{1i} f_{1t} + \beta_{2i} f_{2t} + \cdots + \beta_{Ki} f_{Kt} + \varepsilon_{it} \quad (1) \\ &= \alpha_i + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{it} \end{aligned}$$

- R_{it} is the simple return (real or in excess of the risk-free rate) on asset i ($i = 1, \ldots, N$) in time period t ($t = 1, \ldots, T$),

- f_{kt} is the k^{th} *common factor* $(k = 1, \ldots, K)$,
- β_{ki} is the *factor loading* or *factor beta* for asset i on the k^{th} factor,
- ε_{it} is the asset *specific factor*.

Assume the following: the factor realizations, $\mathbf{f}_t$, are stationary with unconditional moments

$$\begin{aligned} E[\mathbf{f}_t] &= \boldsymbol{\mu}_f \\ cov(\mathbf{f}_t) &= E[(\mathbf{f}_t - \boldsymbol{\mu}_f)(\mathbf{f}_t - \boldsymbol{\mu}_f)'] = \underset{K\times K}{\boldsymbol{\Omega}_f} \end{aligned}$$

Asset specific error terms, ε_{it}, are uncorrelated with each of the common factors, f_{kt},

$$cov(f_{kt}, \varepsilon_{it}) = 0, \text{ for all } k,\ i \text{ and } t.$$

Error terms ε_{it} are serially uncorrelated and contemporaneously uncorrelated across assets

$$\begin{aligned} cov(\varepsilon_{it}, \varepsilon_{js}) &= \sigma^2_{\varepsilon,i} \text{ for all } i = j \text{ and } t = s \\ &= 0, \text{ otherwise} \end{aligned}$$

1. Show that

$$\begin{aligned} var(R_{it}) &= \boldsymbol{\beta}_i' \boldsymbol{\Omega}_f \boldsymbol{\beta}_i + \sigma^2_{\varepsilon,i} \\ cov(R_{it}, R_{jt}) &= \boldsymbol{\beta}_i' \boldsymbol{\Omega}_f \boldsymbol{\beta}_j \end{aligned}$$

2. Using

$$\underset{(N\times 1)}{\mathbf{R}_t} = \underset{(N\times 1)}{\boldsymbol{\alpha}} + \underset{(N\times K)}{\mathbf{B}} \underset{(K\times 1)}{\mathbf{f}_t} + \underset{(N\times 1)}{\boldsymbol{\varepsilon}_t},\ t = 1, \ldots, T$$

and the assumptions of the factor model, show that the $(N \times N)$ covariance matrix of asset returns has the form

$$cov(\mathbf{R}_t) = \boldsymbol{\Omega}_{FM} = \mathbf{B}\boldsymbol{\Omega}_f\mathbf{B}' + \mathbf{D}$$

where $\mathbf{D} = diag(\sigma^2_{\varepsilon,1}, \ldots, \sigma^2_{\varepsilon,N})$

4 Estimation of Macroeconomic Factor Model

For the remaining exercises you will use the managers data set from the PerformanceAnalytics package. Follow the Powerpoint examples in factorModelRiskAnalysisPowerpoint and the code in the R script factorModelrisk-Analysis.r. Additionally, you will need to install the factorAnalytics package from Rforge. First you will estimate a factor model for hedge funds of the form

$$R_{it} = \alpha_i + \beta_{1i} f_{1t} + \beta_{2i} f_{2t} + \beta_{3i} f_{3t} + \varepsilon_{it},\ i = 1, \ldots, 6 \tag{2}$$

where R_{it} = return in excess of 3-month T-Bill rate on hedge fund i in month t; $f_{1t} = EDEC.LS.EQ_t$ = excess total return on EDHEC long-short equity index; f_{2t} = excess total return on S&P 500 index; f_{3t} = excess total return on US 10 year T-Note.

1. Estimate (2) using ordinary least squares regression for each of the six hedge funds (HAM1,...,HAM6) over the period January 1997 through December 2006. Summarize the factor model estimates in a table of the form

Fund	α	β_1	β_2	β_3	σ	R^2
HAM1						
$\vdots$						
HAM6						

Briefly comment on the signs and magnitudes of the estimated coefficients. Which funds have the best and worst fits? which funds have positive and negative α's.

2. For HAM1, compare graphically the in-sample time series of actual and fitted excess returns. Does the fit look reasonable? Plot the histogram and normal qq-plots of the fitted values and the residuals. Do they look normally distributed?

1. Assume a fund of hedge fund portfolio with weight vector $\mathbf{w} = (0.3, 0.2, 0.15, .0.15, 0.2)'$. Derive the estimated factor model parameters for this portfolio. Add these estimates to the table from question 1 above.

5 Factor Model Risk Analysis

1. Using the estimated parameters of the factor model (2) for each asset and the portfolio, estimate $\sigma_{i,FM}$, $VaR_{i,.95}$ and $ES_{i,95}$. Estimate $VaR_{i,.95}$ and $ES_{i,95}$ assuming the factors and residuals are normally distributed. Summarize the risk estimates in a table of the form

Fund	σ_{FM}	$VaR_{.95}$	$ES_{.95}$
HAM1			
⋮			
HAM6			
PORT			

Which assets have the highest and lowest risk measures?

2. Using the estimated parameters of the factor model (2) for each asset, estimate the Euler decomposition of $\sigma_{i,FM}(\boldsymbol{\beta}_i, \sigma_\varepsilon)$

$$\begin{aligned}\sigma_{i,FM}(\boldsymbol{\beta}_i, \sigma_\varepsilon) &= \beta_{1i}\frac{\partial\sigma_{i,FM}(\boldsymbol{\beta}_i, \sigma_{\varepsilon,i})}{\partial\beta_{1i}} + \cdots + \beta_{3i}\frac{\partial\sigma_{i,FM}(\boldsymbol{\beta}_i, \sigma_{\varepsilon,i})}{\partial\beta_{3i}} \\ &\quad +\sigma_\varepsilon\frac{\partial\sigma_{i,FM}(\boldsymbol{\beta}, \sigma_{\varepsilon,i})}{\partial\sigma_{\varepsilon,i}} \\ &= CR_{1i}^\sigma + CR_{2i}^\sigma + CR_{3i}^\sigma + CR_{\varepsilon i}^\sigma\end{aligned}$$

Summarize the factor risk budgets in a table of the form

Fund	σ_{FM}	CR_{1i}^σ	CR_{2i}^σ	CR_{3i}^σ	$CR_{\varepsilon i}^\sigma$	CR_{1i}^σ	PCR_{1i}^σ	PCR_{2i}^σ	PCR_{3i}^σ	$PCR_{\varepsilon i}^\sigma$
HAM1										
⋮										
HAM6										
PORT										

where $PCR_{ji}^\sigma = CR_{ji}^\sigma/\sigma_{i,FM}$. For each asset briefly comment on the factors that have the highest and lowest contributions to factor model volatility.

3. Using the estimated parameters of the factor model (2) for each asset, estimate the Euler decomposition of $ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)$

$$\begin{aligned} ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)(\boldsymbol{\beta}_i, \sigma_\varepsilon) &= \beta_{1i}\frac{\partial ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)}{\partial \beta_{1i}} + \cdots + \beta_{3i}\frac{\partial ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)}{\partial \beta_{3i}} \\ &\quad + \sigma_\varepsilon \frac{\partial \sigma_{i,FM}(\boldsymbol{\beta}, \sigma_{\varepsilon,i})}{\partial \sigma_{\varepsilon,i}} \\ &= CR^{\sigma}_{1i} + CR^{\sigma}_{2i} + CR^{\sigma}_{3i} + CR^{\sigma}_{\varepsilon i} \end{aligned}$$

Use the historical non-parametric estimates of the form

$$\hat{E}^{HS}[\tilde{f}_{jt}|R_t \leq VaR_\alpha] = \frac{1}{[T\alpha]}\sum_{t=1}^{T}\tilde{f}_{jt}\cdot 1\left\{\widehat{VaR}_\alpha^{HS} \leq R_t\right\}$$

where $\tilde{\mathbf{f}}_t = (\mathbf{f}_t, z_t)'$ and $z_t = \varepsilon_t/\sigma_\varepsilon$. Summarize the factor risk budgets in a table of the form

Fund	σ_{FM}	CR^{ES}_{1i}	CR^{ES}_{2i}	CR^{ES}_{3i}	$CR^{ES}_{\varepsilon i}$	CR^{ES}_{1i}	PCR^{ES}_{1i}	PCR^{ES}_{2i}	PCR^{ES}_{3i}	$PCR^{ES}_{\varepsilon i}$
HAM1										
⋮										
HAM6										

where $PCR^{\sigma}_{ji} = CR^{\sigma}_{ji}/ES_{i,.95}$. For each asset briefly comment on the factors that have the highest and lowest contributions to expected shortfall.

6 Factor Model Monte Carlo

The estimated factor model can be used to generate simulated pseudo returns for each asset. These simulations can be used for stress testing and scenario analysis. In order to use the factor model for simulating returns, assumptions must be made for the distributions of $\mathbf{f}_t$ and ε_{it} $(i = 1, \ldots, 6)$.

1. Assume that

$$\begin{aligned} \mathbf{f}_t &\sim iid\ N(\boldsymbol{\mu}_f, \boldsymbol{\Omega}_f) \\ \varepsilon_{it} &\sim iid\ N(0, \sigma^2_{\varepsilon,i}) \end{aligned}$$

Use the sample mean and covariance of $\mathbf{f}_t$ for $\boldsymbol{\mu}_f$ and $\boldsymbol{\Omega}_f$, and use the OLS residual variance estimates for $\sigma^2_{\varepsilon,i}$. Using the factor model Monte Carlo algorithm simulate 1000 pseudo returns for each hedge fund. Use these simulated values to compute the Euler decomposition of $ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)$.

2. Extra credit. The factor model Monte Carlo algorithm can also be used to create conditional simulations. For example, the factors can modeled conditionally using the normal-DCC(1,1) model

$$
\begin{aligned}
\mathbf{f}_t &= \boldsymbol{\mu}_f + \boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma}_t^{1/2}\mathbf{z}_t,\ \mathbf{z}_t \sim N(\mathbf{0}, \mathbf{I}_3), \\
\boldsymbol{\Sigma}_t &= \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t, \\
\mathbf{D}_t &= \begin{pmatrix} \sigma_{1t} & 0 & 0 \\ 0 & \sigma_{2t} & 0 \\ 0 & 0 & \sigma_{3t} \end{pmatrix},\ \mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} & \rho_{13,t} \\ \rho_{12,t} & 1 & \rho_{23,t} \\ \rho_{13,t} & \rho_{13,t} & 1 \end{pmatrix},
\end{aligned}
$$

where σ_{it} are univariate normal-GARCH(1,1) volatilities and $\rho_{ij,t}$ are the GARCH(1,1) conditional correlations between the univariate GARCH(1,1) standardized residuals. The factor model residuals ε_{it} can be modeled as independent univariate normal-GARCH(1,1) processes. Use the rmgarch package to estimate the DCC model for f_t and the rugarch package to estimate the univariate GARCH models for the residuals ε_{it}. Use the fitted models to simulate 1000 pseudo return observations for each hedge fund. Use these simulated values to compute the Euler decomposition of $ES_{i,.95}(\boldsymbol{\beta}_i, \sigma_\varepsilon)$.