AMATH546/ECON589 HW5

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1 Reading

- FRF chapter 1 (section 8) and 3.
- QRM chapter 3, sections 1-3; chapter 5
- FMUND chapter 6.

2 Data and Programs

Download daily adjusted closing prices on Microsoft and the S&P 500 over the period 2000-01-03 to 2012-04-10 and compute the continuously compound returns. You will find my R scripts on the class webpage helpful for this assignment.

3 Forecasting Portfolio VaR with Multivariate GARCH

Consider an equally weighted portfolio of MSFT and GSPC $r_{p,t} = \mathbf{w}'\mathbf{r}_t$ where $\mathbf{w} = (0,5,0.5)'$. There are two ways to compute VaR for a portfolio. The first method is to use the univariate methods for the portfolio returns $r_{p,t}$. The second method is to use multivariate methods.

1. Fit a normal-GARCH(1,1) model to the portfolio returns $r_{p,t}$ and compute 1-day ahead 99% VaR using the formula

$$\widehat{VaR}^{GARCH}_{T+1|T,.99} = \hat{\mu}_p + \hat{\sigma}_{p,T+1|T} \times q^Z_{.01}, \tag{1}$$

where $\hat{\sigma}_{p,T+1|T}$ is the 1-day ahead GARCH(1,1) predicted conditional volatility.

2. Using the `ddcfit()` function from the rmgarch package, estimate the normal-DCC(1,1) model

$$\begin{aligned}
\mathbf{r}_t &= \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t = \boldsymbol{\Sigma}_t^{1/2}\mathbf{z}_t,\ \mathbf{z}_t \sim N(\mathbf{0}, \mathbf{I}_2), \\
\boldsymbol{\Sigma}_t &= \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t, \\
\mathbf{D}_t &= \begin{pmatrix} \sigma_{1t} & 0 \\ 0 & \sigma_{2t} \end{pmatrix},\ \mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} \\ \rho_{12,t} & 1 \end{pmatrix},
\end{aligned}$$

where σ_{1t} and σ_{2t} are univariate GARCH(1,1) volatilities and $\rho_{12,t}$ is the GARCH(1,1) conditional correlation between the univariate GARCH(1,1) standardized residuals. Compute 1-day ahead 99% VaR using the 1-day ahead DCC forecast of $\boldsymbol{\Sigma}_{T+1}$:

$$\widehat{VaR}^{DCC}_{T+1|T,.99} = \hat{\mu}^{DCC}_p + \hat{\sigma}^{DCC}_{p,T+1|T} \times q^Z_{.01},$$

where

$$\hat{\sigma}^{DCC}_{p,T+1|T} = \left(\mathbf{w}'\boldsymbol{\Sigma}^{DCC}_{T+1|T}\mathbf{w}\right)^{1/2}.$$

Is your VaR estimate the same as what you got for the previous question? Should it be?

3. Extra credit. You can also do multivariate historical simulation with the DCC model. How would you do that?

4 Compare Different Copulas

You will simulate from bivariate distributions defined by copulas in this exercise.

1. Use the functions **tCopula()** and **rCopula()** to simulate 3 sets of 2 uniform variates with a sample size of 500. Set df=3 and (correlation) param equal to 0.9,0,-0.9 respectively. Show the scatterplot of the simulated values and comment.

2. Plot the copula CDF, pdf and contours. What is the main difference between these 3 copulas?

3. Simulate another 3 set of copulas using **normalCopula()** with the same param values and show the scatterplots. Plot the copula CDF, pdf and contours. What is the main difference between t-copula and normal copula. Also simulate the independent copula and compare with t copula and normal copula.

4. Calculate Pearson's linear correlation, Kendall's tau and Spearman's rho in each case.

5. Simulate data from the Gumbel copula with $\delta = 1, 4, 10$ and create the scatterplots. Plot the copula CDF, pdf and contours, and point out the differences.

6. Do the same thing for the Clayton copula.

5 Fit and Simulate Copulas

1. You will create and simulate from custom bivariate distributions defined by margins and copulas in this exercise.

2. Using the function **mvdc**(), create a custom bivariate distribution with marginal distributions N(4,4) and t with df=3, and a Gumbel copula with $\delta = 5$.

3. Plot the copula CDF, pdf and contours. Compare with the Gumbel copula you simulated in the previous question.

4. Simulate 500 samples from the custom distribution and create qq-plots for the margins and a scatterplot for the bivariate distribution. Briefly describe your result.

5. Using the simulated data, estimate the bivariate distribution with full MLE. Using the fitted distribution, simulate 500 samples and create qq-plots for the margins and a scatterplot for the bivariate distribution.

6. Using the simulated data,estimate the bivariate distribution by the 2-step IFM procedure. First estimate the marginal distributions and use

the corresponding CDF to create uniform variates. Then estimate the copula.

7. Finally, use the 2-step IFM procedure to fit a custom distribution for Microsoft and SP500 returns. First, fit both series by univariate skew-t distributions. Transform the data to uniforms via the estimated CDF and then estimate a normal copula. Simulate 500 samples from fitted bivariate distribution create qq-plots for the margins and a scatterplot for the bivariate distribution.

8. Use t copula and redo the exercise above. What is the main difference by using the normal and t copula?