Econ 584 Lab 3
Spring 2006

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Due: Wednesday, May 17

1 Reading

1. Hamilton, J. (1993), *Time Series Analysis*, chapters 15 and 17
2. Hayashi, F. (2000), *Econometrics*, chapter 9.
3. Zivot, E. and J. Wang (2002), chapter 4 in *Modeling Financial Time Series with S-PLUS*. Springer-Verlag.
4. Zivot, E. (2005). Lecture notes: "Asymptotics for nonstationary data", and "Unit root tests".
5. Lothian, J. and M. Taylor (1996), "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries," *Journal of Political Economy*, 104, 488-509.
6. Elliot, G., T. Rothenberg, and J. Stock (1996), "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, 64, 813-836.
7. Eviews help topics: Unit root tests.

2 Analytic Questions

1. Consider testing the hypotheses

$$\begin{aligned} H_0 &: y_t \sim I(1) \text{ without drift} \\ H_1 &: y_t \sim I(0) \text{ with non-zero mean} \end{aligned}$$

using statistics computed from the test regression

$$\begin{aligned} y_t &= c + \phi y_{t-1} + \varepsilon_t,\ t = 0, 1 \ldots, T \\ &= \mathbf{x}_t' \boldsymbol{\delta} + \varepsilon_t \\ & \quad \varepsilon_t \sim WN(0, \sigma^2) \end{aligned}$$

where $\mathbf{x}_t = (1, y_{t-1})'$ and $\delta = (c, \phi)'$. In particular, consider the t-statistic and normalized bias:

$$t_{\phi=1} = \frac{\hat{\phi} - 1}{SE(\hat{\phi})}$$
$$T(\hat{\phi} - 1)$$

where

$$\begin{aligned}
\hat{\boldsymbol{\delta}} &= \begin{pmatrix} \widehat{c} \\ \widehat{\phi} \end{pmatrix} = \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t' \right)^{-1} \sum_{t=1}^{T} \mathbf{x}_{t-1} y_t \\
SE(\hat{\phi}) &= \left[\hat{\sigma}^2 \left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t' \right)^{-1}_{(2,2)} \right]^{1/2} \\
\hat{\sigma}^2 &= T^{-1} \sum_{t=1}^{T} (y_t - \mathbf{x}_t' \hat{\boldsymbol{\delta}})^2
\end{aligned}$$

Using the following results

$$\begin{aligned}
T^{-3/2} \sum_{t=1}^{T} y_{t-1} &\Rightarrow \sigma \int_0^1 W(r) dr \\
T^{-2} \sum_{t=1}^{T} y_{t-1}^2 &\Rightarrow \sigma^2 \int_0^1 W(r)^2 dr \\
T^{-1} \sum_{t=1}^{T} y_{t-1} \varepsilon_t &\Rightarrow \sigma^2 \int_0^1 W(r) dW(r)
\end{aligned}$$

and the CMT answer the following questions (Hint: see Hamilton pages 490-494 and Hayashi section 9.2) :

(a) Using the scaling matrix

$$\mathbf{D}_T = \begin{pmatrix} T^{1/2} & 0 \\ 0 & T \end{pmatrix}$$

show that

$$\mathbf{D}_T(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}) \Rightarrow \begin{pmatrix} 1 & \sigma \int_0^1 W(r) dr \\ \sigma \int_0^1 W(r) dr & \sigma^2 \int_0^1 W(r)^2 dr \end{pmatrix}^{-1} \begin{pmatrix} N(0, \sigma^2) \\ \sigma^2 \int_0^1 W(r) dW(r) \end{pmatrix}$$

(b) The second row of $\mathbf{D}_T(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta})$ is $T(\hat{\phi} - 1)$. Using simple matrix inversion, show that

$$T(\hat{\phi} - 1) \Rightarrow \frac{\frac{1}{2}\left[W(1)^2 - 1\right] - W(1)\int_0^1 W(r)dr}{\int_0^1 W(r)^2 dr - \left[\int_0^1 W(r)dr\right]^2}$$

(c) Define $W^\mu(r) = W(r) - \int_0^1 W(r)dr$ to be a de-meaned Wiener process. Show that

$$\begin{aligned} \int_0^1 W^\mu(r)dW(r) &= \frac{1}{2}\left[W(1)^2 - 1\right] - W(1)\int_0^1 W(r)dr \\ \int_0^1 W^\mu(r)^2 dr &= \int_0^1 W(r)^2 dr - \left[\int_0^1 W(r)dr\right]^2 \end{aligned}$$

(d) Using the result in part (c) above show that

$$\begin{aligned} T(\hat{\phi} - 1) &\Rightarrow \frac{\int_0^1 W^\mu(r)dW(r)}{\int_0^1 W^\mu(r)^2 dr} \\ t_{\phi=1} &\Rightarrow \frac{\int_0^1 W^\mu(r)dW(r)}{\left(\int_0^1 W^\mu(r)^2 dr\right)^{1/2}} \end{aligned}$$

2. Hamilton, chapter 17, exercise 17.2. (Hint: see the back of the book)

3 Computer Exercises

For this part of the lab you will do some of the empirical exercises from Hayashi, chapter 9 (pages 613 – 618)

1. Empirical exercise (a) page 614.
2. Empirical exercise (b) page 615.
3. Empirical exercise (c) page 615.
4. Empirical exercise (e) page 616. Eviews implements the DF-GLS test so you do not have to compute this as described in Hayashi. See the online help for unit root tests in Eviews for details. See also the discussion of the efficient unit root tests in chapter 4 of MFTS. For the lag length, use the automatic Ng-Perron modified AIC procedure. Also, compute the ERS point optimal unit root tests as well and use the modified AIC procedure to choose the lag length.