Econ 584 Final Exam
Spring 2006

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Exam is due Friday June 9 at 9:00 am in my office or in my mailbox.

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Question 1

1. Give state space representations of the form

$$\begin{gathered}
\mathbf{y}_t = \mathbf{Z}_t \boldsymbol{\alpha}_t + \mathbf{d}_t + \boldsymbol{\varepsilon}_t, \ \boldsymbol{\varepsilon}_t \sim \text{iid } N(\mathbf{0}, \mathbf{H}_t) \\
\boldsymbol{\alpha}_t = \mathbf{T}_t \boldsymbol{\alpha}_{t-1} + \mathbf{c}_t + \mathbf{R}_t \boldsymbol{\eta}_t, \ \boldsymbol{\eta}_t \sim \text{iid } N(\mathbf{0}, \mathbf{Q}_t) \\
E[\boldsymbol{\varepsilon}_t \boldsymbol{\eta}_t'] = \mathbf{0} \\
\boldsymbol{\alpha}_0 \sim N(\mathbf{a}_0, \mathbf{P}_0)
\end{gathered}$$

for the following models. Make sure to describe the distribution of the initial state vector $\boldsymbol{\alpha}_0$.

a. ARMA(1,1) model

$$\begin{aligned}
y_t &= \phi y_{t-1} + u_t + \theta u_{t-1} \\
& \quad u_t \sim \text{iid } N(0, \sigma^2)
\end{aligned}$$

where $|\phi| < 1$ and $|\theta| < 1$.

b. Time varying parameter model

$$\begin{aligned}
y_t &= x_{1t}\beta_{1t} + x_{2t}\beta_{2t} + u_t, \ u_t \sim \text{iid } N(0, \sigma^2) \\
\beta_{1t} &= \beta_{1t-1} + v_{1t}, \ v_{1t} \sim \text{iid } N(0, \sigma_1^2) \\
\beta_{2t} - \bar{\beta}_2 &= \phi(\beta_{2t-1} - \bar{\beta}_2) + v_{2t}, \ v_{2t} \sim \text{iid } N(0, \sigma_2^2) \\
cov(v_{1t}, v_{2t}) &= 0
\end{aligned}$$

where $|\phi| < 1$ and $\bar{\beta}_2 = E[\beta_{2t}]$.

c. Dynamic factor coincident indicator model

$$\begin{aligned}
\Delta y_{it} &= \gamma_i \Delta c_t + e_{it},\ i = 1, 2, 3, 4, \\
\Delta c_t &= \phi_1 \Delta c_{t-1} + \phi_2 \Delta c_{t-2} + w_t,\ w_t \sim \text{iid } N(0, 1) \\
e_{it} &= \psi_{i1} e_{it-1} + \psi_{i2} e_{it-2} + v_t,\ w_t \sim \text{iid } N(0, \sigma_i^2),\ 1, 2, 3, 4,
\end{aligned}$$

where Δy_{it} denotes the de-meaned growth rate of the observable i^{th} coincident macro variable (e.g., industrial production, personal income, manufacturing sales, etc.), and Δc_t denotes the de-meaned growth rate of the unobserved common component, and e_{it} denotes the unobserved idiosyncratic component of Δy_{it}.

2. Briefly discuss how you would estimate a model put in state space form using the method of maximum likelihood. That is, describe how you would compute the log-likelihood function and how you would maximize the log-likelihood.

Question 2. This question is based on the second half of Stock and Watson's 1988 JEP paper "Variable Trends in Economic Time Series", which is available on the class syllabus page. You will find it most helpful to read the second half of paper before answering this question.

Consider the following simple model for aggregate consumption

$$\begin{aligned}
Y_t &= Y_t^P + Y_t^S \\
Y_t^P &= Y_{t-1}^P + u_t, u_t \sim \text{iid } N(0, 1) \\
& \quad Y_t^S \sim \text{iid } N(0, 1) \\
C_t &= Y_t^P \\
P_t &= P_{t-1} + v_t, v_t \sim \text{iid } N(0, 1)
\end{aligned}$$

where Y_t is the log of disposable income, $Y_t^P$is the log of permanent income, Y_t^S is the log of transitory income, C_t is the log of aggregate consumption, P_t is the log of the price level, and u_t, v_t, and Y_t^S are mutually independent. Suppose $T = 150$ observations are observed on this simple economy and the following regression results are reported by econometrician A:

Estimated Equation	R^2	Durbin-Watson
$C_t = 9.16 + 0.40 P_t$	0.15	0.08
(28.7) (5.12)		
$C_t = 2.48 + 0.069 \cdot t$	0.66	0.16
(6.35) (16.9)		
$\Delta C_t = 0.048 + \Delta 0.28 \cdot Y_t$	0.31	2.27
(0.81) (8.06)		
$\Delta C_t = 0.41 - 0.041 \cdot C_{t-1}$	0.03	1.98
(2.33) (-2.15)		

where t-statistics are in parentheses. Econometrician A's naive interpretation of the results are:

- consumers have money illusion
- consumption contains a linear trend
- the marginal propensity to consume is 0.28
- past consumption is useful for predicting future consumption

(a) In light of the results on regressions with integrated variables, critically evaluate the results given by econometrician A.

Now suppose econometrician B, who is more familiar with modern time series methods than econometrican A, runs the following regressions on the same data

Estimated Equation	R^2	Durbin-Watson
$C_t = 0.51 + 0.94 \cdot Y_t$	0.93	1.88
(2.60) (-2.74)		
$C_t = 0.45 + 0.97 \cdot C_{t-1} - 0.01 \cdot C_{t-2}$	0.94	2.01
(2.52) (11.7) (-0.13)		
$C_t = 0.41 + 1.03 \cdot C_{t-1} - 0.07 \cdot Y_{t-1}$	0.94	1.97
(2.36) (14.3) (-0.97)		
$C_t = 0.47 + 0.95 \cdot C_{t-1} + 0.004 \cdot P_{t-1} + 0.06 \cdot \Delta P_{t-1}$	0.94	2.00
(2.24) (45.0) (0.17) (0.87)		

where t-statistics are in parentheses (note: for the first regression the t-statistic is for testing the hypothesis that the true coefficient on Y_t is equal to one and not zero). Based on the above results, econometrician B concludes

- Hall's interpretation of the permanent income model is largely correct
- the marginal propensity to consume is less than 1

(b) In light of the results on regressions with integrated and cointegrated variables, critically evaluate econometrician B's interpretation of the above results.

Question 3. This question is based on King and Watson's 1996 Fed Review paper "Testing Long-Run Neutrality", which is available on the class syllabus page. You will find it particularly useful to look at the Appendix to this paper before answering the following questions.

King and Watson (KW) show that testing long-run neutrality within a SVAR framework requires the data to be $I(1)$. They characterize long-run neutrality of money using the SMA representation for $\Delta\mathbf{y}_t = (\Delta y_t, \Delta m_t)'$ written as

$$\begin{aligned}
\text{output:} \quad & \Delta y_t = \mu_y + \theta_{yy}(L)\varepsilon_{yt} + \theta_{ym}(L)\varepsilon_{mt} \\
\text{money} \;: \quad & \Delta m_t = \mu_m + \theta_{my}(L)\varepsilon_{yt} + \theta_{mm}(L)\varepsilon_{mt}
\end{aligned}$$

where ε_{yt} represents exogenous shocks to output that are uncorrelated with exogenous shocks to nominal money ε_{mt}. If an unexpected and exogenous permanent change in the level of money leads to a permanent change in the level of output then money is not long-run neutral towards output.

With the data in logs, the long-run elasticity of output with respect to permanent changes in money is

$$\gamma_{ym} = \frac{\theta_{ym}(1)}{\theta_{mm}(1)}$$

Money is neutral in the long-run when

$$\theta_{ym}(1) = 0 \text{ or } \gamma_{ym} = 0$$

Assume that the SMA representation is derived from the simple SVAR(1) model

$$\begin{aligned}
\Delta y_t &= c_y + \lambda_{ym}\Delta m_t + \alpha_{1,yy}\Delta y_{t-1} + \alpha_{1,ym}\Delta m_{t-1} + \varepsilon_{yt} \\
\Delta m_t &= c_m + \lambda_{my}\Delta y_t + \alpha_{1,my}\Delta y_{t-1} + \alpha_{1,mm}\Delta m_{t-1} + \varepsilon_{mt}
\end{aligned}$$

which has the form

$$\mathbf{B}\Delta\mathbf{y}_t = \mathbf{c} + \mathbf{\Gamma}_1\Delta\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$$

$$\mathbf{B} = \begin{pmatrix} 1 & -\lambda_{ym} \\ -\lambda_{my} & 1 \end{pmatrix}, \; \mathbf{\Gamma}_1 = \begin{pmatrix} \alpha_{1,yy} & \alpha_{1,ym} \\ \alpha_{1,my} & \alpha_{1,mm} \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{mt} \end{pmatrix} \sim \text{iid } N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_m^2 \end{pmatrix}\right)$$

Assume that the long-run elasticity of money wrt output

$$\gamma_{my} = \frac{\theta_{my}(1)}{\theta_{yy}(1)}$$

is known. For example, one might assume $\gamma_{my} = 1$ which is consistent with long-run price stability under the assumption that velocity is stable.

(a) Show how the SVAR model parameters may be consistently estimated when γ_{my} is known.

(b) Given the estimates of the SVAR model parameters when γ_{my} is known, show how γ_{ym} may be estimated.

(c) Briefly discuss how you would compute a standard error for the estimate of γ_{ym}? You do not have to give explicit details of this calculation.