



Econ 584

Time Series Econometrics

Cointegration

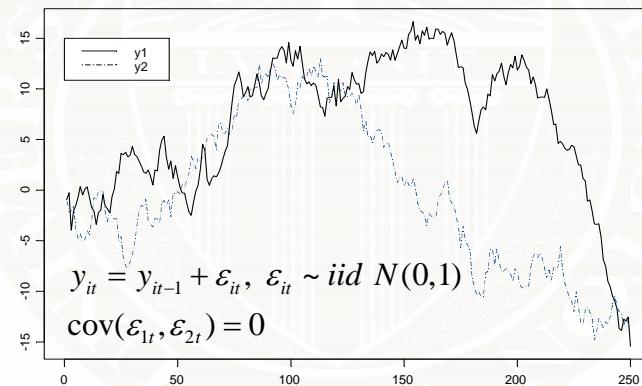
Eric Zivot

May 22, 2006

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Spurious Regression

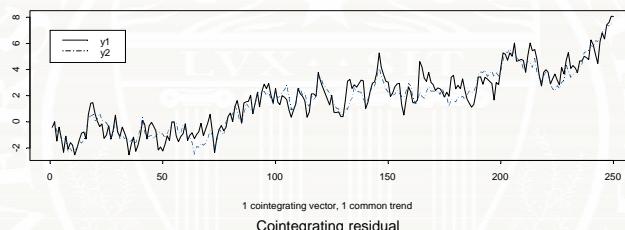


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1 cointegrating vector, 1 common trend

Simulated bivariate cointegrated system

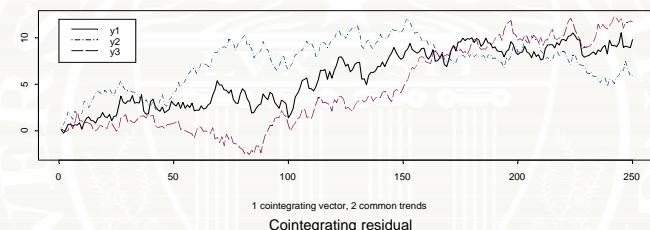


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1 cointegrating vector, 2 common trends

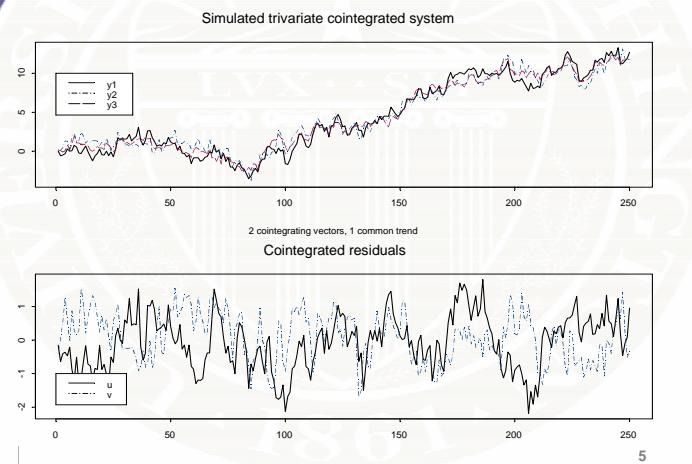
Simulated trivariate cointegrated system



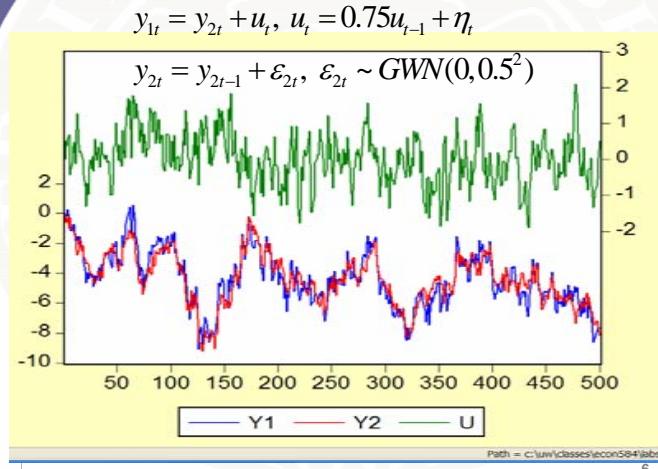
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2 cointegrating vectors, 1 common trend



Cointegration test: known β



ADF test on u_t

Null Hypothesis: U has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=5)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.097037	0.0000
Test critical values:		
1% level	-3.443228	
5% level	-2.867112	
10% level	-2.569800	

*MacKinnon (1996) one-sided p-values.

Reject H_0 : y_1 and y_2 are not cointegrated at 1% level

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Cointegration Test: Estimate β

Step 1: estimate β by OLS

Dependent Variable: Y1
Method: Least Squares
Date: 05/24/06 Time: 13:30
Sample: 1 500
Included observations: 500

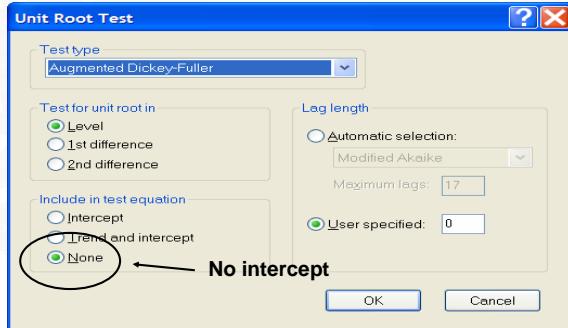
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.015124	0.084955	-0.178019	0.8588
Y2	0.993235	0.017938	55.36959	0.0000
R-squared	0.860261	Mean dependent var	-4.382984	
Adjusted R-squared	0.859981	S. D. dependent var	1.884345	
S.E. of regression	0.705106	Akaike info criterion	2.143056	
Sum squared resid	247.5931	Schwarz criterion	2.159914	
Log likelihood	-533.7639	F-statistic	3065.792	
Durbin-Watson stat	0.572230	Prob(F-statistic)	0.000000	

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Step 2: ADF test on OLS residual

$$\Delta \hat{u}_t = \pi \hat{u}_{t-1} + \eta_t$$



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Use PO Critical Values

Null Hypothesis: UHAT has a unit root
Exogenous: None
Lag Length: 0 (Fixed)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.105974	0.0000
Test critical values:		
1% level	-2.569604	
5% level	-1.941459	← Do not use!
10% level	-1.616273	

*MacKinnon (1996) one-sided p-values.

N-1 = 1, constant in 1st step regression

PO 5% critical value is -3.36

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Stock-Watson DOLS

$$y_{t1} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + \varepsilon_t$$

Dependent Variable: Y1

Method: Least Squares

Date: 05/24/06 Time: 14:05

Sample (adjusted): 3 499

Included observations: 497 after adjustments

Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.019383	0.167521	-0.115705	0.9079
Y2	0.991895	0.035268	28.12417	0.0000
D(Y2(1))	0.103777	0.067753	1.531687	0.1262
D(Y2)	0.064669	0.064762	0.998574	0.3185
D(Y2(-1))	0.080376	0.069807	1.151412	0.2501
R-squared	0.858116	Mean dependent var	-4.393828	
Adjusted R-squared	0.856963	S.D. dependent var	1.864151	
S.E. of regression	0.705027	Akaike info criterion	2.148847	
Sum squared resid	244.5548	Schwarz criterion	2.191187	
Log likelihood	-528.9885	F-statistic	743.9081	
Durbin-Watson stat	0.583469	Prob(F-statistic)	0.000000	

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Stock-Watson DGLS

$$y_{t1} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Dependent Variable: Y1

Method: Least Squares

Date: 05/24/06 Time: 14:09

Sample (adjusted): 4 499

Included observations: 496 after adjustments

Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.121389	0.206434	-0.588028	0.5568
Y2	0.970262	0.042988	22.57072	0.0000
D(Y2(1))	0.002622	0.045515	0.057618	0.9541
D(Y2)	0.007644	0.054670	0.139816	0.8889
D(Y2(-1))	0.044936	0.045376	0.990307	0.3225
AR(1)	0.714675	0.031811	22.46608	0.0000
R-squared	0.929317	Mean dependent var	-4.402307	
Adjusted R-squared	0.928595	S.D. dependent var	1.856414	
S.E. of regression	0.496064	Akaike info criterion	1.447801	
Sum squared resid	120.5792	Schwarz criterion	1.498687	
Log likelihood	-353.0547	F-statistic	1288.465	
Durbin-Watson stat	2.034233	Prob(F-statistic)	0.000000	
Inverted AR Roots				
.71				

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Error Correction Model

$$\Delta y_{1t} = c_1 + \alpha_1 u_{t-1} + \psi_1 \Delta y_{1t-1} + \psi_2 \Delta y_{2t-1} + \varepsilon_{1t}$$

$$u_t = y_{1t} - y_{2t}$$

Dependent Variable: D(Y1)

Method: Least Squares

Date: 05/24/06 Time: 14:22

Sample (adjusted): 3 500

Included observations: 498 after adjustments

Significant error correction coef

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.011964	0.030793	-0.388540	0.6978
U(-1)	0.211256	0.047246	-4.471368	0.0000
D(Y1(-1))	0.060483	0.062492	-1.287900	0.1984
D(Y2(-1))	0.098793	0.085872	1.150462	0.2505

R-squared	0.058119	Mean dependent var	-0.015140
Adjusted R-squared	0.052400	S.D. dependent var	0.705377
S.E. of regression	0.686648	Akaike info criterion	2.094009
Sum squared resid	232.9137	Schwarz criterion	2.127829
Log likelihood	-517.4083	F-statistic	10.16088
Durbin-Watson stat	1.999842	Prob(F-statistic)	0.000002

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Error Correction Model

$$\Delta y_{2t} = c_2 + \alpha_2 u_{t-1} + \psi_1 \Delta y_{1t-1} + \psi_2 \Delta y_{2t-1} + \varepsilon_{2t}$$

$$u_t = y_{1t} - y_{2t}$$

Dependent Variable: D(Y2)

Method: Least Squares

Date: 05/24/06 Time: 14:26

Sample (adjusted): 3 500

Included observations: 498 after adjustments

Error correction coef close to zero

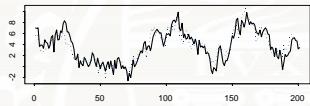
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.016337	0.021825	-0.748542	0.4545
U(-1)	0.069809	0.033486	2.084698	0.0376
D(Y1(-1))	-0.060433	0.044292	-1.357644	0.1752
D(Y2(-1))	0.043862	0.060863	0.720673	0.4715

R-squared	0.009543	Mean dependent var	-0.015137
Adjusted R-squared	0.003528	S.D. dependent var	0.487531
S.E. of regression	0.486670	Akaike info criterion	1.405540
Sum squared resid	117.0028	Schwarz criterion	1.439360
Log likelihood	-345.9793	F-statistic	1.586532
Durbin-Watson stat	2.001857	Prob(F-statistic)	0.191726

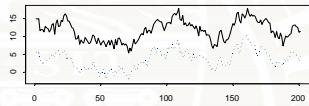
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Trend Cases: Data

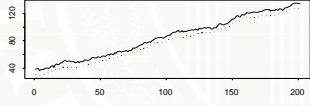
Case 1: No constant



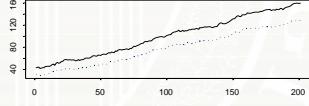
Case 2: Restricted constant



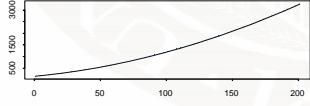
Case 3: Unrestricted constant



Case 4: Restricted trend



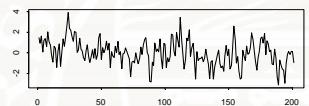
Case 5: Unrestricted trend



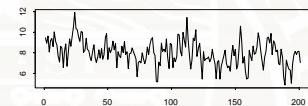
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Trend Cases: Residuals

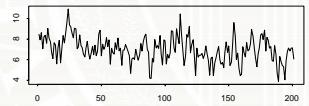
Case 1: No constant



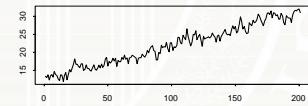
Case 2: Restricted constant



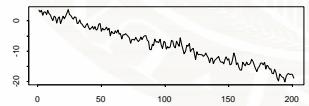
Case 3: Unrestricted constant



Case 4: Restricted trend



Case 5: Unrestricted trend



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Test for Cointegration

$$y_{1t} = y_{2t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}$$

$$\varepsilon_{1t} \sim iid N(0, 0.5^2)$$

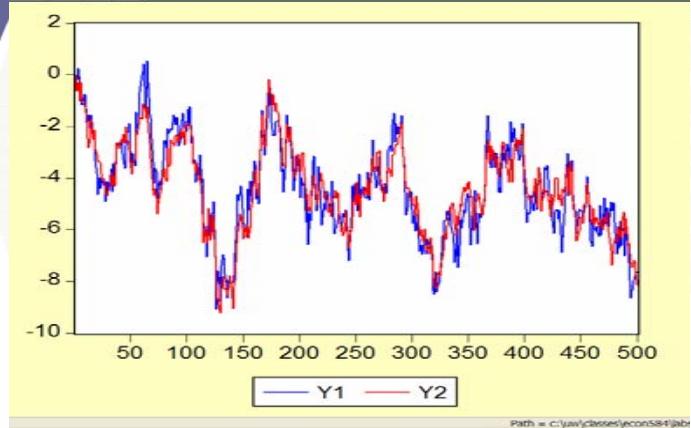
$$\varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$\beta = (1, -1)'$$

Note: No drift and residuals have mean zero

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UW Bivariate VAR: 1 cointegrating vector



Path = C:\javi\classes\ycon584\lab1

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Johansen LR Tests based on Levels VAR(1) and No Deterministic Terms

Johansen Cointegration Test

Cointegration Test Specification

Deterministic trend assumption of test:

Assume no deterministic trend in data:
 ① No intercept or trend in CE or test VAR
 ② Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:
 ③ Intercept (no trend) in CE and test VAR
 ④ Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:
 ⑤ Intercept and trend in CE - linear trend in VAR

Summary:
 ⑥ Summarize all 5 sets of assumptions

* Critical values may not be valid with exogenous variables; do not include C or Trend.

Exog variables*:

Lag intervals: 0 0

Lag spec for differenced endogenous

Critical Values:
 MHM Size 0.05
 Osterwald-Lenum

OK Cancel

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Johansen LR Tests

Date: 05/31/06 Time: 13:36

Sample (adjusted): 2 500

Included observations: 499 after adjustments

Trend assumption: No deterministic trend

Series: Y1 Y2

Lags interval (in first differences): No lags

Trace test finds 1 cointegrating vector

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob. **
None *	0.145974	79.02705	12.32090	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob. **
None *	0.145974	78.73880	11.22480	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

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Johansen Test with Misspecified Trend

Johansen Cointegration Test

No lagged differences

Deterministic trend assumption of test:
 1) No intercept or trend in CE or test VAR
 2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:
 3) Intercept (no trend) in CE and test VAR
 4) Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:
 5) Intercept and trend in CE - linear trend in VAR

Summary:
 6) Summarize all 5 sets of assumptions

* Critical values may not be valid with exogenous variables: do not include C or Trend.

Exog variables*:
Lag intervals: 0 0
Lag spec for differenced endogenous
Critical Values: M-HM
Size: 0.05
 Qsternwald-Lenmen

OK Cancel

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Misspecifying trend leads to wrong conclusion! Test finds 2 cointegrating vectors!

Data: 05/31/06 Time: 13:33
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: Y1 Y2
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.145989	90.23008	20.26184	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.145989	78.74771	15.89210	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

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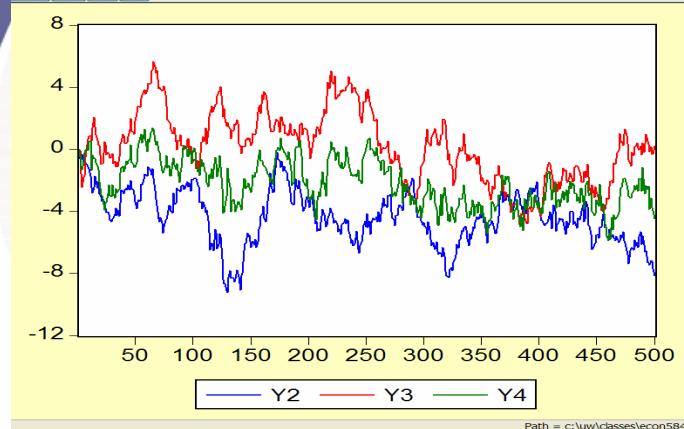
Test for Cointegration

$$\begin{aligned}
y_{4t} &= 0.5y_{2t} + 0.5y_{3t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t} \\
\varepsilon_{1t} &\sim iid N(0, 0.5^2) \\
y_{2t} &= y_{2t-1} + \varepsilon_{2t}, \quad \varepsilon_{2t} \sim iid N(0, 0.5^2) \\
y_{3t} &= y_{3t-1} + \varepsilon_{3t}, \quad \varepsilon_{3t} \sim iid N(0, 0.5^2) \\
Y_t &= (y_{4t}, y_{2t}, y_{3t})' \\
\beta &= (1, -0.5, -0.5)'
\end{aligned}$$

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Trivariate VAR: 1 cointegrating vector



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Johansen LR Tests

Date: 05/31/06 Time: 14:17

Sample (adjusted): 2 500

Included observations: 499 after adjustments

Trend assumption: No deterministic trend

Series: Y2 Y3 Y4

Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.111588	65.77018	24.27596	0.0000
At most 1	0.012827	6.728387	12.32090	0.3530
At most 2	0.000574	0.286414	4.129906	0.6535

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.111588	59.04179	17.79730	0.0000
At most 1	0.012827	6.441973	11.22480	0.3022
At most 2	0.000574	0.286414	4.129906	0.6535

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Trace test finds 1
cointegrating vector



Test for Cointegration

$$y_{1t} = y_{2t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$y_{5t} = y_{2t} + v_t, \quad v_t = 0.75v_{t-1} + \varepsilon_{5t}$$

$$\varepsilon_{1t} \sim iid N(0, 0.5^2), \quad \varepsilon_{5t} \sim iid N(0, 0.5^2)$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}, \quad \varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$Y_t = (y_{1t}, y_{5t}, y_{2t})'$$

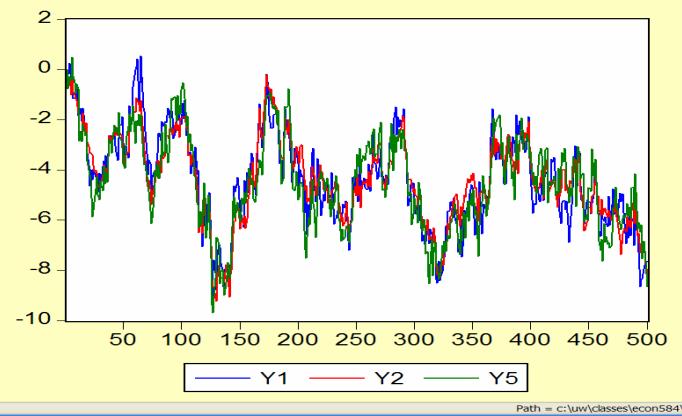
$$\beta_1 = (1, 0, -1)'$$

$$\beta_2 = (0, 1, -1)'$$

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Trivariate VAR: 2 cointegrating vectors



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Johansen LR Tests

Date: 05/31/06 Time: 14:30

Sample (adjusted): 2 500

Included observations: 499 after adjustments

Trend assumption: No deterministic trend

Series: Y1 Y5 Y2

Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.159565	140.6944	24.27596	0.0001
At most 1 *	0.101909	53.95064	12.32090	0.0000
At most 2	0.000634	0.316276	4.129906	0.6358

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.159565	86.74372	17.79730	0.0000
At most 1 *	0.101909	53.63437	11.22480	0.0000
At most 2	0.000634	0.316276	4.129906	0.6358

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