Econ 583 Lab 9

Eric Zivot

Winter 2013
Due: Monday, March 18

1 Reading

1. Hayashi, Chapters 4 and 5 .

2 Hayahsi Exercises

1. Chapter 5, Page 337, Questions for Review 6; Page 341, Questions for Review 3.

2. Chapter 5, Page 356, Analytic Exercise 4.

3 Weak Instruments

Consider the instrumental variables regression set-up of the previous problem under the weak instrument asymptotic framework of Staiger and Stock (1997, Etca) which assumes that $\boldsymbol{\pi}$ in (**??**) is local-to-zero:

$$\begin{aligned} \boldsymbol{\pi} &= \boldsymbol{\pi}_n = \frac{1}{\sqrt{n}} \cdot \mathbf{g}, \qquad (1) \\ \mathbf{g} &= \text{fixed vector of constants.} \end{aligned}$$

1. Let $\hat{\boldsymbol{\pi}}$ denote the OLS estimate of $\boldsymbol{\pi}$ in (**??**). Under (1), show that

$$\sqrt{n}\hat{\boldsymbol{\pi}} \xrightarrow{d} N(\mathbf{g}, \sigma_v^2 \boldsymbol{\Sigma}_{xx}^{-1}) \Rightarrow \hat{\boldsymbol{\pi}} \overset{A}{\sim} N\left(\frac{\mathbf{g}}{\sqrt{n}}, \frac{1}{n}\sigma_v^2 \boldsymbol{\Sigma}_{xx}^{-1}\right).$$

2. Show that the asymptotic distribution of the Wald statistic for testing $H_0 : \boldsymbol{\pi} = \mathbf{0}$ in (**??**) is $\chi^2(k, d)$ where $d = \mathbf{g}'\boldsymbol{\Sigma}_{xx}\mathbf{g}/\sigma_v^2$. Hint: Use the following result. Let the $n \times 1$ vector $\mathbf{Y}$ have a multivariate normal distribution: $\mathbf{Y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Then $(\mathbf{Y} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{Y} - \boldsymbol{\mu}) \sim \chi^2(n)$ and $\mathbf{Y}'\boldsymbol{\Sigma}^{-1}\mathbf{Y} \sim \chi^2(n, d)$ where $d = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$

is the non-centrality parameter. Here $\chi^2(n, d)$ is called a non-central chi-square distribution with n degrees of freedom and non-centrality parameter d. The quantity $d = \mathbf{g}'\mathbf{\Sigma}_{xx}\mathbf{g}/\sigma_v^2$ is a unit-free measure of instrument quality called the concentration parameter. The closer d is to zero, the weaker are the instruments.

3. Consider testing the null hypothesis $H_0 : \delta = \delta_0$ vs. $H_1 : \delta \neq \delta_0$. The Anderson-Rubin (AR) statistic is computed as the F-statistic for testing $H_0 : \boldsymbol{\psi} = \mathbf{0}$ from the regression

$$y_i - z_i\delta_0 = \mathbf{x}_i'\boldsymbol{\psi} + w_i,$$

   where $\boldsymbol{\psi} = \boldsymbol{\pi}(\delta - \delta_0)$ and $w_i = v_i(\delta - \delta_0) + \varepsilon_i$. Show that

$$\text{AR}(\delta_0) = \frac{(\mathbf{y} - \mathbf{z}\delta_0)'\mathbf{P}_X(\mathbf{y} - \mathbf{z}\delta_0)/k}{(\mathbf{y} - \mathbf{z}\delta_0)'\mathbf{Q}_X(\mathbf{y} - \mathbf{z}\delta_0)/(n-k)},$$

   where $\mathbf{P}_X = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ and $\mathbf{Q}_X = I_n - \mathbf{P}_X$.

4. Under (1) and $H_0 : \delta = \delta_0$, show that $k \cdot \text{AR}(\delta_0) \xrightarrow{d} \chi^2(k)$.

5. $\text{AR}(\delta_0)$ is closely related to the CU-GMM J-statistic (under conditional homoskedasticity) evaluated at $\delta = \delta_0$:

$$J(\delta_0, S^{-1}(\delta_0)) = \frac{(\mathbf{y} - \mathbf{z}\delta_0)'\mathbf{P}_X(\mathbf{y} - \mathbf{z}\delta_0)}{(\mathbf{y} - \mathbf{z}\delta_0)'(\mathbf{y} - \mathbf{z}\delta_0)/n}.$$

   Under (1) and $H_0 : \delta = \delta_0$, show that $J(\delta_0, S^{-1}(\delta_0)) \xrightarrow{d} \chi^2(k)$.

6. Under (1) and $H_0 : \delta = \delta_0$, show that $k \cdot \text{AR}(\delta_0) - J(\delta_0, S^{-1}(\delta_0)) \xrightarrow{p} 0$ so that $k \cdot \text{AR}(\delta_0)$ and $J(\delta_0, S^{-1}(\delta_0))$ are asymptotically equivalent. This result, shows that AR statistic is really a joint test of $H_0 : \delta = \delta_0$ and the validity of the overidentifying restrictions $E[\mathbf{x}_i\varepsilon_i] = 0$.