updated: February 17, 2001; January 16, 2002 (minor correction on 3.3.8); February 4, 2003 (correction on 3.1.4); February 23, 2004 (correction on 3.3.8)

Hayashi *Econometrics*: Answers to Selected Review Questions

Chapter 3

Section 3.1

1. By (3.1.3a),

$$\text{Cov}(p_i, u_i) = \frac{\text{Cov}(v_i, u_i) - \text{Var}(u_i)}{\alpha_1 - \beta_1}.$$

The numerator can be positive.

2. The plim of the OLS estimator equals

$$\alpha_0 + \left(\alpha_1 - \frac{\text{Cov}(p_i, u_i)}{\text{Var}(p_i)}\right) \text{E}(p_i).$$

4. By (3.1.10a), $\text{Cov}(p_i, u_i) = -\text{Var}(u_i)/(\alpha_1 - \beta_1) \neq 0$ and $\text{Cov}(p_i, \zeta_i) = \text{Var}(\zeta_i)/(\alpha_1 - \beta_1) \neq 0$. x_i remains a valid instrument without the assumption that demand and supply shifters are uncorrelated.

Section 3.2

2. After the substitution inidcated in the hint, you should find that the log labor coefficient is unity in the output equation.

3. The demand for labor is now

$$L_i = \left(\frac{w}{p}\right)^{\frac{1}{\phi_1 - 1}} (A_i)^{\frac{1}{1-\phi_1}} (\phi_1)^{\frac{1}{1-\phi_1}} \exp\left(\frac{v_i}{1-\phi_1}\right).$$

Substitute this into the production function to obtain

$$Q_i = \left(\frac{w}{p}\right)^{\frac{\phi_1}{\phi_1 - 1}} (A_i)^{\frac{1}{1-\phi_1}} (\phi_1)^{\frac{\phi_1}{1-\phi_1}} \exp\left(\frac{v_i}{1-\phi_1}\right).$$

So the ratio of Q_i to L_i doesn't depend on A_i or v_i.

Section 3.3

1. The demand equation in Working's model without observable supply shifter cannot be identified because the order condition is not satisfied. With the observable supply shifter, the demand equation is exactly identified because the rank condition is satisfied, as explained in the text, and the order condition holds with equality.

2. Yes.

3. The orthogonality condition is

$$\mathrm{E}[\log(Q_i)] - \phi_0 - \phi_1 \,\mathrm{E}[\log(L_i)] = 0.$$

4. In Haavelmo's example, $y_i = C_i, \mathbf{z}_i = (1, Y_i)', \mathbf{x}_i = (1, I_i)'$. In Friedman's PIH, $y_i = C_i, \mathbf{z}_i = Y_i, \mathbf{x}_i = 1$. In the production function example, $y_i = \log(Q_i), \mathbf{z}_i = (1, \log(L_i))', \mathbf{x}_i = 1$.

5. $\boldsymbol{\sigma}_{\mathbf{xy}}$ is a linear combination of the L columns of $\boldsymbol{\Sigma}_{\mathbf{xz}}$ (see (3.3.4)). So adding $\boldsymbol{\sigma}_{\mathbf{xy}}$ to the columns of $\boldsymbol{\Sigma}_{\mathbf{xz}}$ doesn't change the rank.

6. Adding extra rows to $\boldsymbol{\Sigma}_{\mathbf{xz}}$ doesn't reduce the rank of $\boldsymbol{\Sigma}_{\mathbf{xz}}$. So the rank condition is still satisfied.

7. The linear dependence between AGE_i, $EXPR_i$, and S_i means that the number of instruments is effectively four, instead of five. The rank of $\boldsymbol{\Sigma}_{\mathbf{xz}}$ could still be four. However, the full-rank (non-singularity) condition in Assumption 3.5 no longer holds. For $\boldsymbol{\alpha} = (0, 1, -1, -1, 0)'$,

$$\boldsymbol{\alpha}' \mathbf{g}_i \mathbf{g}_i' = \varepsilon_i^2 (\boldsymbol{\alpha}' \mathbf{x}_i) \mathbf{x}_i' = \mathbf{0}'.$$

So $\boldsymbol{\alpha}' \,\mathrm{E}(\mathbf{g}_i \mathbf{g}_i') = \mathbf{0}'$, which means $\mathrm{E}(\mathbf{g}_i \mathbf{g}_i')$ is singular.

8. $\boldsymbol{\Sigma}_{\widehat{\mathbf{x}}\mathbf{z}} \equiv \mathrm{E}(\widehat{\mathbf{x}}\mathbf{z}') = \mathbf{A}\boldsymbol{\Sigma}_{\mathbf{xz}}$, which is of full column rank. $\mathrm{E}(\varepsilon_i^2 \widehat{\mathbf{x}}\widehat{\mathbf{x}}') = \mathbf{A}\,\mathrm{E}(\mathbf{g}_i \mathbf{g}_i')\mathbf{A}'$. This is nonsingular because $\mathbf{A}$ is of full row rank and $\mathrm{E}(\mathbf{g}_i \mathbf{g}_i')$ is positive definite.

Section 3.4

2. 0.

Section 3.5

3. The expression in brackets in the hint converges in probability to zero. $\sqrt{n}\overline{\mathbf{g}}$ converges in distribution to a random variable. So by Lemma 2.4(b), the product converges to zero in probability.

4. The three-step GMM estimator is consistent and asymptotically normal by Proposition 3.1. Since the two-step GMM estimator is consistent, the recomputed $\widehat{\mathbf{S}}$ is consistent for $\mathbf{S}$. So by Proposition 3.5 the three-step estimator is asymptotically efficient.

Section 3.6

1. Yes.

3. The rank condition for $\mathbf{x}_{1i}$ implies that $K_1 \geq L$.

4. No, because $J_1 = 0$.

Section 3.7

2. They are asymptotically chi-squared under the null because $\widehat{\mathbf{S}}$ is consistent. They are, however, no longer numerically the same.

Section 3.8

1. Yes.

2. Without conditional homoskedasticity, 2SLS is still consistent and asymptotically normal, if not asymptotically efficient, because it is a GMM estimator. Its Avar is given by (3.5.1) with $\mathbf{W} = (\sigma^2 \mathbf{\Sigma_{xx}})^{-1}$.

5. $\mathbf{S_{xz}}$ is square.

7. No.