Department of Economics
University of Washington

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Economics 584
Computer Lab #2
Suggested Solutions

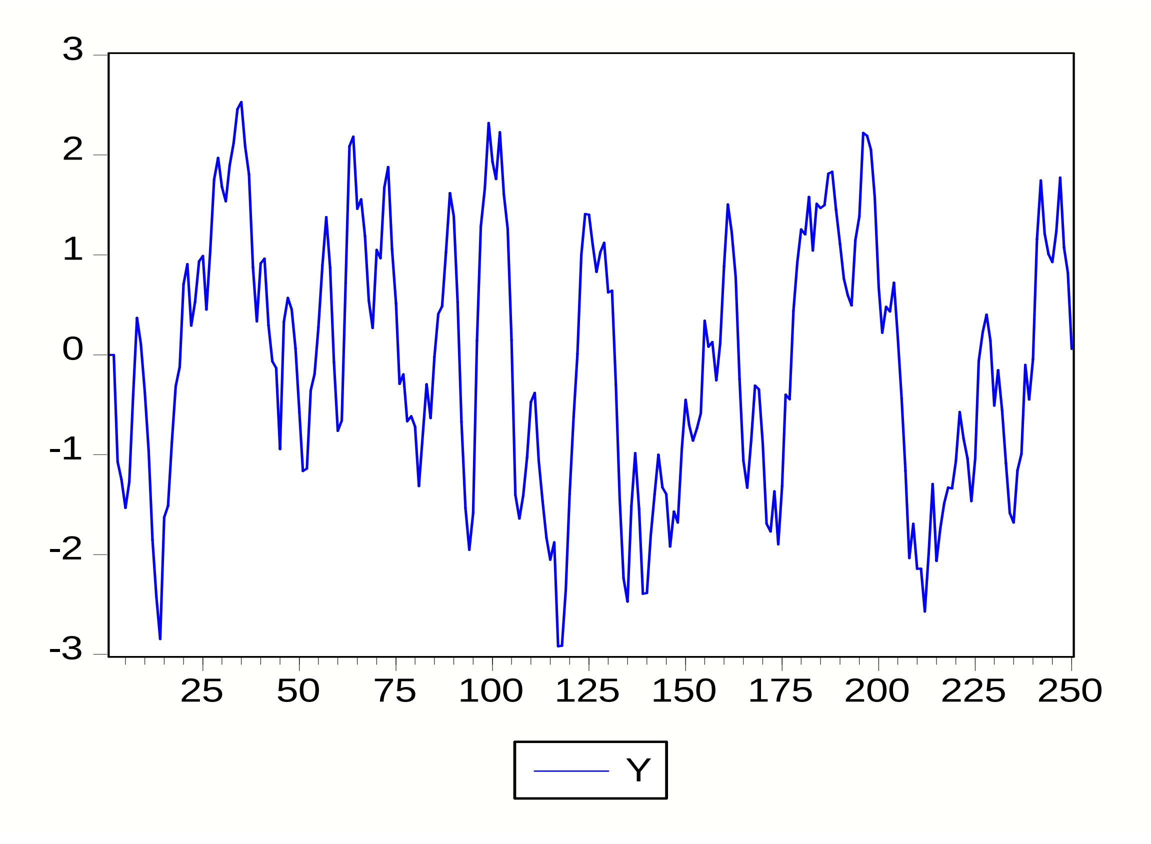
Empirical Exercises

Comparing forecasting models

Simulated values from the model

$$y_t = 1.2 y_{t-1} - 0.4 y_{t-2} + \varepsilon_t,\ \varepsilon_t \sim iid\ N(0,(0.5)^2)$$
$$y_1 = y_2 = 0$$

are illustrated below.

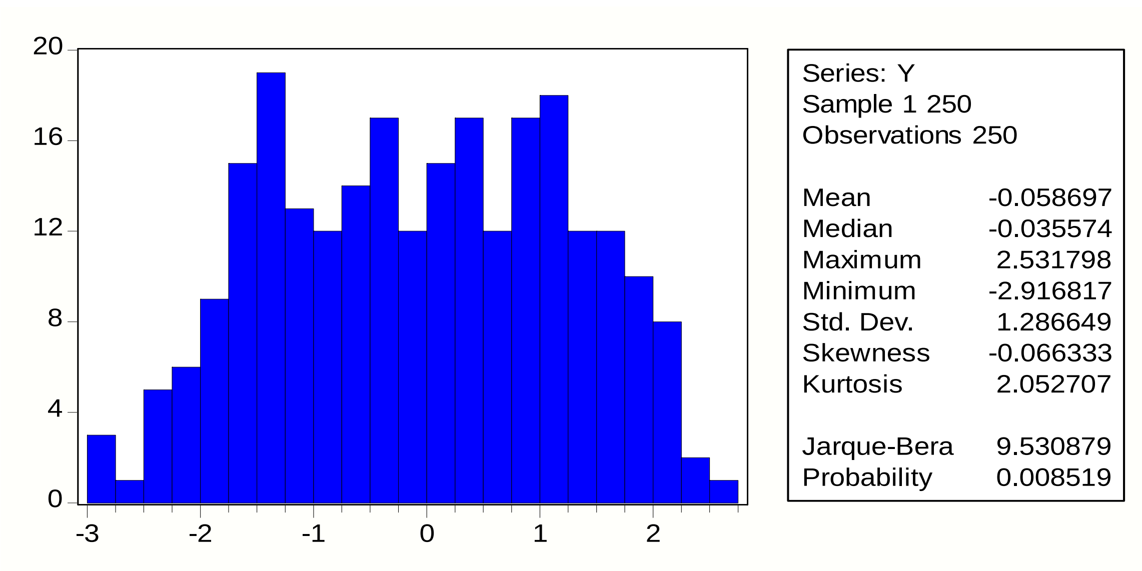


The series looks stationary with a high degree of persistence (note: the sum of the AR coefficients is 0.8). The SACF and PACF are illustrated below.

```
Date: 05/24/05   Time: 09:06
Sample: 1 250
Included observations: 250
===========================================================================
  Autocorrelation    Partial Correlation       AC      PAC    Q-Stat   Prob
===========================================================================
      .|*******|          .|*******|      1   0.904   0.904   206.59   0.000
      .|******  |       ***|.      |      2   0.738  -0.431   344.76   0.000
      .|****    |         .|.      |      3   0.565   0.006   426.22   0.000
      .|***     |         .|.      |      4   0.424   0.065   472.20   0.000
      .|***     |         .|*      |      5   0.334   0.110   500.96   0.000
      .|**      |         *|.      |      6   0.270  -0.093   519.84   0.000
      .|**      |         *|.      |      7   0.210  -0.069   531.23   0.000
      .|*       |         .|.      |      8   0.145  -0.026   536.72   0.000
      .|*       |         .|.      |      9   0.084   0.015   538.57   0.000
      .|.       |         *|.      |     10   0.026  -0.064   538.75   0.000
      .|.       |         .|.      |     11  -0.022  -0.017   538.88   0.000
      .|.       |         .|.      |     12  -0.055   0.007   539.69   0.000
===========================================================================
```

The SACF decays geometrically to zero and the PACF cuts off at lag 2. This is consistent with an AR(2) model.

Descriptive statistics are given below



The mean is close to zero. Interestingly, the JB statistic rejects normality for the data. This could be due to the fact that the JB statistic was designed for iid data.

3. Using the first 200 observations to fit the AR(2) model gives

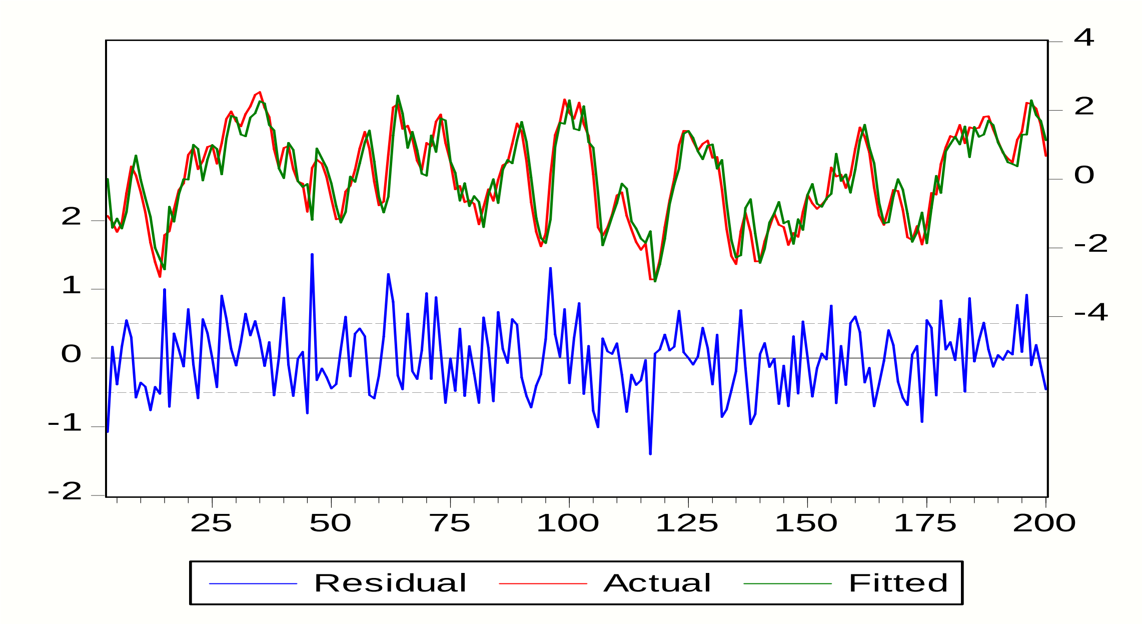
Dependent Variable: Y
Method: Least Squares
Date: 05/24/05 Time: 09:12
Sample (adjusted): 3 200
Included observations: 198 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.041071	0.244807	0.167769	0.8669
AR(1)	1.327540	0.063503	20.90501	0.0000
AR(2)	-0.473806	0.063727	-7.434910	0.0000

R-squared	0.853581	Mean dependent var	0.047120
Adjusted R-squared	0.852079	S.D. dependent var	1.309917
S.E. of regression	0.503800	Akaike info criterion	1.481761
Sum squared resid	49.49381	Schwarz criterion	1.531583
Log likelihood	-143.6943	F-statistic	568.3973
Durbin-Watson stat	1.958502	Prob(F-statistic)	0.000000

Inverted AR Roots	.66-.18i	.66+.18i

The estimated results are similar to the actual values. The inverted roots of the characteristic polynomial $\hat{\phi}(z) = 1 - 1.328z + 0.474z^2 = 0$ are complex and have modulus inside the complex unit circle so that the fitted model is stationary and ergodic. The plot of the actual, fitted and residuals indicate that the model tracks the simulated data well. The correlogram of the residuals (not shown) reveals no omitted serial correlation.



4. Using the first 200 observations to fit a mis-specified MA(1) gives

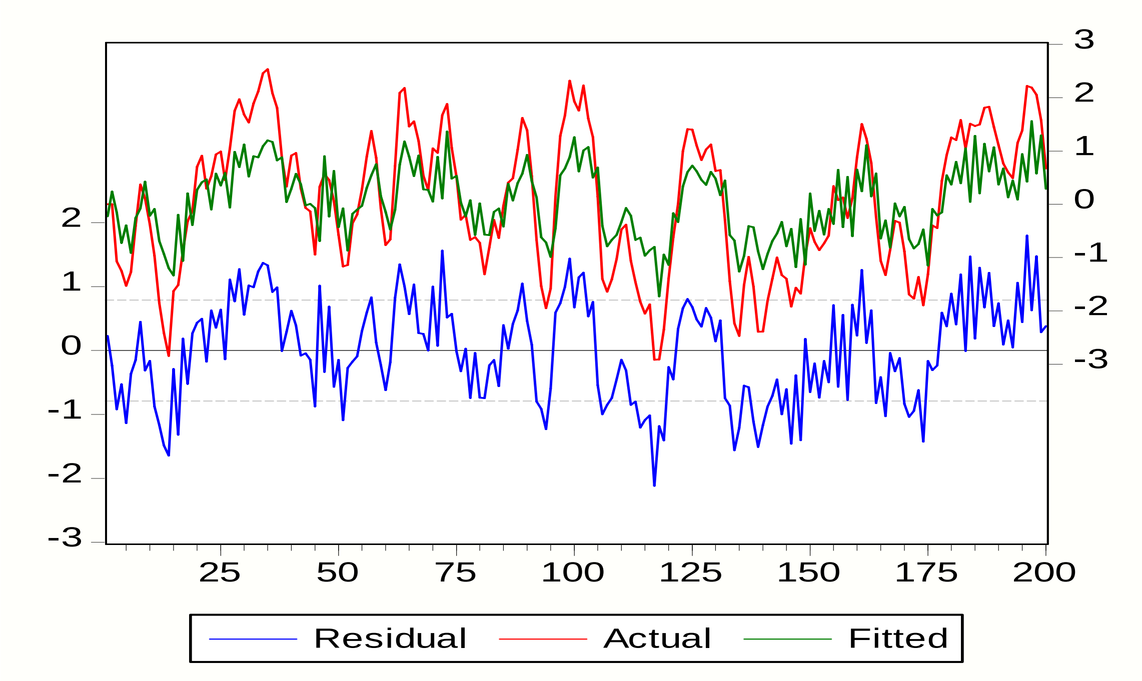
Dependent Variable: Y
Method: Least Squares
Date: 05/24/05 Time: 09:32
Sample: 1 200
Included observations: 200
Convergence achieved after 13 iterations
Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.049147	0.102946	0.477405	0.6336
MA(1)	0.841426	0.039271	21.42622	0.0000

R-squared	0.633518	Mean dependent var	0.046649
Adjusted R-squared	0.631667	S.D. dependent var	1.303326
S.E. of regression	0.790994	Akaike info criterion	2.378897
Sum squared resid	123.8829	Schwarz criterion	2.411880
Log likelihood	-235.8897	F-statistic	342.2726
Durbin-Watson stat	0.783957	Prob(F-statistic)	0.000000

Inverted MA Roots	-.84

The MA coefficient is close to one which is required to capture the large first order sample autocorrelation. The small DW statistic indicates omitted positive serial correlation in the residuals. The SACF and PACF of the residuals (not shown) indicates omitted serial correlation. The modified Q-statistics are large for all lags. The plot of the actual, fitted and residuals below indicates that the model does not track the simulated data as well as the AR(2) model.



5. Forecasts from the rolling 1-step ahead forecasts from the AR(2) and MA(1) are displayed in the tables below.

Forecast: YF – AR2

Actual: Y

Forecast sample: 201 250

Included observations: 50

Root Mean Squared Error	0.494905
Mean Absolute Error	0.422211
Mean Absolute Percentage Error	125.4196
Theil Inequality Coefficient	0.212080
Bias Proportion	0.025806
Variance Proportion	0.030056
Covariance Proportion	0.944139

Forecast: YF – MA1
Actual: Y
Forecast sample: 201 250
Included observations: 50

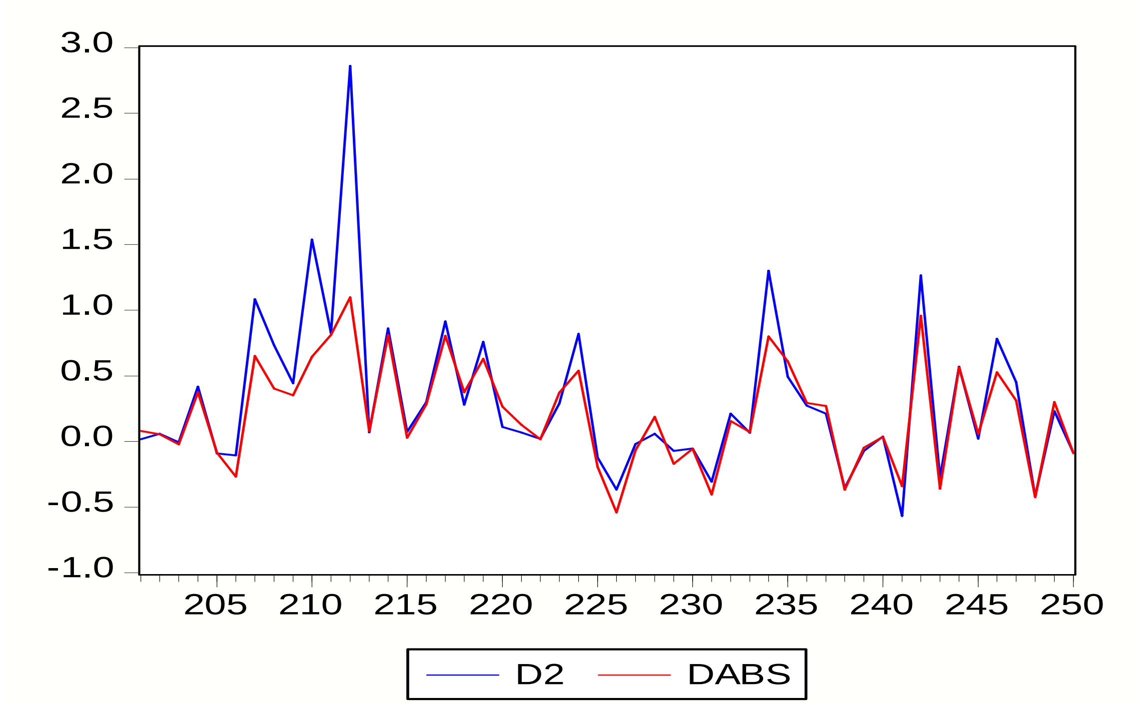
Root Mean Squared Error	0.746679
Mean Absolute Error	0.633243
Mean Absolute Percentage Error	107.9825
Theil Inequality Coefficient	0.407445
Bias Proportion	0.155979
Variance Proportion	0.524707
Covariance Proportion	0.319314

The RMSE and RAE are both smaller for the AR(2) model indicating a superior fit.

6. To statistically compare the forecasting accuracy of the AR(2) and MA(1) models, we may compute Diebold-Mariano (DM) statistics using the squared error and absolute error loss functions. The DM statistics are based on the following loss differentials

$$d_{sq,t} = \left(\hat{\varepsilon}_t^{MA1}\right)^2 - \left(\hat{\varepsilon}_t^{AR2}\right)^2$$
$$d_{abs,t} = \left|\hat{\varepsilon}_t^{MA1}\right| - \left|\hat{\varepsilon}_t^{AR2}\right|$$

computed using the rolling 1-step ahead forecast errors from the AR(2) and MA(1) models, respectively. A time plot of these loss differentials are shown below



In general both loss differentials are positive indicating that the MA(1) model produces a larger forecast error than the AR(2) model. The DM statistic

$$DM = \frac{\bar{d}}{SE(\bar{d})}$$

may be computed by regressing the loss differential on a constant and choosing the NW correction to the standard error.

Dependent Variable: D2
Method: Least Squares
Date: 05/24/05 Time: 09:53
Sample: 201 250
Included observations: 50
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.312600	0.103399	3.023237	0.0040

Dependent Variable: DABS
Method: Least Squares
Date: 05/24/05 Time: 10:10
Sample: 201 250
Included observations: 50
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.211032	0.066393	3.178508	0.0026

The DM statistic has an asymptotic standard normal distribution. Using both the squared and absolute value loss functions we reject the null hypothesis that the AR(2) and MA(1) models have equally forecasting accuracy. Since the t-statistics are positive we conclude that the AR(2) model is more accurate than the MA(1) model.

Working with State Space Models

In this exercise, a simple AR(2) model is estimated by conditional MLE and by exact MLE via state space methods. The AR(2) model has the form

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t,\ \varepsilon_t \sim iid\ N(0,\sigma^2)$$

The model is fit to detrended quarterly observations on log real GDP over the period 1947:1 through 1999:4, and then dynamic forecasts are produced over the period 2001:1 through 2003:4.

Q1 and Q2. The conditional MLEs for the AR(2) are produced using the following Eviews commands

LS dtlrgdp ar(1) ar(2)

and are given in the table below.

Dependent Variable: DTLRGDP
Method: Least Squares
Date: 05/24/05 Time: 10:40
Sample (adjusted): 1947Q3 1999Q4
Included observations: 210 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.311783	0.064607	20.30410	0.0000
AR(2)	-0.358569	0.064422	-5.565912	0.0000

R-squared	0.945366	Mean dependent var	0.000505
Adjusted R-squared	0.945103	S.D. dependent var	0.040677
S.E. of regression	0.009531	Akaike info criterion	-6.459112
Sum squared resid	0.018894	Schwarz criterion	-6.427235
Log likelihood	680.2068	Durbin-Watson stat	2.078946

Inverted AR Roots	.92	.39

3. The exact MLEs for the AR(2) are produced by first creating a state space form. The Kalman filter is used to create the prediction error decomposition of the log-likelihood, and this likelihood is maximized to give the MLEs. The state space set up allows for the marginal likelihood to be created for the first two initial values. In Eviews, the state space form for the AR(2) model (without a constant) is

```
@signal dtlrgdp = sv1

@state sv1 = c(2)*sv1(-1) + c(3)*sv2(-1) + [var = exp(c(1))]
@state sv2 = sv1(-1)
```

The coefficient c(1) denotes the variance of the error term, c(2) denotes the first AR term and c(3) denotes the second AR term. Notice that there is no constant in the specification because we are modeling the detrended data. The exact MLEs are

Sspace: SSAR2
Method: Maximum likelihood (Marquardt)
Date: 05/24/05 Time: 12:54
Sample: 1947Q1 1999Q4
Included observations: 212
Estimation settings: tol= 0.00010, derivs=accurate numeric
Initial Values: C(1)=0.00000, C(2)=1.31777, C(3)=-0.36195
Convergence achieved after 7 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-9.313497	0.073084	-127.4359	0.0000
C(2)	1.317628	0.055531	23.72799	0.0000
C(3)	-0.361841	0.057508	-6.292044	0.0000
	Final State	Root MSE	z-Statistic	Prob.
SV1	-0.005128	0.009497	-0.539963	0.5892
SV2	-0.008845	0.000000	NA	0.0000

Log likelihood	684.8372	Akaike info criterion	-6.432427
Parameters	3	Schwarz criterion	-6.384928
Diffuse priors	0	Hannan-Quinn criter.	-6.413229

The exact MLEs are close the conditional MLEs. The estimate of the standard deviation of the error term is sqrt(exp(-9.313497)) = 0.009497 which is close to the standard error of the regression reported in the conditional MLE output. The exact log-likelihood is slightly higher than the conditional log-likelihood.

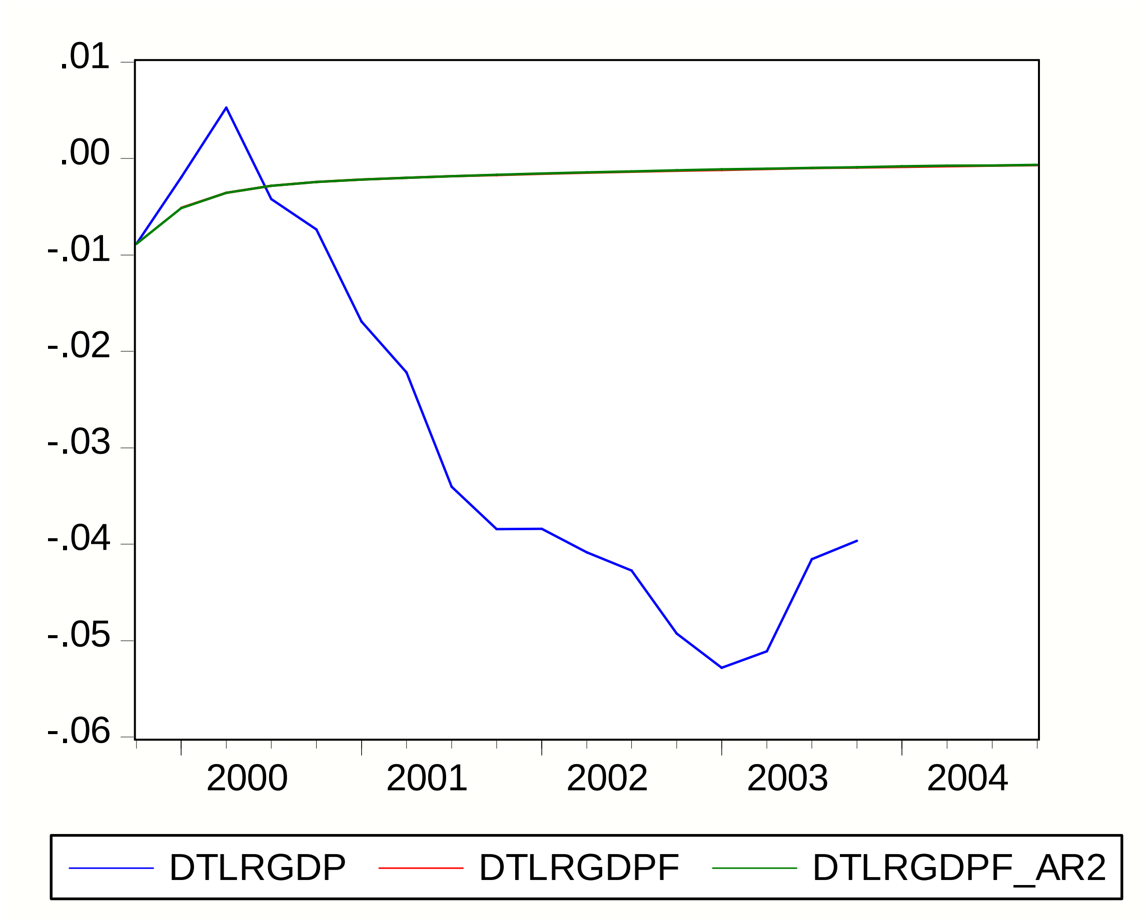
Remark: Good starting values are important for the estimation of state space models. By default, for nonlinear least squares type problems, EViews uses the values in the coefficient vector at the time you begin the estimation procedure as starting values. If you wish to change the starting values, first make certain that the spreadsheet view of the coefficient vector is in edit mode, then enter the coefficient values. When you are finished setting the initial values, close the coefficient vector window and estimate your model. You may also set starting coefficient values from the command window using the

PARAM command. Simply enter the PARAM keyword, followed by pairs of coefficients and their desired values:

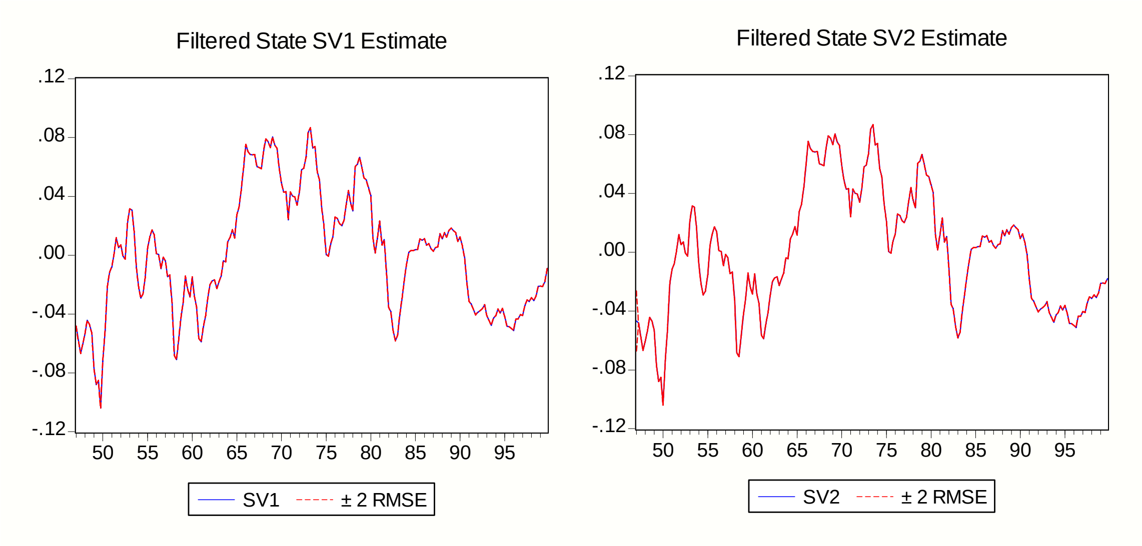
```
param c(1) 153 c(2) .68 c(3) .15
```

sets C(1)=153, C(2)=.68, and C(3)=.15. All of the other elements of the coefficient vector are left unchanged.

The forecasts from the state space model, the conditional AR(2) and the actual values are illustrated below. Notice that the forecasts from the state space model are essentially identical to those from the conditional AR(2) model.



4. The filtered estimates of the state vector from the state space model are illustrated below.



For the AR(2) model, the first state variable is y(t) and the second state variable is y(t-1).

Estimate Simple Unobserved Components Model

1. The state space representation for the Clark model is

```
@signal lrgdp*100 = sv1 + sv2

@state sv1 = c(1) + sv1(-1) + [var = exp(c(2))]
@state sv2 = c(3)*sv2(-1) + c(4)*sv3(-1) + [var  = exp(c(5))]
@state sv3 = sv2(-1)
```

To improve numerical stability, the log of real GDP is multiplied by 100. This is done so that the derivatives of the log-likelihood are more closely scaled.

The starting values for the estimation are set using

```
param c(1) 0 c(2) -1 c(3)  1.2 c(4)  -0.4 c(5) -1
```

The MLEs are given in the table below

Sspace: SSCLARK
Method: Maximum likelihood (Marquardt)
Date: 05/30/05   Time: 11:59
Sample: 1947Q1 2003Q4
Included observations: 228
Estimation settings: tol= 0.00010, derivs=accurate numeric

Initial Values: C(1)=0.00000, C(2)=-1.00000, C(3)=1.20000, C(4)=
-0.40000, C(5)=-1.00000
Convergence achieved after 20 iterations

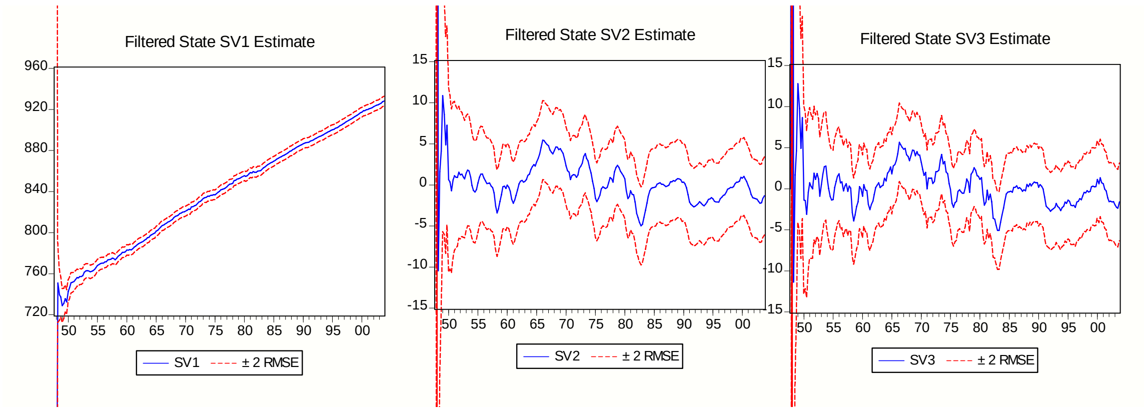
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.826180	0.046946	17.59837	0.0000
C(2)	-1.237703	0.662030	-1.869559	0.0615
C(3)	1.441194	0.135482	10.63750	0.0000
C(4)	-0.493771	0.137166	-3.599809	0.0003
C(5)	-0.644815	0.454801	-1.417796	0.1563
	Final State	Root MSE	z-Statistic	Prob.
SV1	928.7268	2.340589	396.7918	0.0000
SV2	-1.022323	2.311634	-0.442251	0.6583
SV3	-1.221890	2.277786	-0.536437	0.5917

Log likelihood	-325.1962	Akaike info criterion	2.896458
Parameters	5	Schwarz criterion	2.971663
Diffuse priors	3	Hannan-Quinn criter.	2.926801

The MLEs for the AR coefficients are 1.441 and -0.494, respectively. The roots of the characteristic equation $\phi(z) = 1 - 1.441z + 0.494z^2 = 0$ are 1.779 and 1.137, respectively. Since these values are greater than 1, the AR component is covariance stationary.

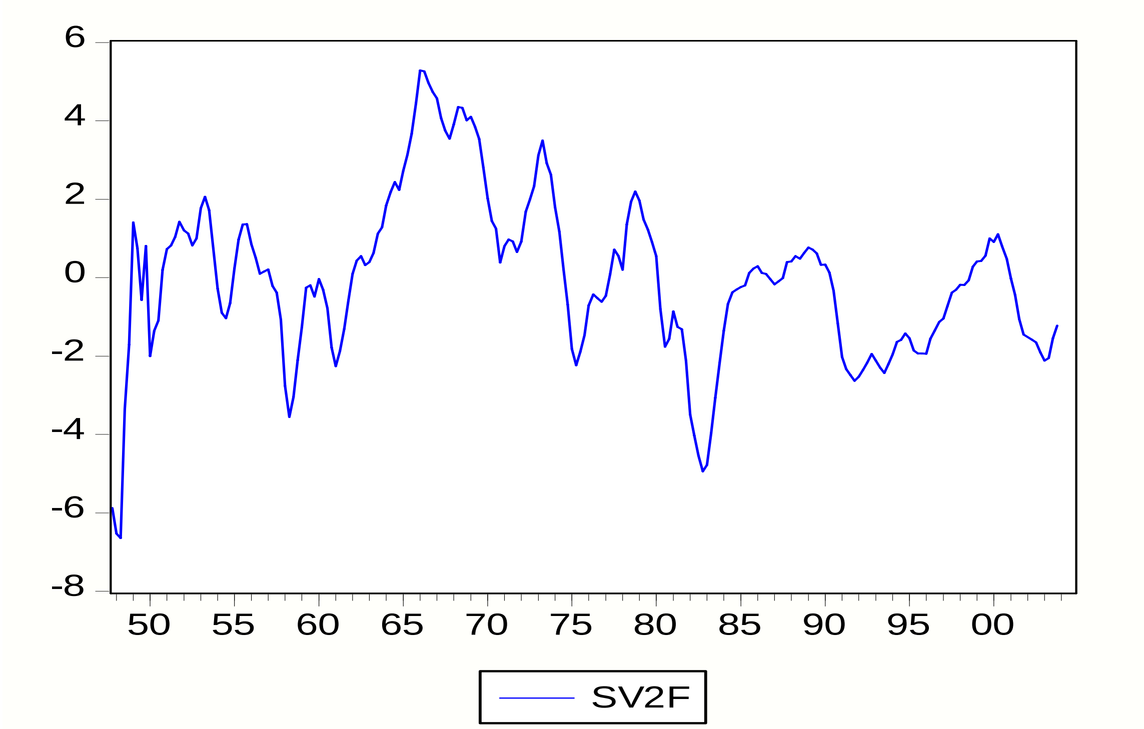
The variance of the permanent component is 0.290050, and the variance of the transitory component is 0.52476. Notice that the transitory component has a higher variance than the permanent component. The ratio of the permanent component variance to the stationary component variance is 0.553 indicating that the stationary component is almost twice as important as the permanent component for explaining the variation of log real GDP.

The filtered state estimates are given below

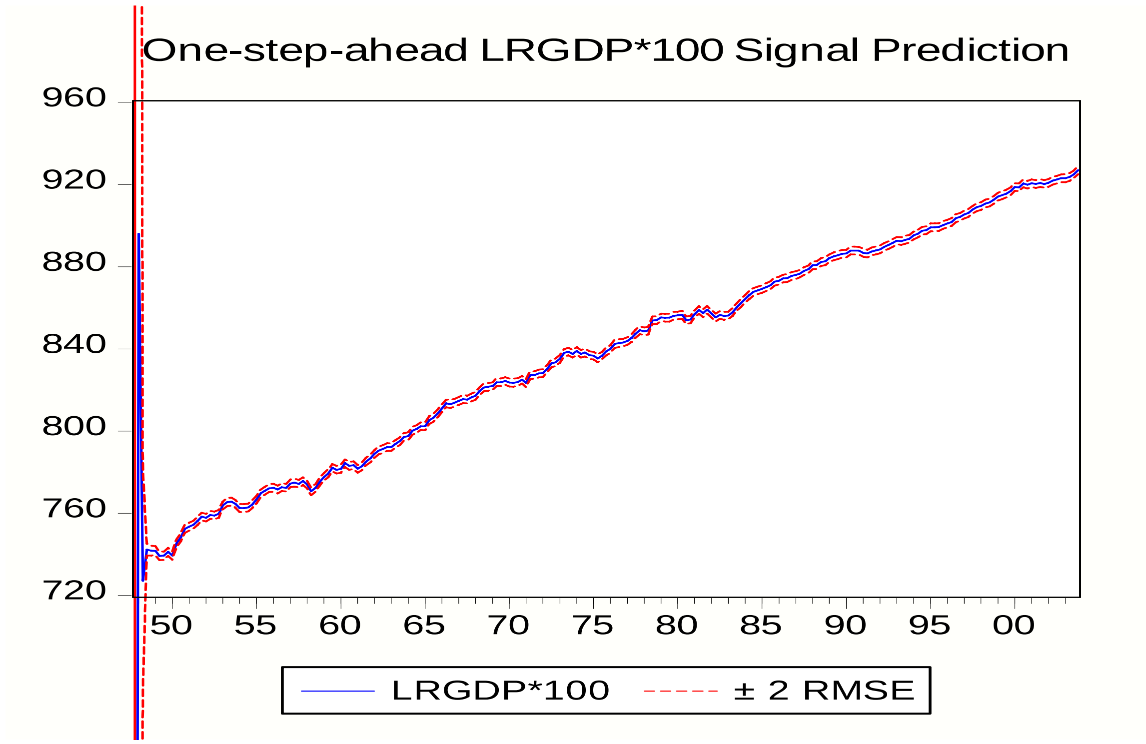


Notice that the filtered trend estimate is very close to a linear trend, and the filtered state estimates are very similar to the filtered estimates of the AR(2) for the linearly detrended data. The graphs have been modified since the initial states are not estimated very precisely, and this results in very large SE values that distort the graphs.

The filtered cycle state without the SE bars and omitting the initial state estimates is illustrated below. This model shows boom periods during the late 60s and late 90s, with recessions in the late 50s, mid 70s, early 80s, early 90s and early 00s.



The 1-step ahead response (signal) is given below. Notice that the Clark model tracks actual output fairly well.


One-step-ahead LRGDP*100 Signal Prediction
960
920
880
840
800
760
720
50
55
60
65
70
75
80
85
90
95
00
LRGDP*100
± 2 RMSE