Econ 583 Lab 6

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Due: Monday, February 18.

1 Reading

1. Hayashi Chapter 7.
2. Hall, Chapter 3.
3. Newey, W.K. and D. McFadden (1994). "Large Sample Estimation and Hypothesis Testing", Chapter 36 in *Handbook of Econometrics, Volume IV*, Edited by R.F Engle and D.L. McFadden.

2 Nonlinear GMM: Analytic Questions

1. Chapter 7, Questions for Review, Section 7.1 (page 455) 2.
2. Chapter 7, Questions for Review, Section 7.2 (page 469) 6 and 7.
3. Chapter 7, Questions for Review, Section 7.3 (page 487) 5 and 6.
4. Let $Y_1, \ldots, Y_n$ be *iid* random variables from a Gamma distribution with parameters α and β. For $\boldsymbol{\theta} = (\alpha, \beta)'$, the pdf has the form

$$f(y; \boldsymbol{\theta}) = \frac{\alpha^{\beta}}{\Gamma(\beta)} e^{-\alpha y} y^{\beta - 1},\ y > 0, \alpha > 0, \beta > 0$$

where $\Gamma(\cdot)$ denotes the gamma function. That is,

$$\Gamma(\beta) = \int_0^{\infty} t^{\beta - 1} e^{-t} dt$$

and $\Gamma(\beta) = (\beta - 1)\Gamma(\beta - 1)$. Some moments of functions of a Gamma random variable Y are

$$\begin{aligned} E[Y] &= \beta/\alpha \\ E[Y^2] &= \beta(\beta+1)/\alpha^2 \\ E[\ln Y] &= \Psi(\beta) - \ln\alpha \\ E[1/Y] &= \alpha/(\beta-1) \end{aligned}$$

where $\Psi(\beta) = \frac{d}{d\beta}\ln\Gamma(\beta)$.

(a) Using the moments described above, determine the vector of moments, $g(\mathbf{w}_t, \boldsymbol{\theta})$, required for GMM estimation. This vector should satisfy $E[g(\mathbf{w}_t, \boldsymbol{\theta}_0)] = \mathbf{0}$ for $\boldsymbol{\theta} = \boldsymbol{\theta}_0$ and $E[g(\mathbf{w}_t, \boldsymbol{\theta})] \neq \mathbf{0}$ for $\boldsymbol{\theta} \neq \boldsymbol{\theta}_0$.

(b) Is $\{g(\mathbf{w}_t, \boldsymbol{\theta}_0), I_t\}$ an ergodic-stationary MDS for an appropriately chosen information set I_t? Why or why not?

(c) Describe how you would estimate $\boldsymbol{\theta}$ using the iterated efficient GMM estimator. In your description, provide the following

   i. Form of the GMM objective function

   ii. Initial weight matrix used

   iii. Estimator for $\mathbf{S} =$ asymptotic variance of $\sqrt{n}\bar{\mathbf{g}}(\boldsymbol{\theta}_0)$

   iv. Give the iteration scheme used to determine the GMM estimate of $\boldsymbol{\theta}$.

3 Nonlinear GMM: Empirical Exercise

In this exercise, you will estimate a typical Euler equation asset pricing model. The data for the exercise consists of monthly observations on gross consumption growth, C_t/C_{t-1}, and returns, R_{jt}, for ten size sorted portfolios and T-Bills (risk free rate). (see Zivot and Wang 2005, chapter 21 for an example using S-PLUS).

Consider an Euler equation asset pricing model that allows the individual to invest in J risky assets with returns $R_{j,t+1}$ $(j = 1, \ldots, J)$, as well as a risk-free asset with certain return $R_{f,t+1}$. Assuming power utility over consumption C

$$\begin{aligned} U(C) &= \frac{C^{1-\alpha_0}}{1-\alpha_0} \\ \alpha_0 &= \text{risk aversion parameter} \end{aligned}$$

the Euler equations may be written as the system of $J+1$ nonlinear equaitons

$$\begin{aligned} E\left[(1+R_{f,t+1})\beta_0\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha_0}|I_t\right] - 1 &= 0, \\ E\left[(R_{j,t+1}-R_{f,t+1})\beta_0\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha_0}|I_t\right] &= 0,\ j = 1,\ldots,J, \end{aligned}$$

where I_t represents information available at time t and β_0 represents the time discount parameter. The parameter vector to be estimated $\boldsymbol{\theta}_0 = (\beta_0, \alpha_0)'$.

1. Using the instrument $x_t = 1$, GMM estimation may be performed using the $J+1$ vector of moments

$$g(\mathbf{w}_{t+1}, \boldsymbol{\theta}) = \begin{pmatrix} (1 + R_{f,t+1})\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} - 1 \\ (R_{1,t+1} - R_{f,t+1})\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \\ \vdots \\ (R_{J,t+1} - R_{f,t+1})\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \end{pmatrix}$$

   Using these moments, estimate $\boldsymbol{\theta}$ using the iterated efficient GMM estimator (with a White-type HC covariance estimator) as well as the continuous updating (CU) efficient GMM estimator. Use $\boldsymbol{\theta} = (1, 5)'$ as the starting values for the optimization for the iterated GMM estimator, and use the iterated GMM estimates as the starting values for the CU GMM estimator. Do the estimates look economically plausible? Does the J-statistic reject the overidentifying restrictions at the 5% significance level?

2. Now consider using the instrument $\mathbf{x}_t = (1, C_t/C_{t-1})'$. What are the $2(J+1)$ moment conditions to be used for GMM estimation? Using these moments, estimate $\boldsymbol{\theta}$ using the iterated efficient GMM estimator (with a White-type HC covariance estimator). Do the estimates look economically plausible? Does the J-statistic reject the overidentifying restrictions at the 5% significance level?